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Recent Developments in the Generalized Finite Element Method for the Simulation of 3-D Hydraulic Fracture Propagation and Interactions

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Acknowledgements

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Hydraulic Fracturing of Gas Shale Reservoirs

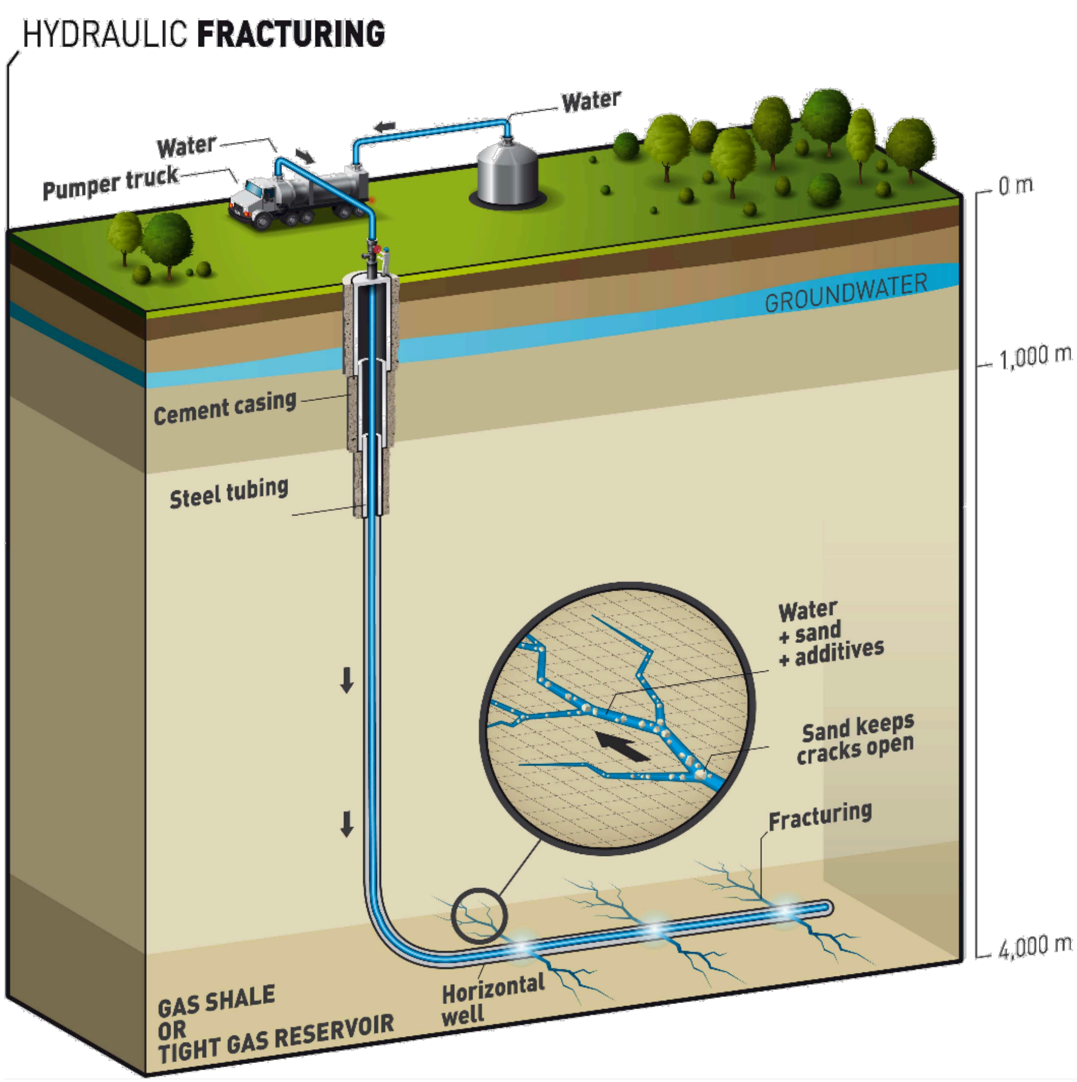
Motivation

- Natural gas and oil production in the US has increased significantly in the past few years thanks to advances in hydraulic fracturing of shale reservoirs
- Yet there are concerns about the environmental impact of toxic fluids used in this process

Objectives: Develop computational methods that can

- provide more realist simulations of hydraulic fracturing treatments
- evaluate the potential environmental impact of interactions between hydraulic fractures and naturally existing fractures in shale reservoirs

What is Hydraulic Fracturing?



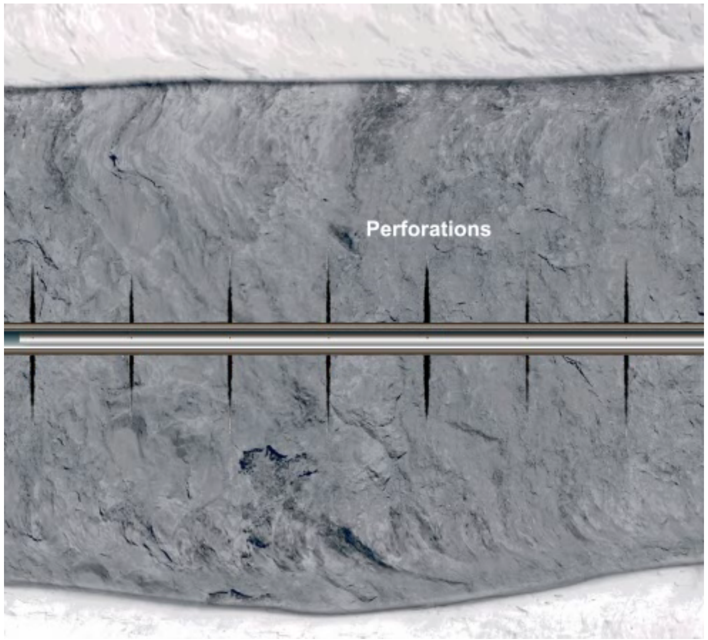
Video

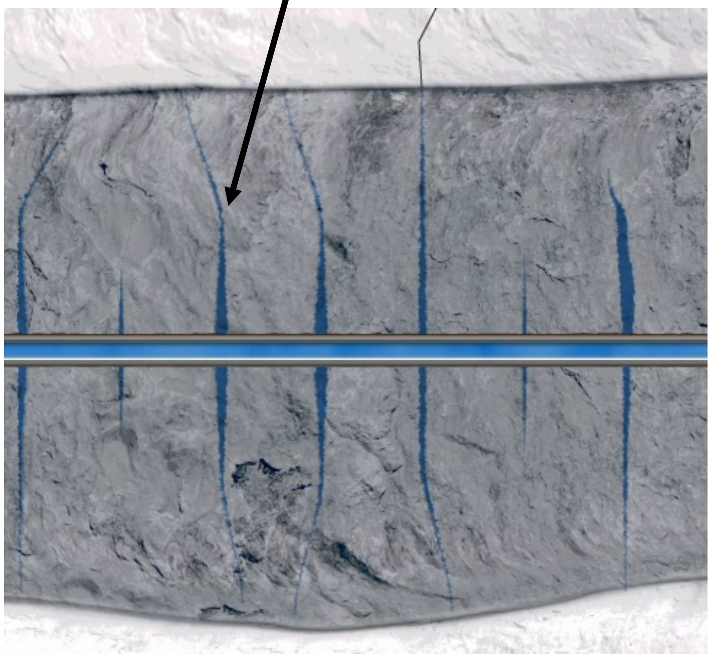
Graham Roberts, New York Times, http://www.nytimes.com/interactive/2011/02/27/us/fracking.html

Hydraulic Fracturing Simulation

Current Focus: 3-D effects not captured by available simulators

- Initial stages of fracture propagation: Fracture re-orientation, interaction and coalescence

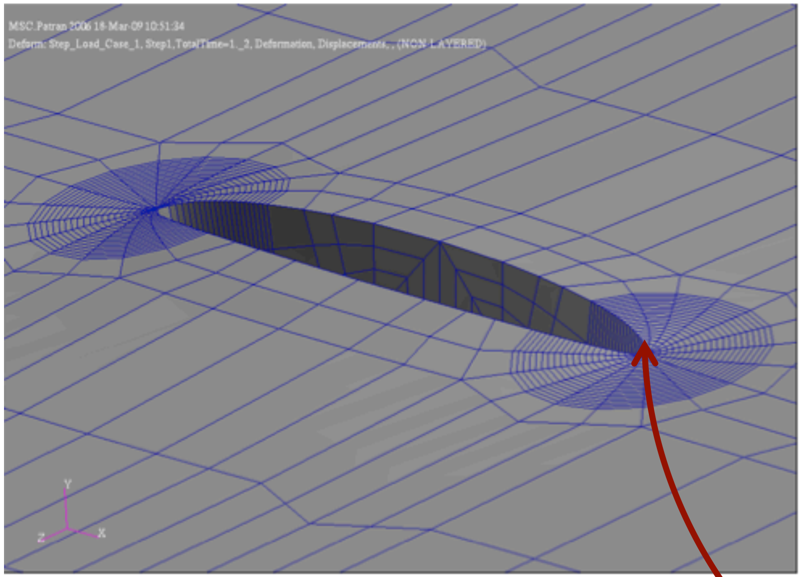


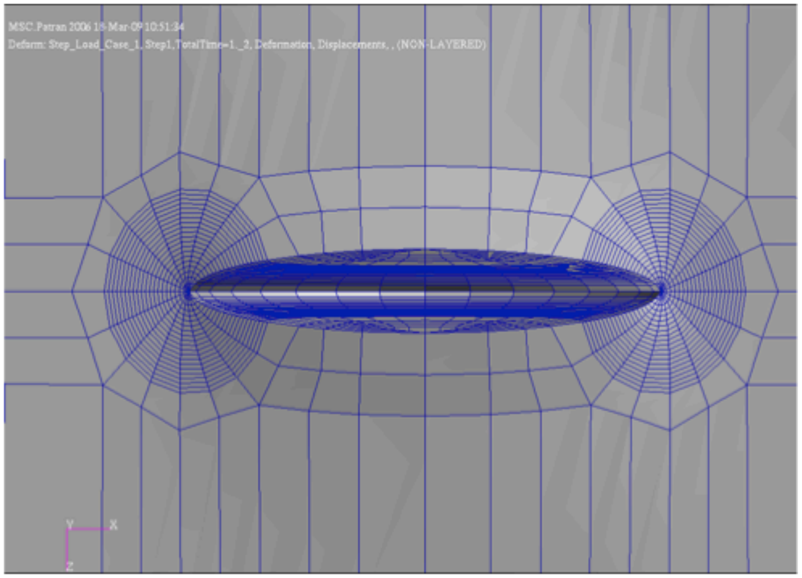


Strategy: Generalized Finite Element Methods

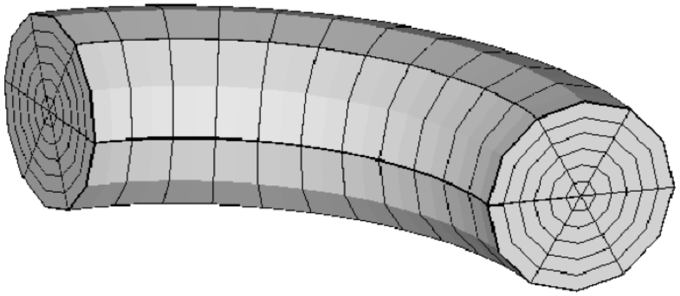
Modeling 3-D Fractures: Limitations of Standard FEM

- It is not "just" fitting the 3-D <u>evolving</u> fracture
- FEM meshes must satisfy special requirements for acceptable accuracy





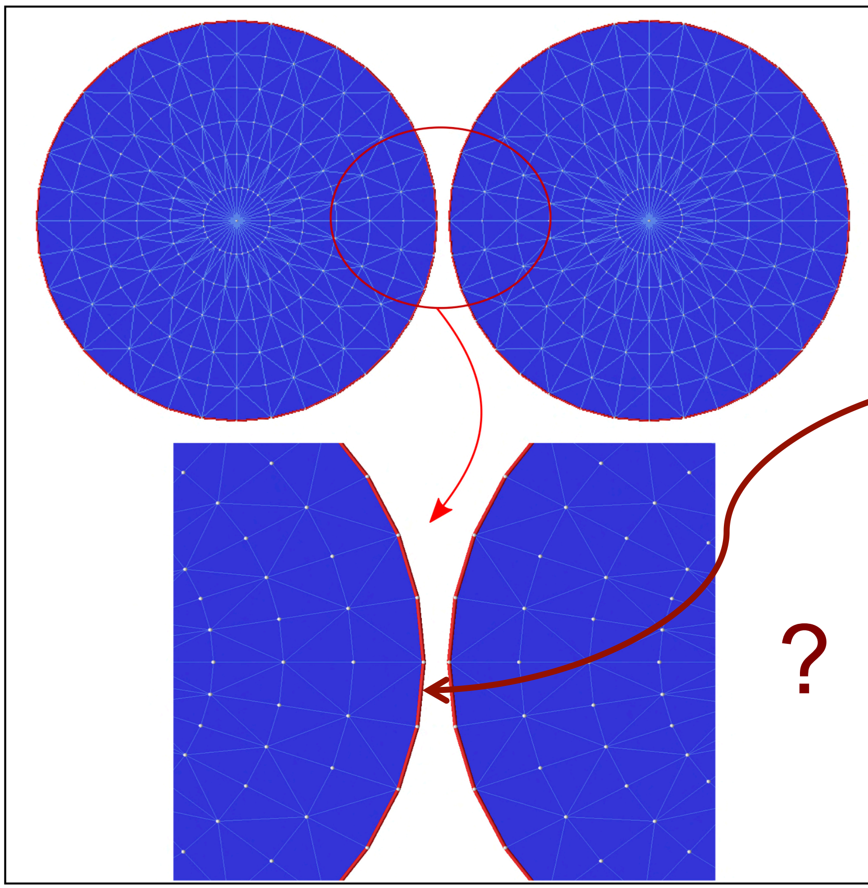
FEM mesh for a surface fracture



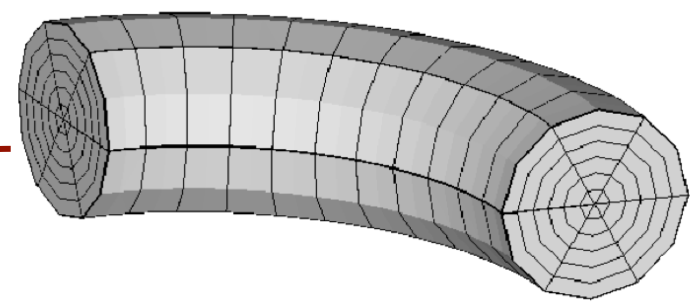
Mesh with quarter-point elements

Limitations of Standard FEM

- Difficulties arise if fracture front is close to complex geometrical features
- Fracture surfaces with sharp turns
- Coalescence of fractures



- Not possible in general to automatically create structured meshes along both fracture fronts when they are in close proximity



- Even with these crafted meshes and quarter-point elements, convergence rate of std FEM is slow (*controlled by singularity at fracture front*)

- **Strategy**: Generalized FEM

Outline

- Motivation and limitations of existing methods
- Basic ideas of GFEM
- GFEM for 3D hydraulic fractures
- Applications
  - Verification
  - Fracture re-orientation
  - Coalescence of 3-D fractures
- Future work and conclusions

Early Works on Generalized FEMs

- Babuska, Caloz and Osborn, 1994 (Special FEM).
- Duarte and Oden, 1995 (Hp Clouds).
- Babuska and Melenk, 1995 (PUFEM).
- Oden, Duarte and Zienkiewicz, 1996 (Hp Clouds/GFEM).
- Duarte, Babuska and Oden, 1998 (GFEM).
- Belytschko et al., 1999 (Extended FEM).
- Strouboulis, Babuska and Copps, 2000 (GFEM).

- Basic idea:
  - Use a partition of unity to build Finite Element shape functions

- Review paper

  Belytschko T., Gracie R. and Ventura G. A review of extended/generalized finite element methods for material modeling, *Mod. Simul. Matl. Sci. Eng.*, 2009

  "The XFEM and GFEM are basically identical methods: the name generalized finite element method was adopted by the Texas school in 1995–1996 and the name extended finite element method was coined by the Northwestern school in 1999."

Generalized Finite Element Method

- GFEM is a Galerkin method with special test/trial space given by

$$\mathbb{S}_{GFEM} = \mathbb{S}_{FEM} + \mathbb{S}_{ENR}$$

Low order FEM space

Enrichment space with functions related to the given problem

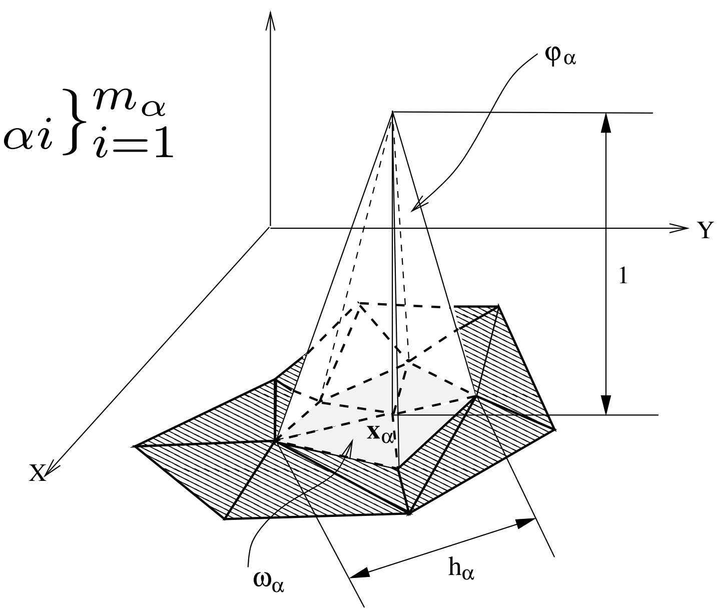
$$\mathbb{S}_{FEM} = \sum_{\alpha \in I_h} c_\alpha \varphi_\alpha, \quad c_\alpha \in \mathbb{R}$$

$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$L_{\alpha i} \in \chi_\alpha(\omega_\alpha)$$

Enrichment function

Patch space



Generalized Finite Element Method

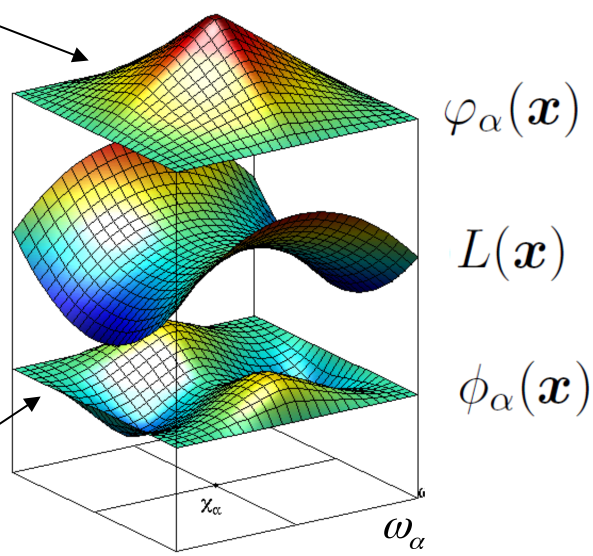
$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$\phi_{\alpha i}(x) = \varphi_\alpha(x) L_{\alpha i}(x) \qquad \sum_\alpha \varphi_\alpha(\boldsymbol{x}) = 1$$

Linear FE shape function

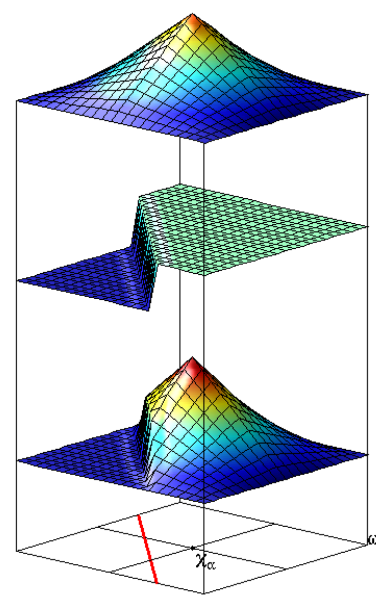
Enrichment function

GFEM shape function



[Oden, Duarte & Zienkiewicz, 1996]

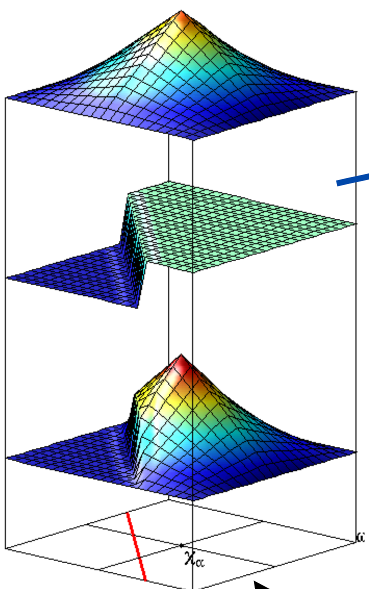
• Allows construction of shape functions incorporating a-priori knowledge about solution



Discontinuous enrichment [Moes et al., 1999]

GFEM Approximation for 3-D Fractures

$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



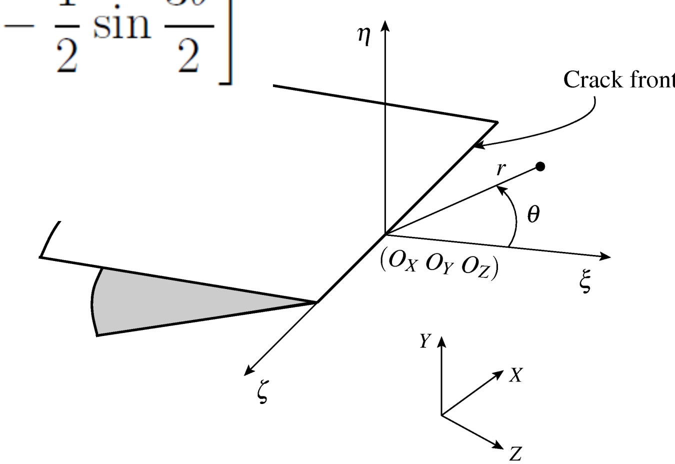
patch ω_α

$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[(\kappa - \frac{1}{2})\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[(\kappa + \frac{1}{2})\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

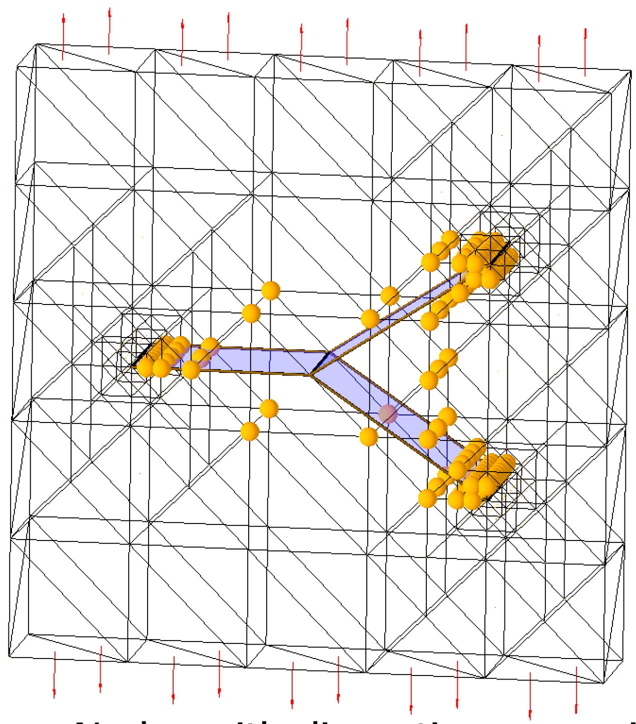
$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

[Duarte & Oden 1996]



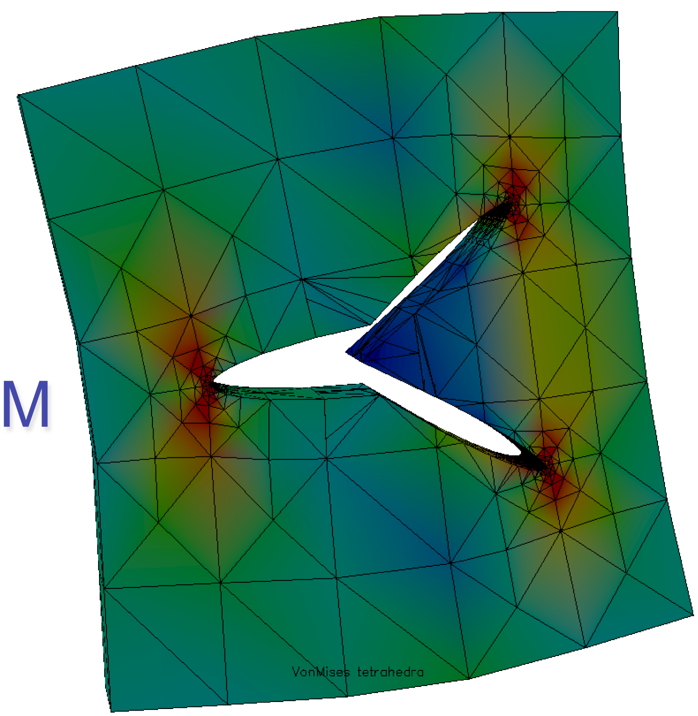
Modeling Fractures with the GFEM

- Fractures are modeled via enrichment functions, *not* the FEM mesh
- Mesh refinement *still required* for acceptable accuracy



● = Nodes with discontinuous enrichments

hp-GFEM

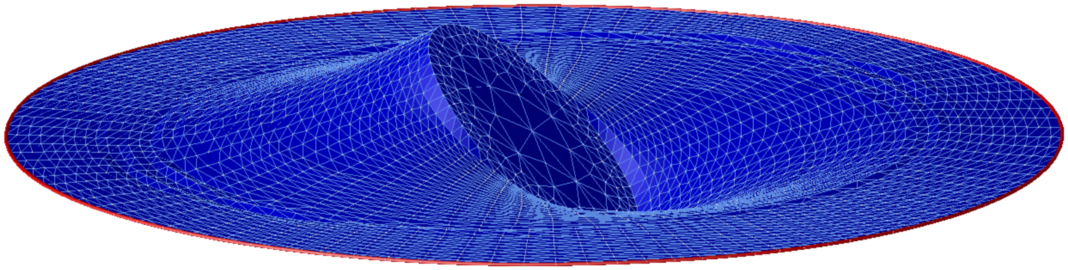


Von Mises stress

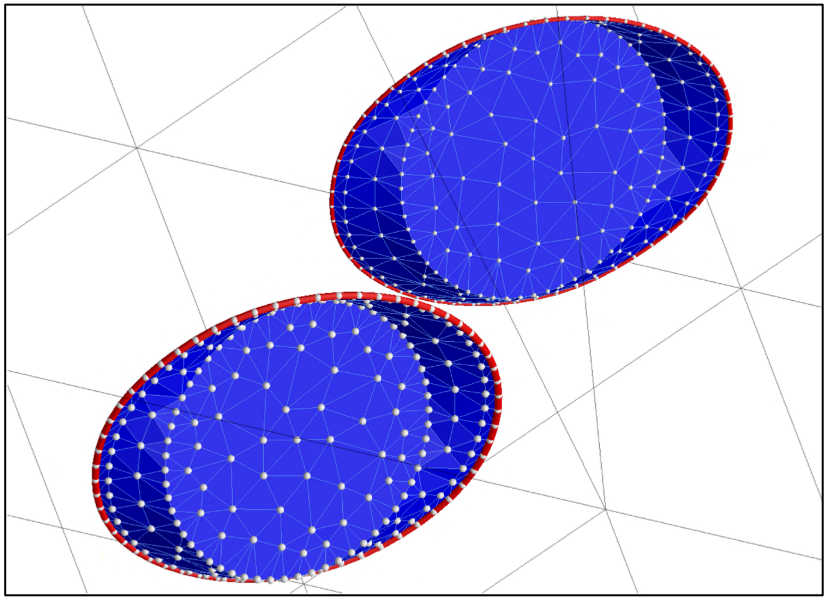
[Duarte et al., International Journal Numerical Methods in Engineering, 2007]

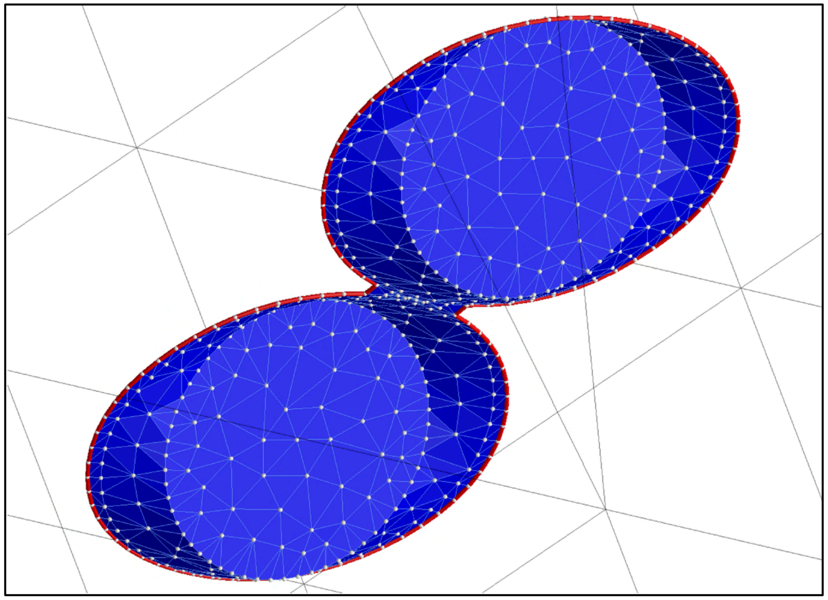
3D Fracture Surface Representation

- High-fidelity explicit representation of fracture surfaces [Duarte et al., 2001, 2009]



- Coalescence of fractures [Garzon et al., 2014]



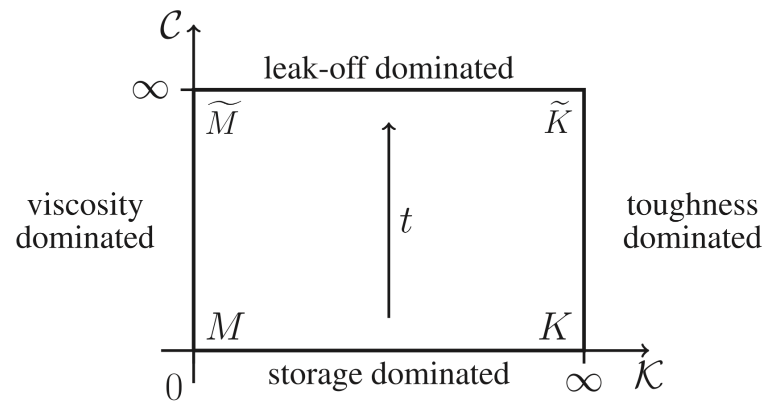


Selection of Enrichment Functions: Hydraulic Fracturing Regimes

- Fracture propagation is governed by
  - two competing energy dissipation mechanisms: Viscous flow and fracturing process;
  - two competing storage mechanisms: In the fracture and in the porous matrix

$\mathcal{K} =$ Dimensionless toughness

$\mathcal{C} =$ Leak-off coefficient



Hydraulic fracture parametric space*

Current Focus: Storage-toughness dominated regime

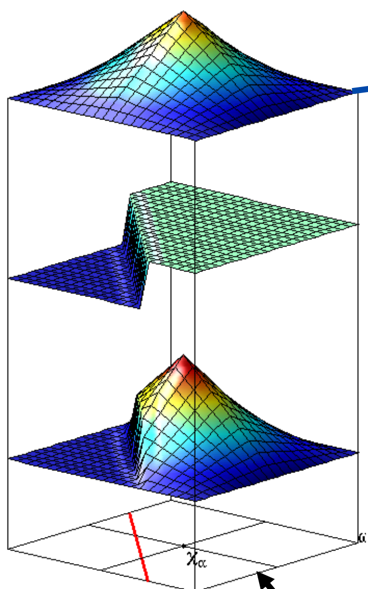
- Low permeability reservoirs: Neglect flow of hydraulic fluid across fracture faces:
  - <u>Storage dominated</u> regime
- High confining stress (no fluid lag) and low viscosity fluid (water):
  - Near constant fluid pressure in fracture; <u>Toughness dominated</u> regime
- Brittle elastic material

*[Carrier & Granet, EFM, 2013]

Selection of Enrichment Functions: Hydraulic Fracturing Regimes

Enrichments for toughness-dominated regime:

$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



patch ω_α

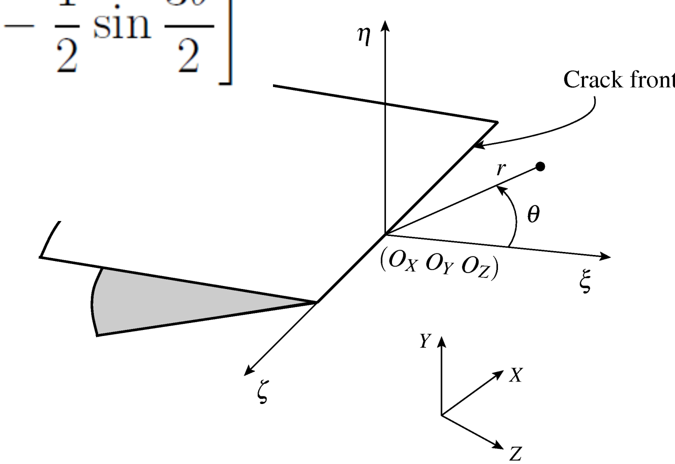
$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

[Duarte & Oden 1996]

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

Valid for toughness-dominated problems

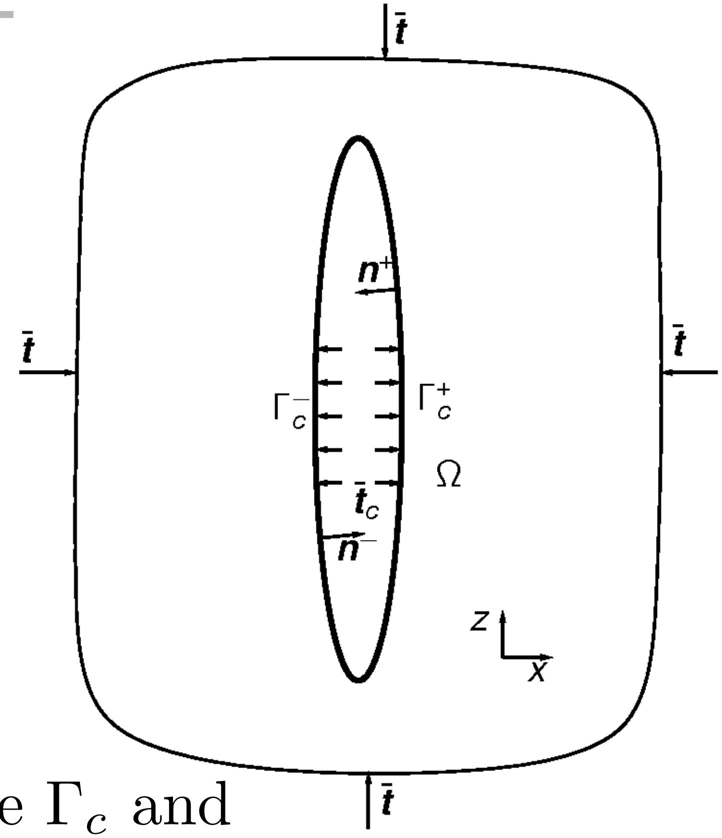


Governing Equations for Coupled Problem

Governing equations for porous medium

$$\int_{\Omega} \nabla_s \boldsymbol{\delta u} : \boldsymbol{\sigma}(\boldsymbol{u}) d\Omega$$

$$= \int_{\partial\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{\delta u} d\Gamma + \int_{\Gamma_c^+} \bar{\boldsymbol{t}}_c^+ \cdot [\![\boldsymbol{\delta u}]\!] d\Gamma_c$$



Cross section of fracture

where $[\![\boldsymbol{\delta u}]\!] = \boldsymbol{\delta u}^+ - \boldsymbol{\delta u}^-$ is the virtual displacement jump across the crack surface Γ_c and

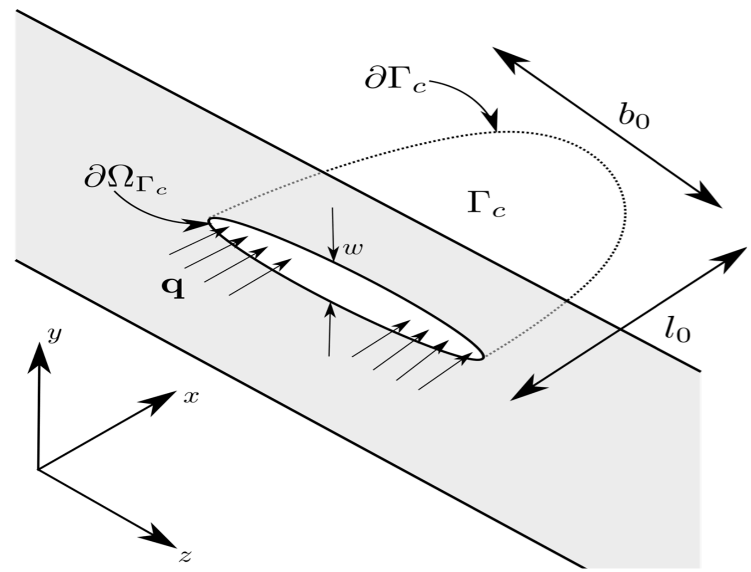
$$\bar{\boldsymbol{t}}_c^+ = -p\boldsymbol{n}^+ = p\boldsymbol{n}^- \text{ (1st coupling cond.)}$$

$$p = \text{Fluid pressure in fracture cavity}$$

Fluid flow in the fracture

Reynold's lubrication theory: Conservation of mass

$$\nabla_{\bar{\boldsymbol{x}}} \cdot \boldsymbol{q} + \frac{\partial w}{\partial t} = Q_I - Q_L$$



Poiseuille's cubic law $\boldsymbol{q} = \frac{w^3}{12\mu} \nabla p$

Fracture opening $w = [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}^-$ (second coupling condition)

Weak form for hydraulic fluid in the fracture

$$\int_{\Gamma_c} \frac{w^3}{12\mu} \nabla_{\bar{\boldsymbol{x}}} \delta p \cdot \nabla_{\bar{\boldsymbol{x}}} p \, d\Gamma_c = \int_{\Gamma_c} \delta p \left[Q_i - Q_L - \frac{\partial w}{\partial t} \right] d\Gamma_c + \int_{\partial\Gamma_c} \bar{q}(s) \, \delta p \, ds$$

Coupled Equations

$$A\left(\boldsymbol{u},\boldsymbol{\delta u}\right)+B\left(p,[\![\boldsymbol{\delta u}]\!]\right)=L_u\left(\boldsymbol{\delta u}\right)$$

$$C\left(\frac{\partial[\![\boldsymbol{u}]\!]}{\partial t},\delta p\right)+D\left(p,\delta p\right)=L_p\left(\delta p\right)$$

where

$$A\left(\boldsymbol{u},\boldsymbol{\delta u}\right)=\int_\Omega \nabla_s\boldsymbol{\delta u}:\boldsymbol{\sigma}(\boldsymbol{u})d\Omega$$

$$B\left(p,[\![\boldsymbol{\delta u}]\!]\right)=-\int_{\Gamma_c} p[\![\boldsymbol{\delta u}]\!]\cdot\boldsymbol{n}^- d\Gamma_c$$

$$C\left(\frac{\partial[\![\boldsymbol{u}]\!]}{\partial t},\delta p\right)=\int_{\Gamma_c}\delta p\left[\frac{\partial[\![\boldsymbol{u}]\!]}{\partial t}\right]\cdot\boldsymbol{n}^-\,d\Gamma_c$$

$$D\left(p,\delta p\right)=\int_{\Gamma_c}\frac{w^3}{12\mu}\nabla_{\bar{\boldsymbol{x}}}\delta p\cdot\nabla_{\bar{\boldsymbol{x}}}p\,d\Gamma_c$$

Coupled Equations

$$L_u(\boldsymbol{\delta u}) = \int_{\partial\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{\delta u} \, d\Gamma$$

$$L_p(\delta p) = \int_{\Gamma_c} [Q_I - Q_L] \, \delta p \, d\Gamma + \int_{\partial\Gamma_c} \bar{q}(s) \, \delta p \, ds$$

Discretizing in time and space

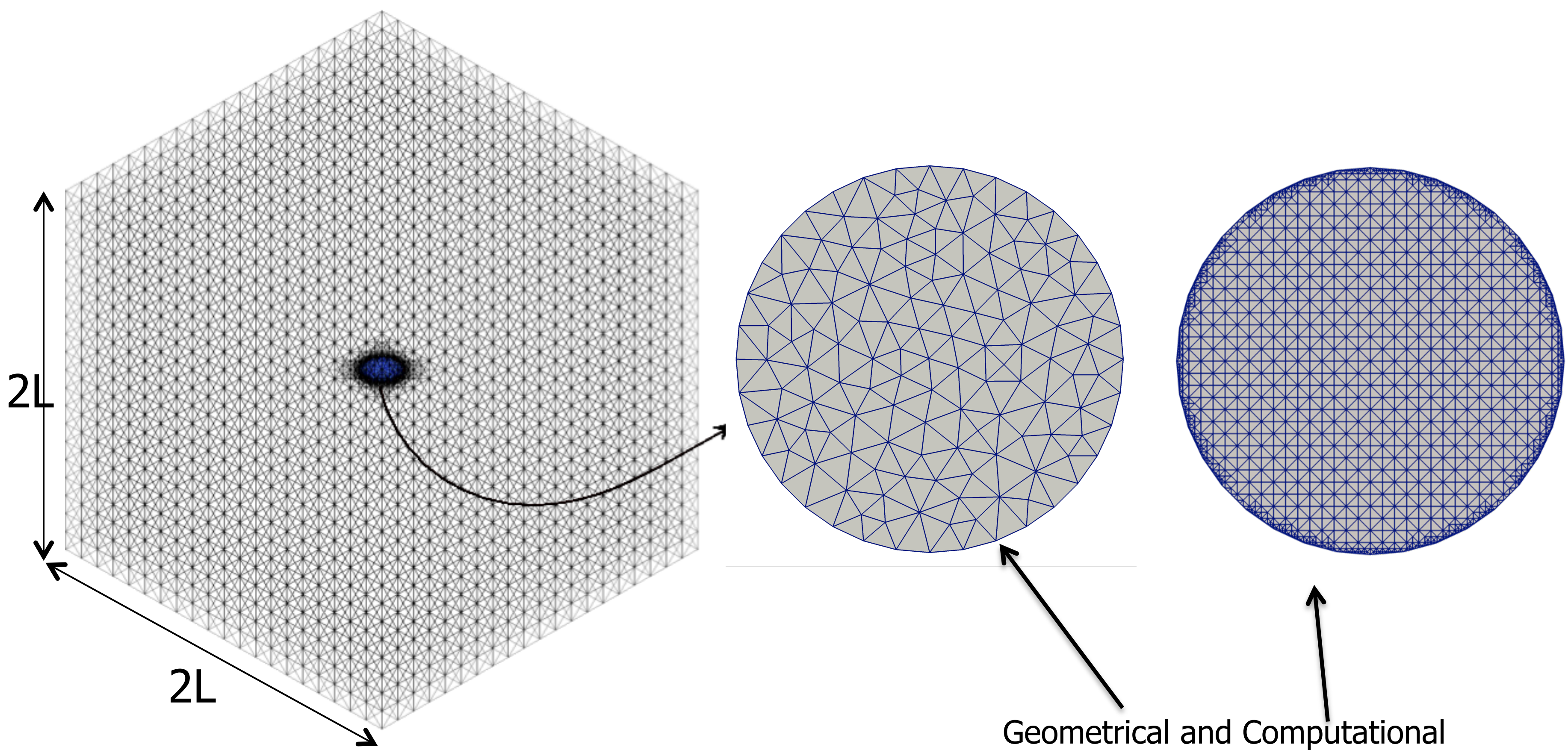
$$\begin{bmatrix} \boldsymbol{K}_u^{n+1} & -\left(\boldsymbol{K}_c^{n+1}\right)^T \\ \boldsymbol{K}_c^{n+1} & \Delta t \boldsymbol{K}_p^{n+1} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{u}}^{n+1} \\ \hat{\boldsymbol{p}}^{n+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{t}_u^{n+1} \\ \left(\boldsymbol{K}_c^{n+1,n}\right) \hat{\boldsymbol{u}}^n + \Delta t \boldsymbol{Q}_p^{n+1} + \Delta t \bar{\boldsymbol{q}}_p^{n+1} \end{bmatrix}$$

- Solved at each time step, within each crack propagation increment.
- System is PD: Unique solution without need of auxiliary conditions adopted in staggered schemes or restrictions on time step.

Outline

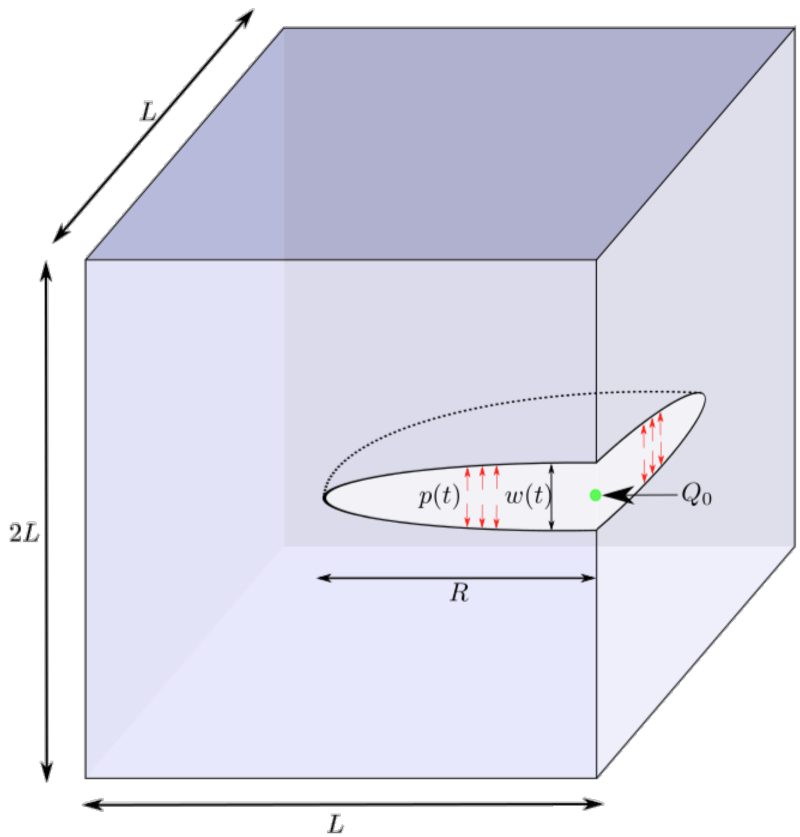
- Motivation and limitations of existing methods
- Basic ideas of GFEM
- GFEM for 3D hydraulic fractures
- Applications
  - Verification
  - Fracture re-orientation
  - Coalescence of 3-D fractures
- Future work and conclusions

Circular Fracture



Geometrical and Computational fracture surface for fluid pressure computation

Circular Fracture



1/4th Schematic Cutaway

Adopt [Zielonka et al. 2014]:

$L = 5\,m \qquad R = 0.5\,m$

Incomp. Newtonian fluid with viscosity

$\mu = 25, 50, 100$ cPoise

Injection rate at center of fracture

$Q = 0.00005\,m^3/s$

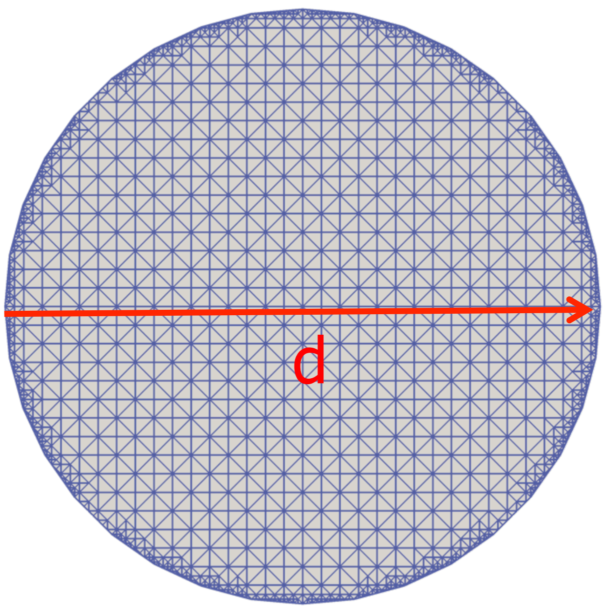
$E = 17\,\mathrm{GPa}$

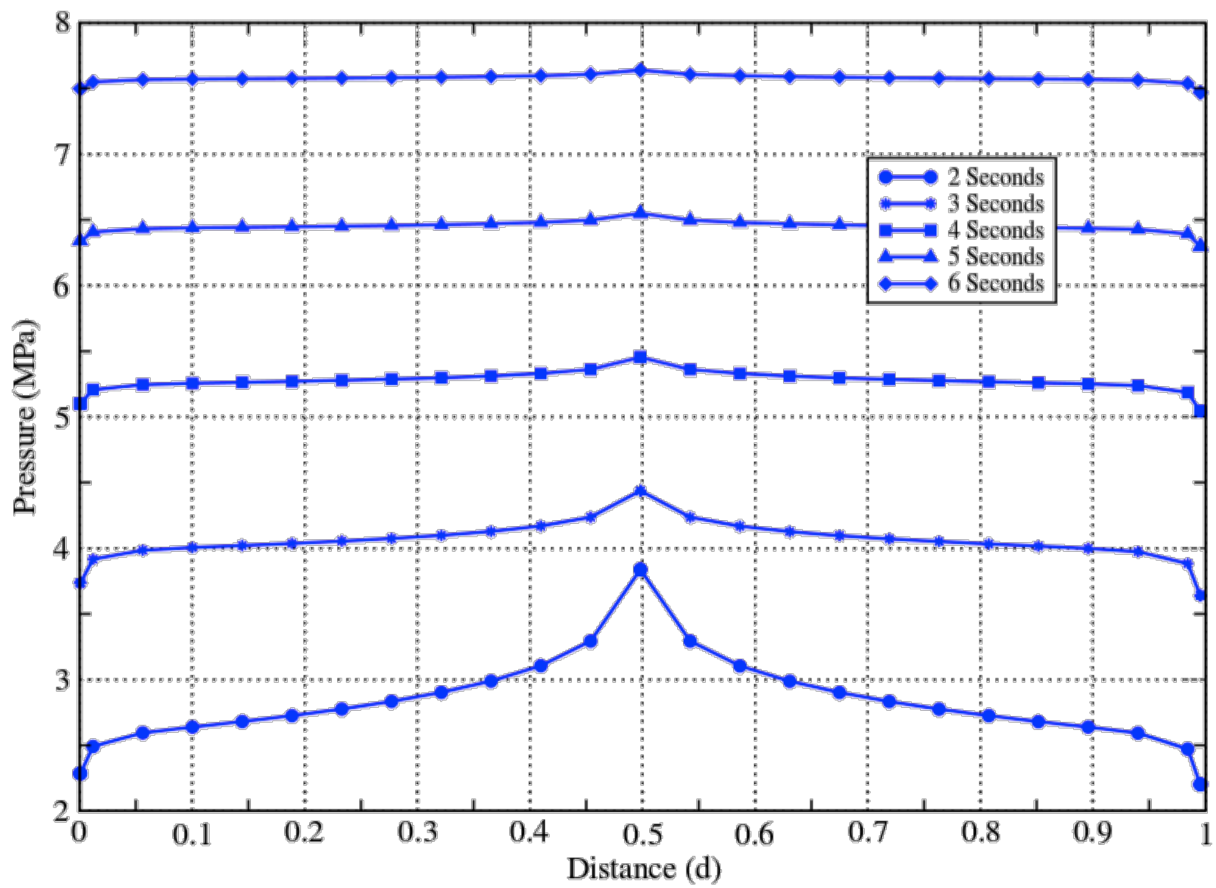
$\nu = 0.2$

$K_{Ic} = 1.46\ \mathrm{MPa}\sqrt{m}$

Circular Fracture

Pressure distribution as a function of time

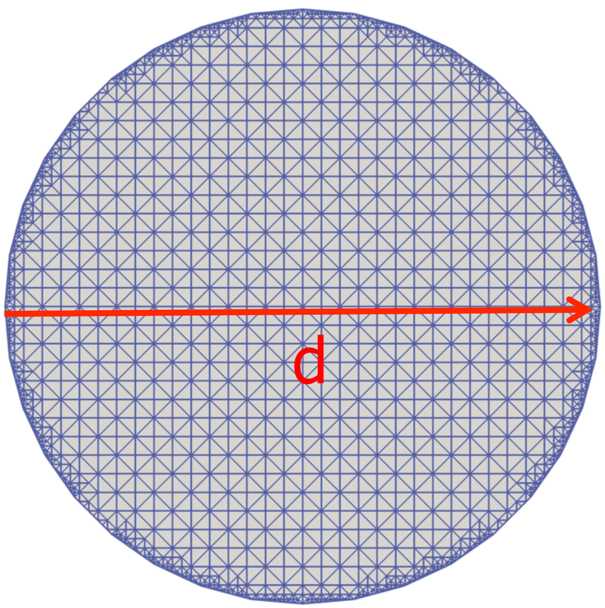


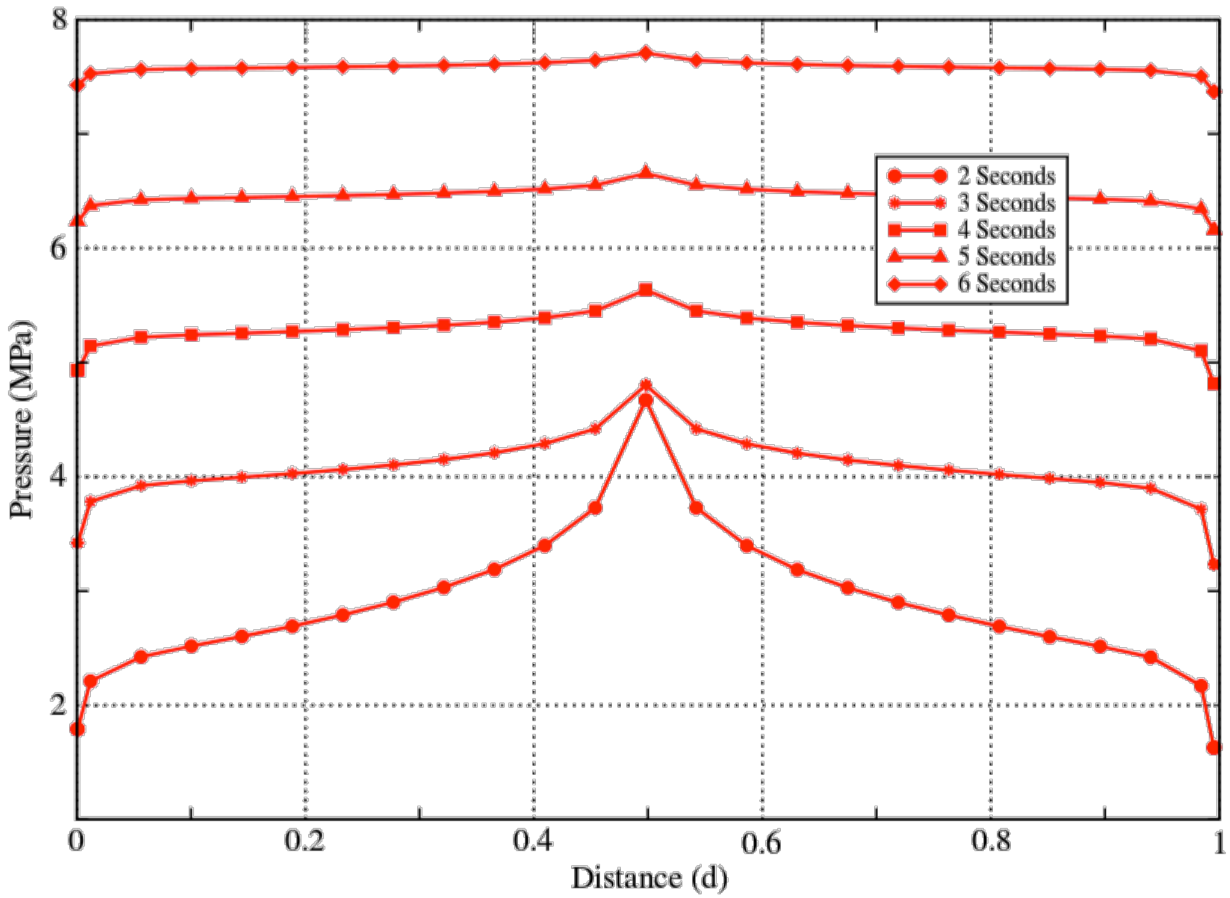


$\mu = 25$ cPoise

Circular Fracture

Pressure distribution as a function of time

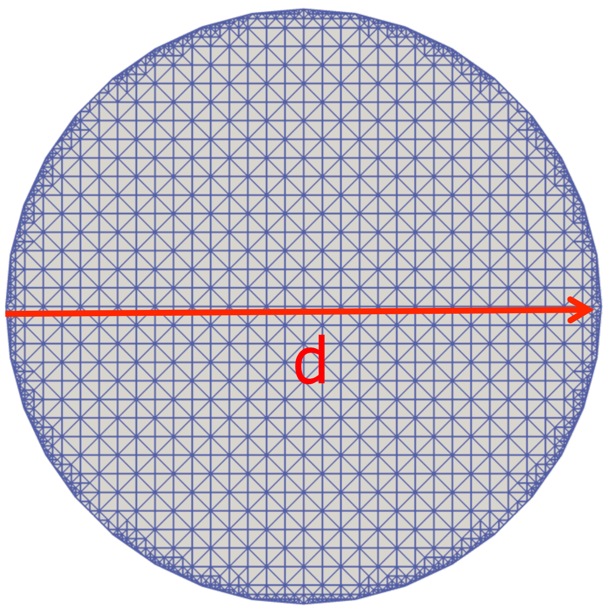


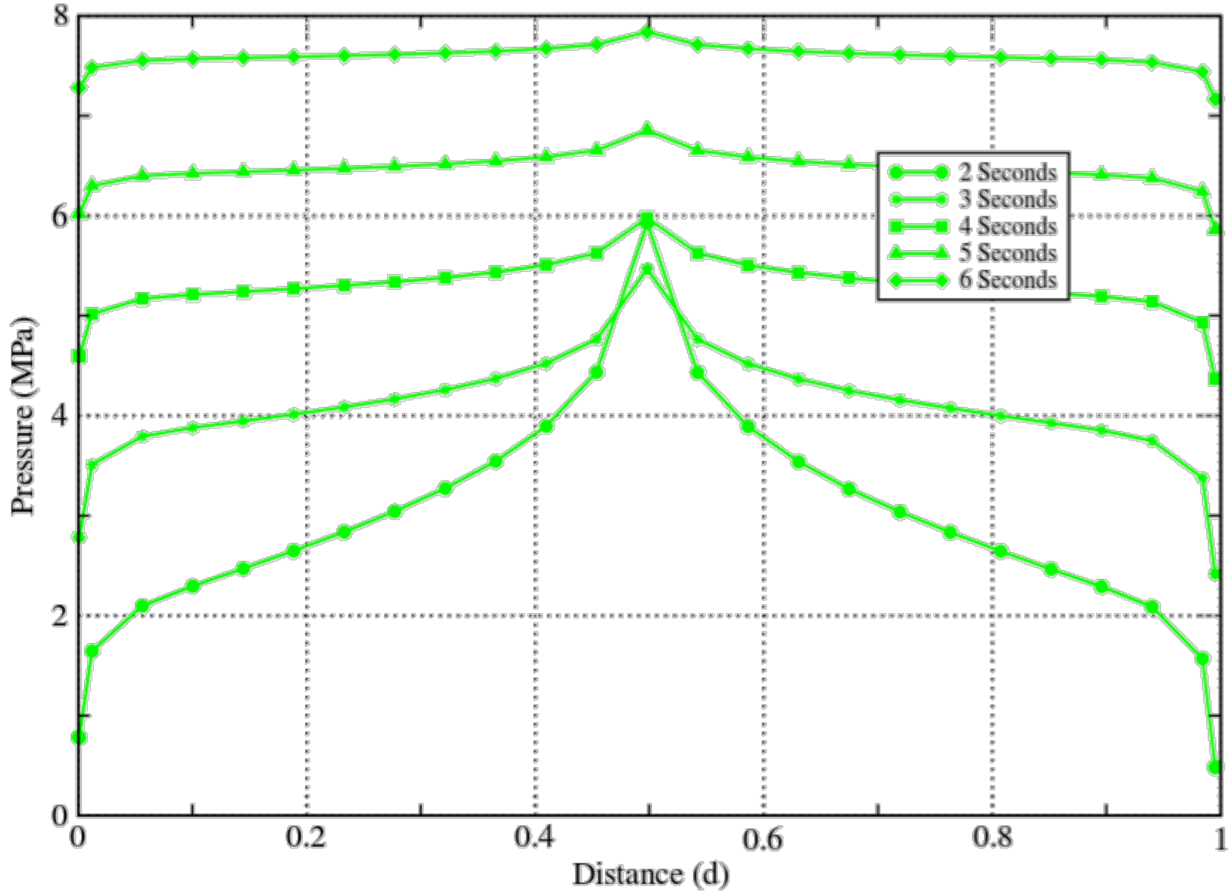


$\mu = 50$ cPoise

Circular Fracture

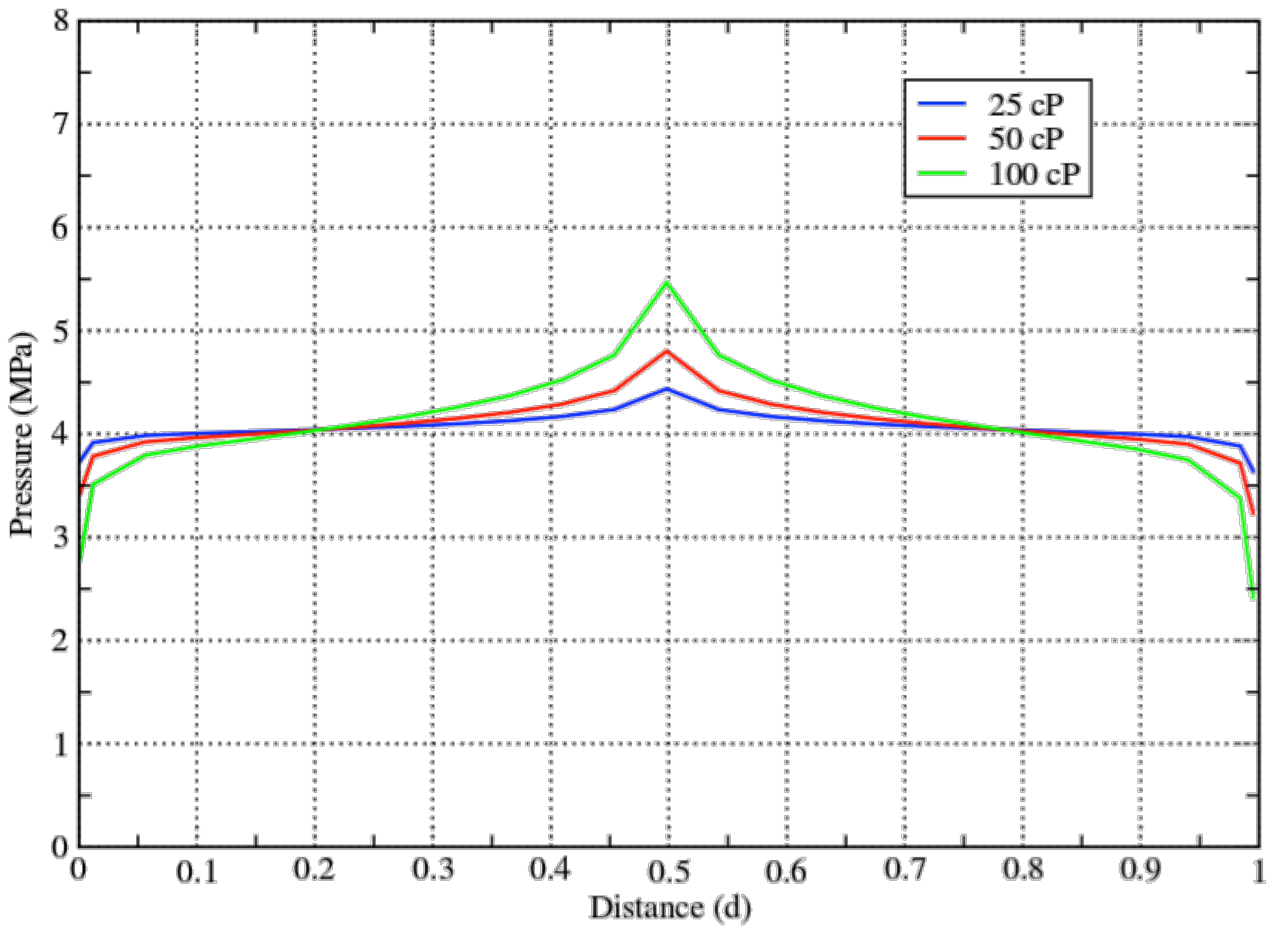
Pressure distribution as a function of time





$$\mu = 100 \text{ cPoise}$$

Circular Fracture



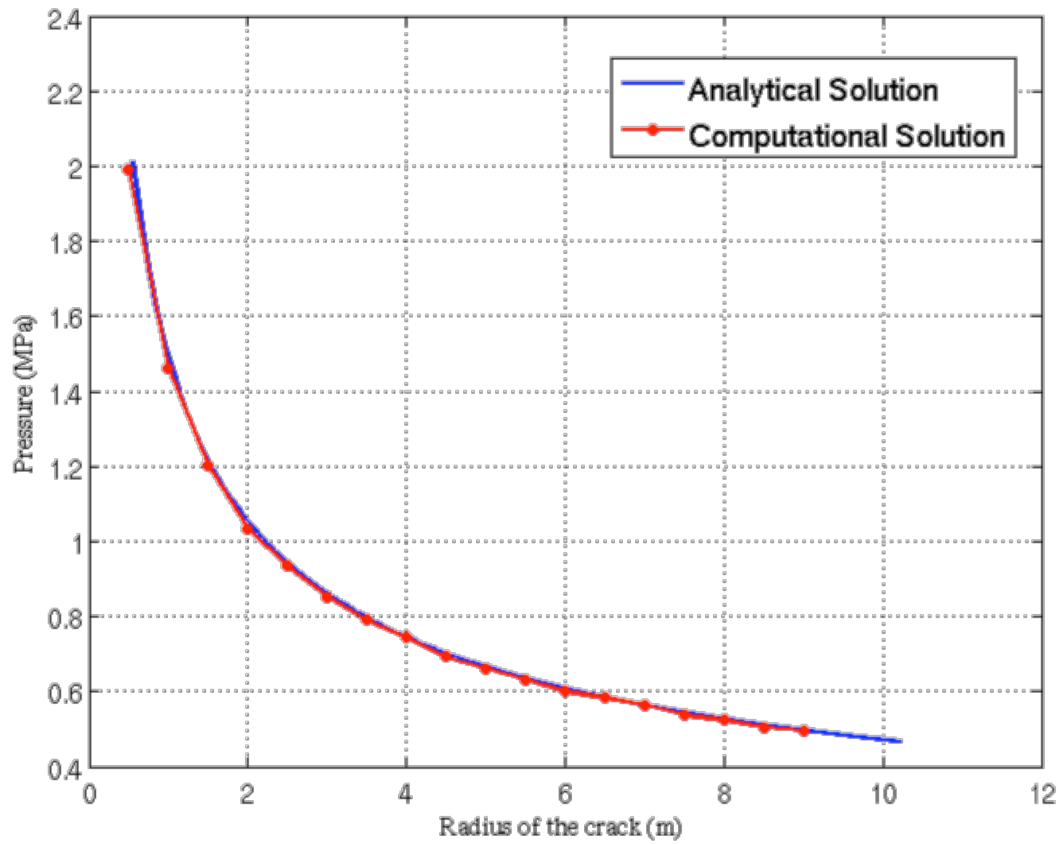
Time = 3 Seconds

Verification: Propagation of Circular Fracture

Critical pressure for a continuously propagating crack with $K_I = K_{Ic}$

Incomp. Newtonian fluid with viscosity $\mu = 1.0$ cPoise

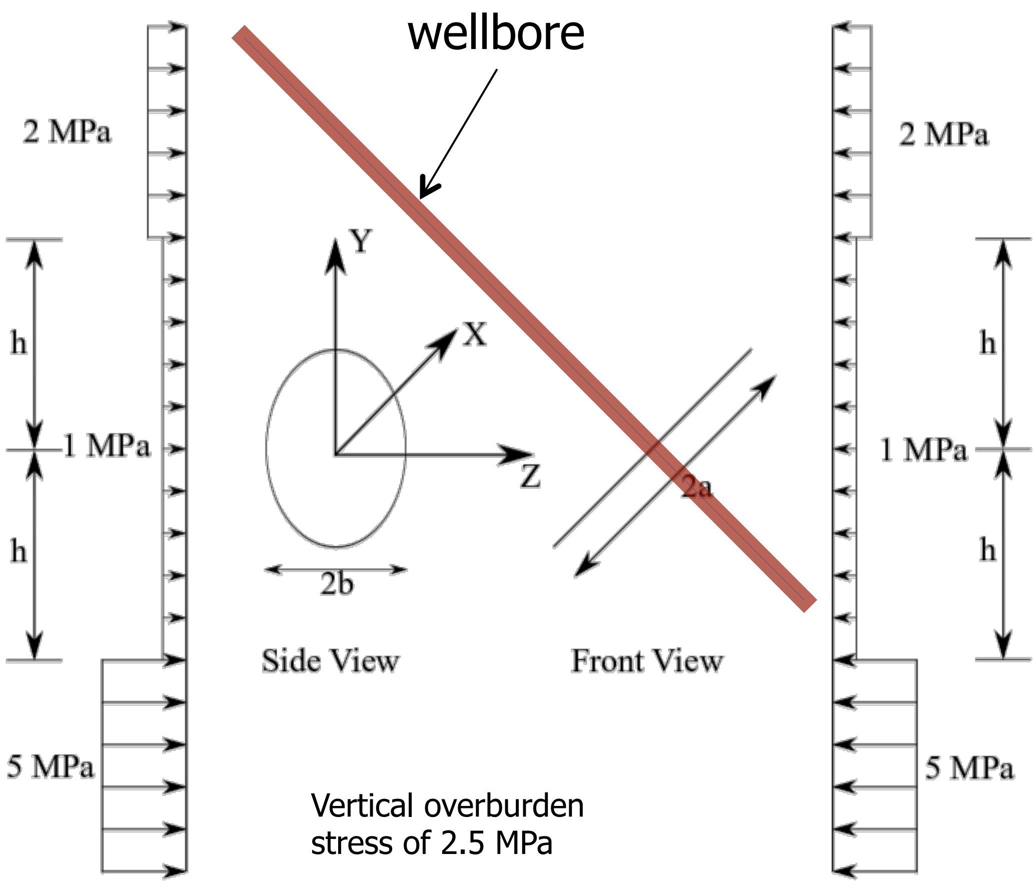
Injection rate at center of fracture $Q = 0.001\, m^3/s$

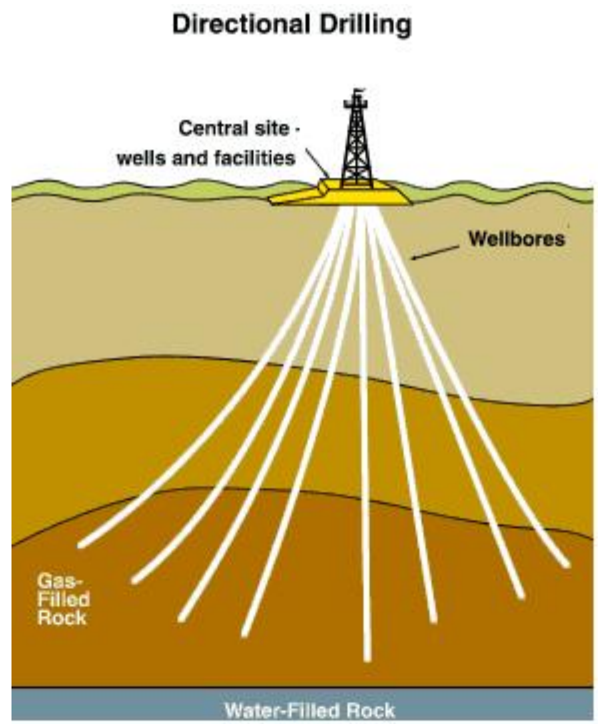


Analytical solution: Propagation of a penny-shaped fluid-driven fracture in an impermeable rock: Asymptotic solutions, Savitski and Detournay, 2002.

Application: Fracture Re-Orientation*

- Fracture starts in a direction not perpendicular to minimum in-situ stress
- Misalignment of fracture and confining in-situ stresses

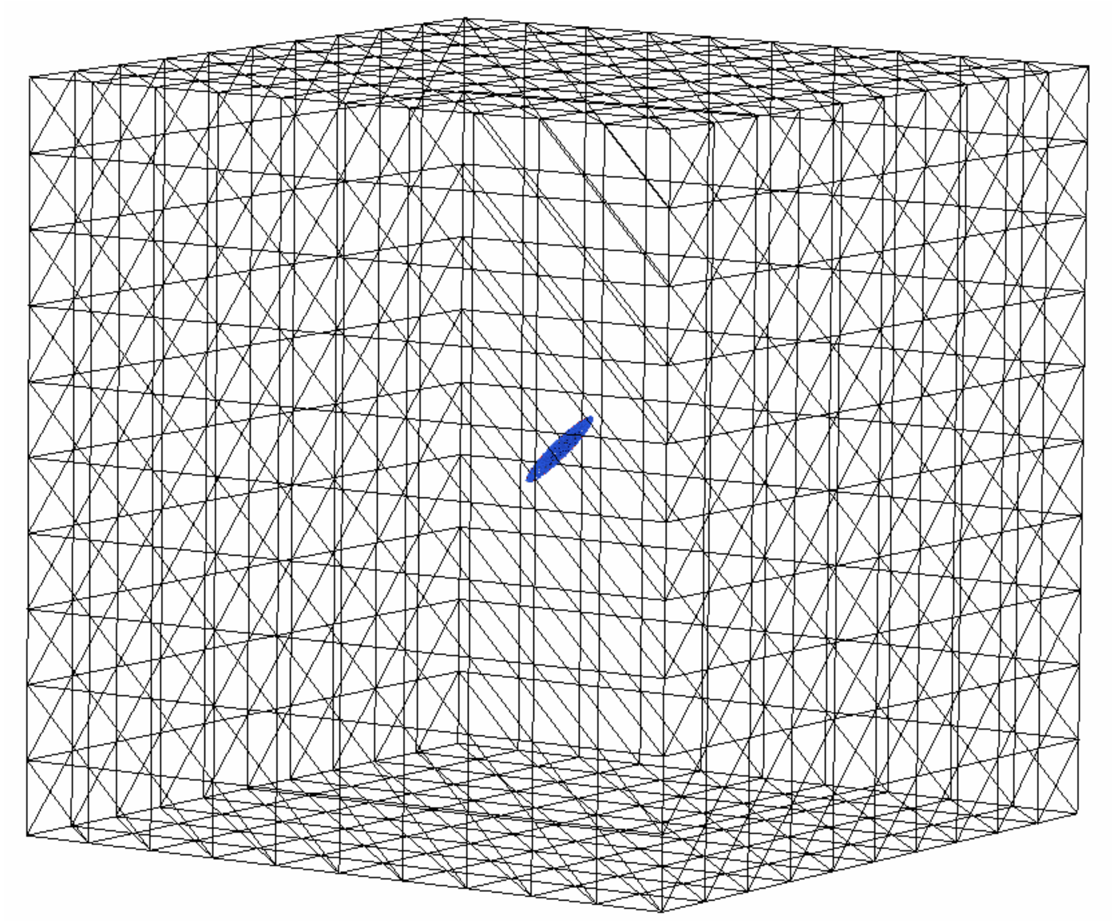


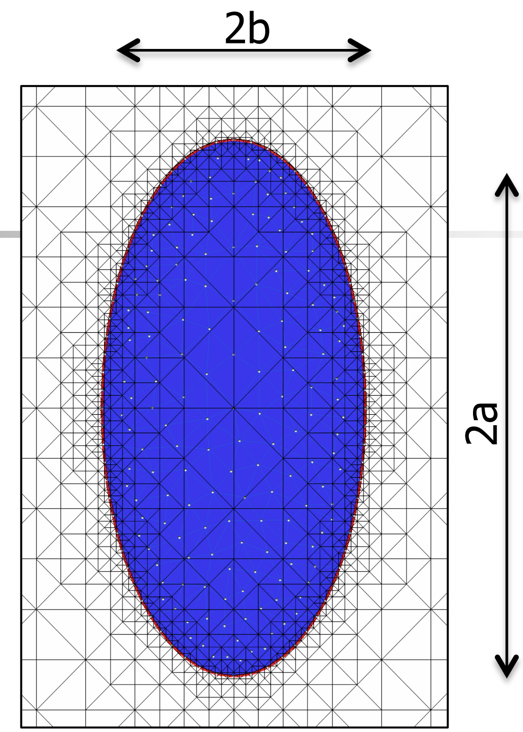


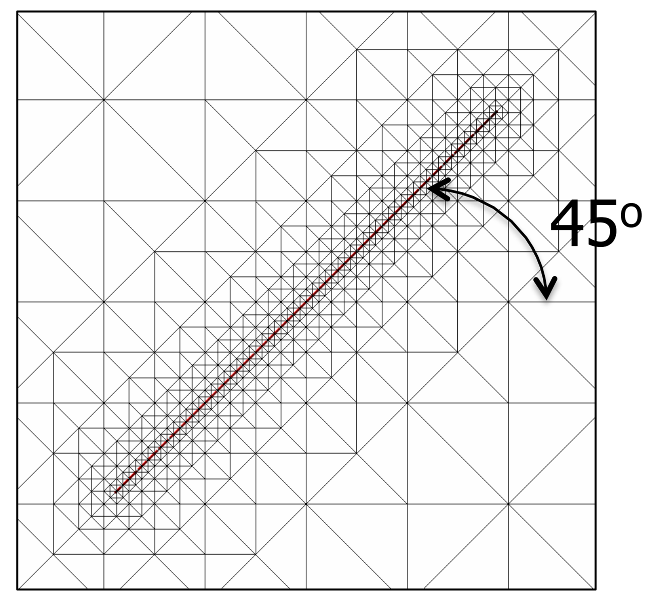
a = 10m
b = 5m
h = 15m
p = 3.5 MPa

*[Rungamornrat et al., 2005; Gupta & Duarte, 2014]

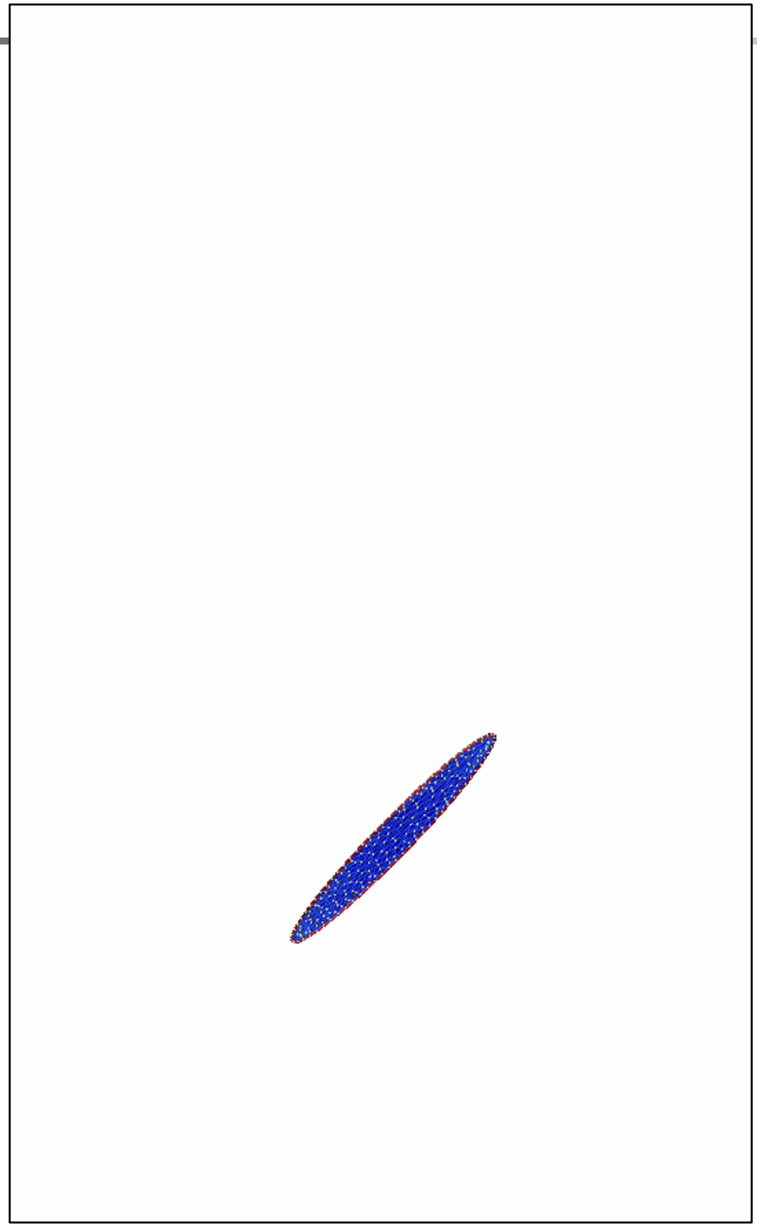
Fracture Re-Orientation



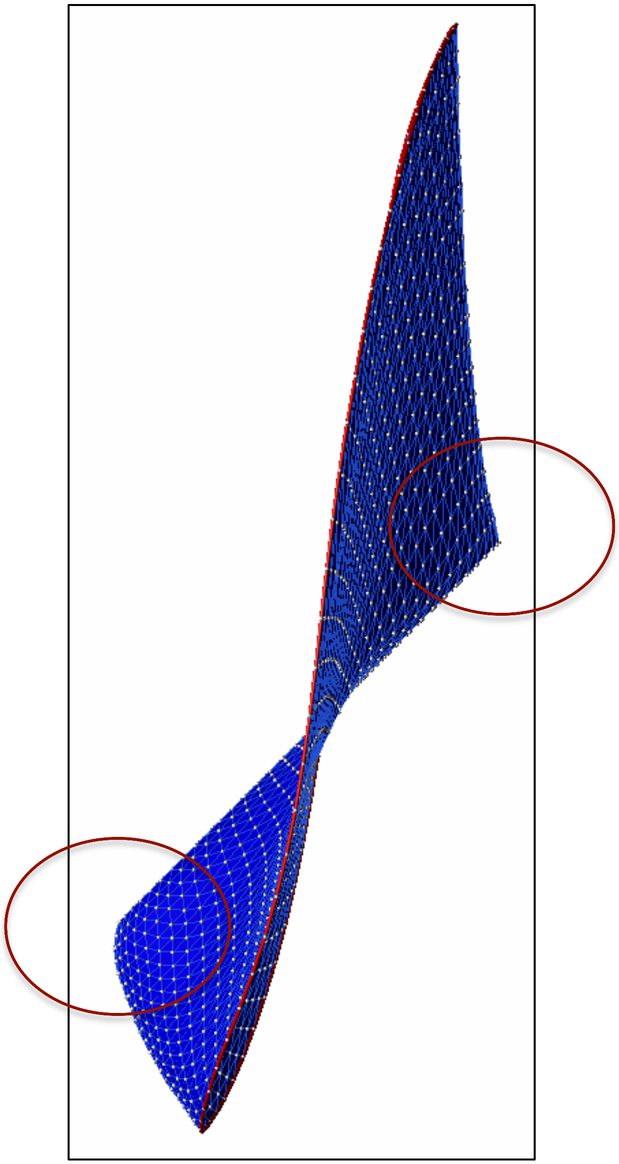


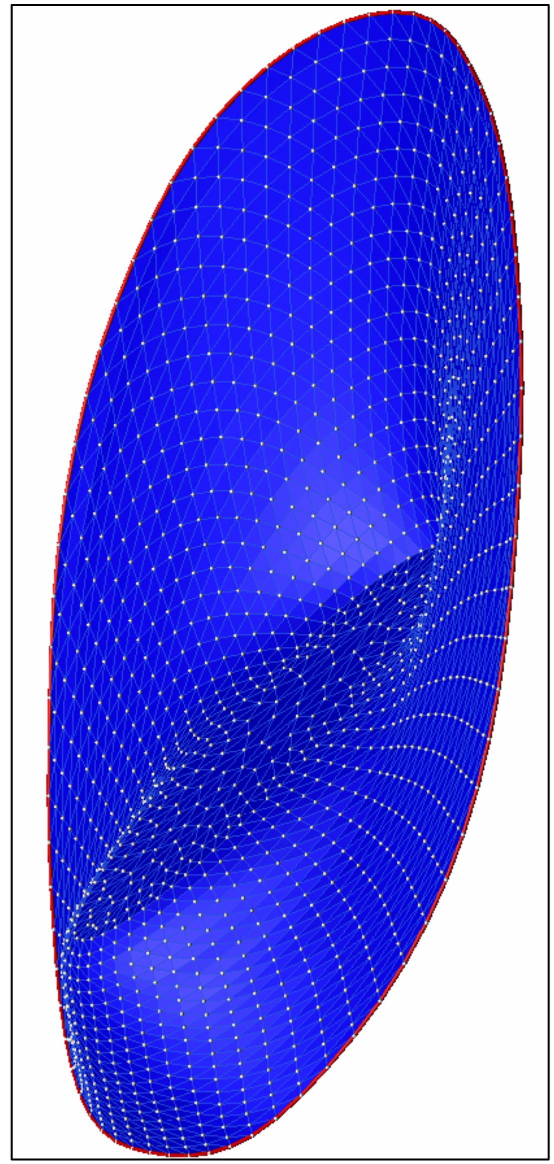


Fracture Re-Orientation

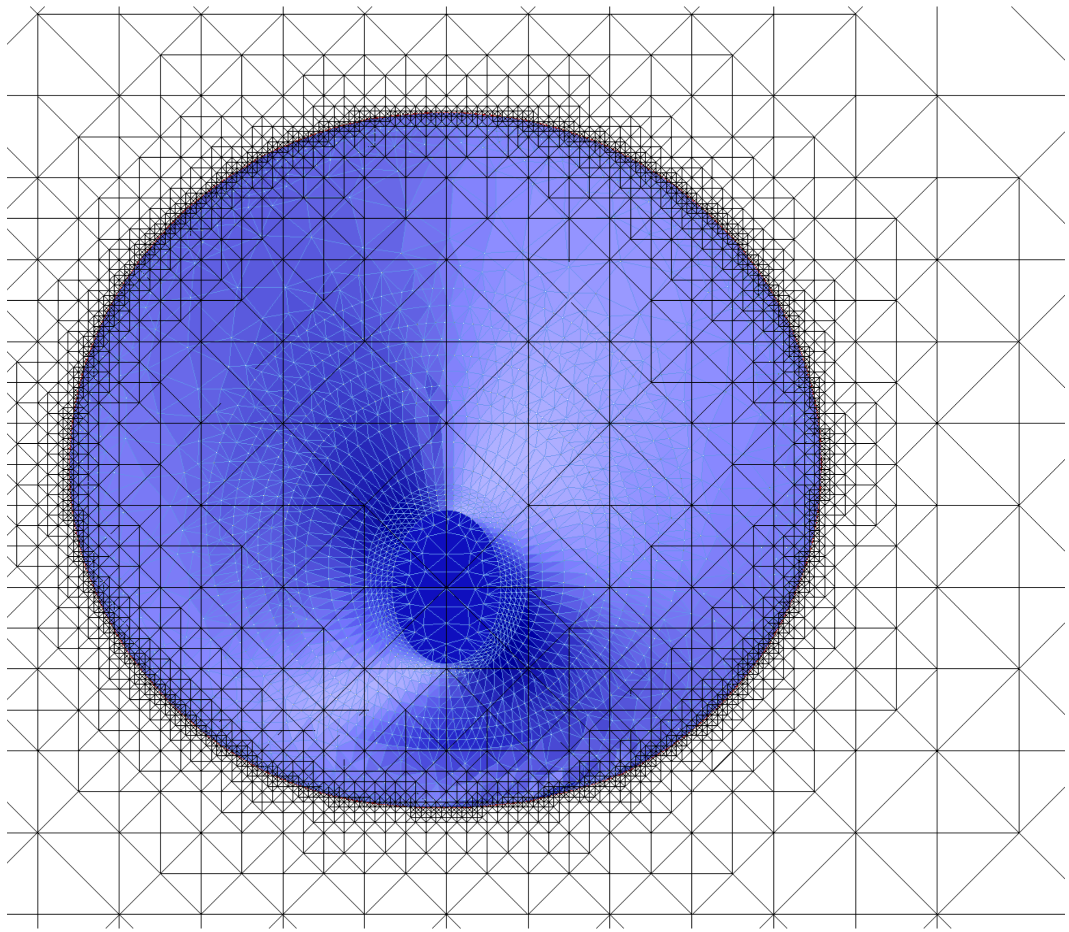


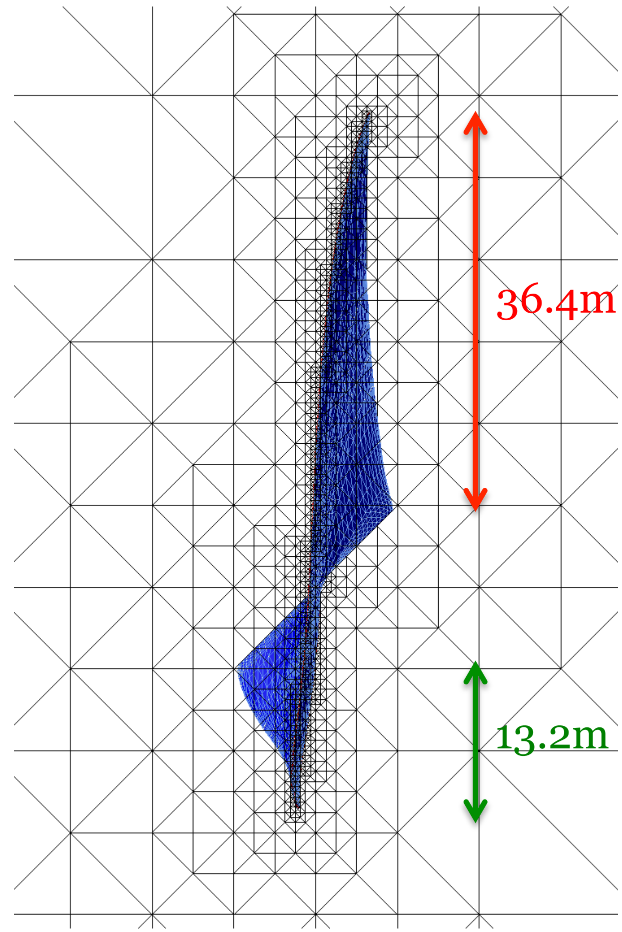
Fracture Re-Orientation: Step 20





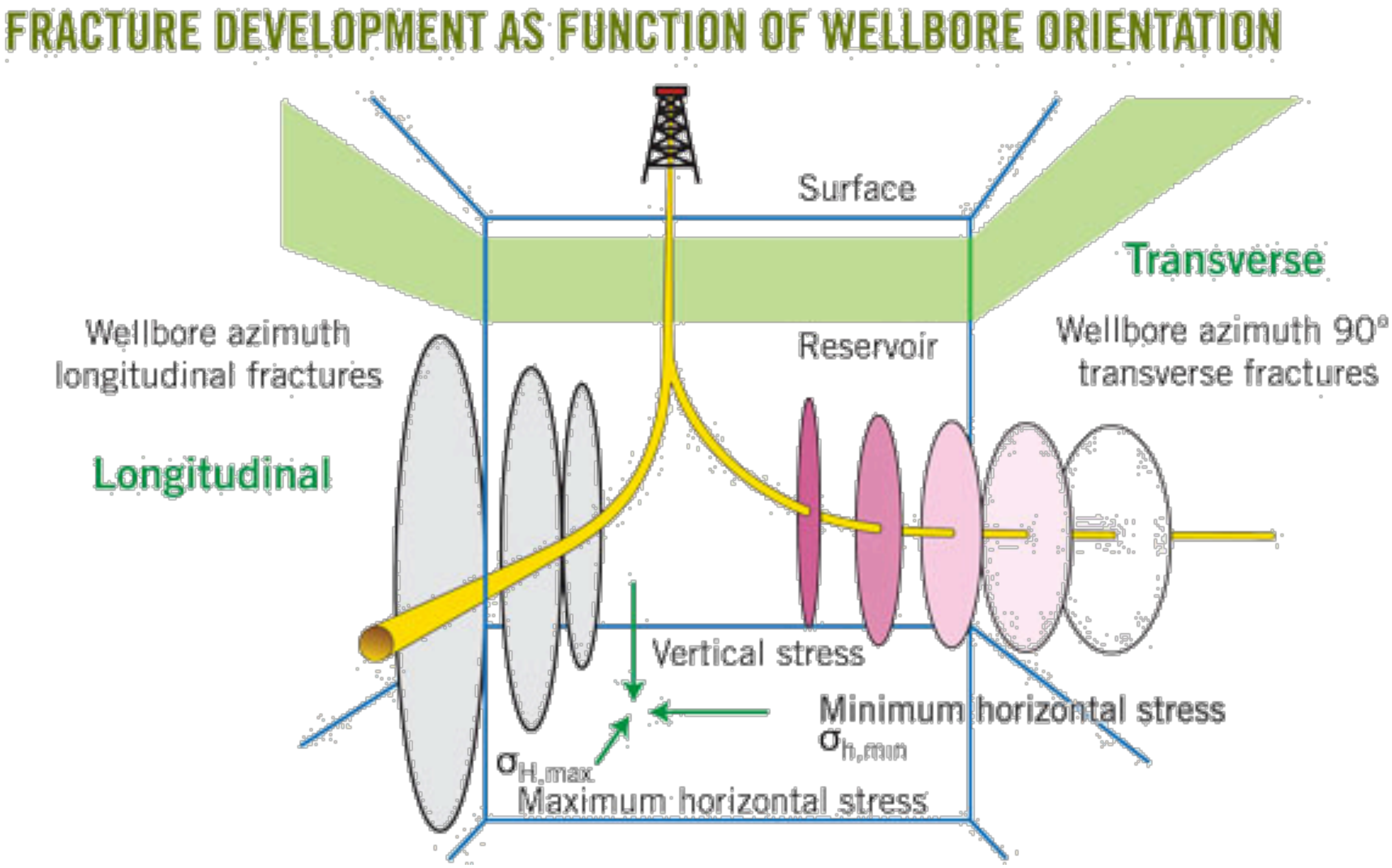
Fracture Re-Orientation: Adaptive Mesh





- Adaptive refinement along fracture front
- Sharp features are preserved
- High fidelity of fracture surface, regardless of computational mesh

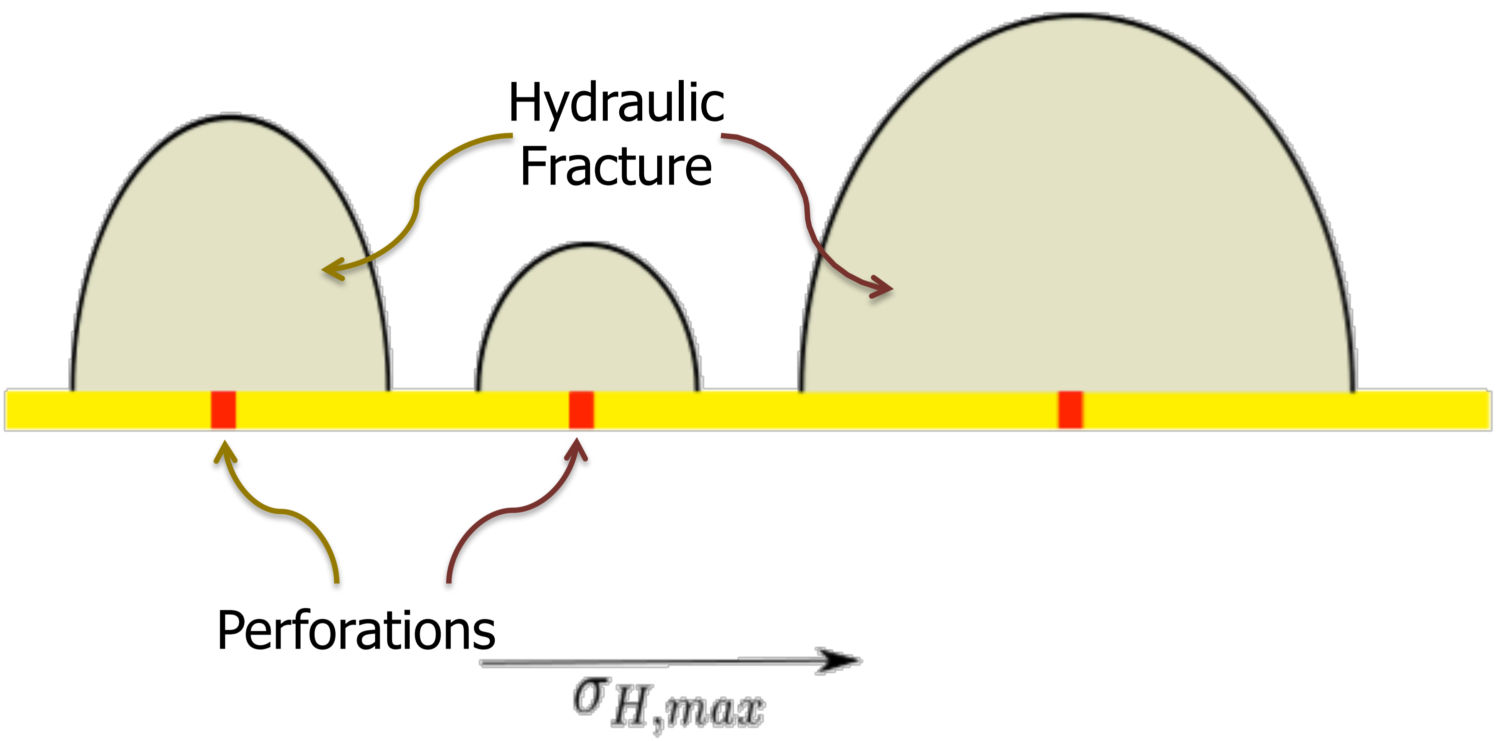
Typical Hydraulic Fracturing



[Z. Rahim et al., 2012]

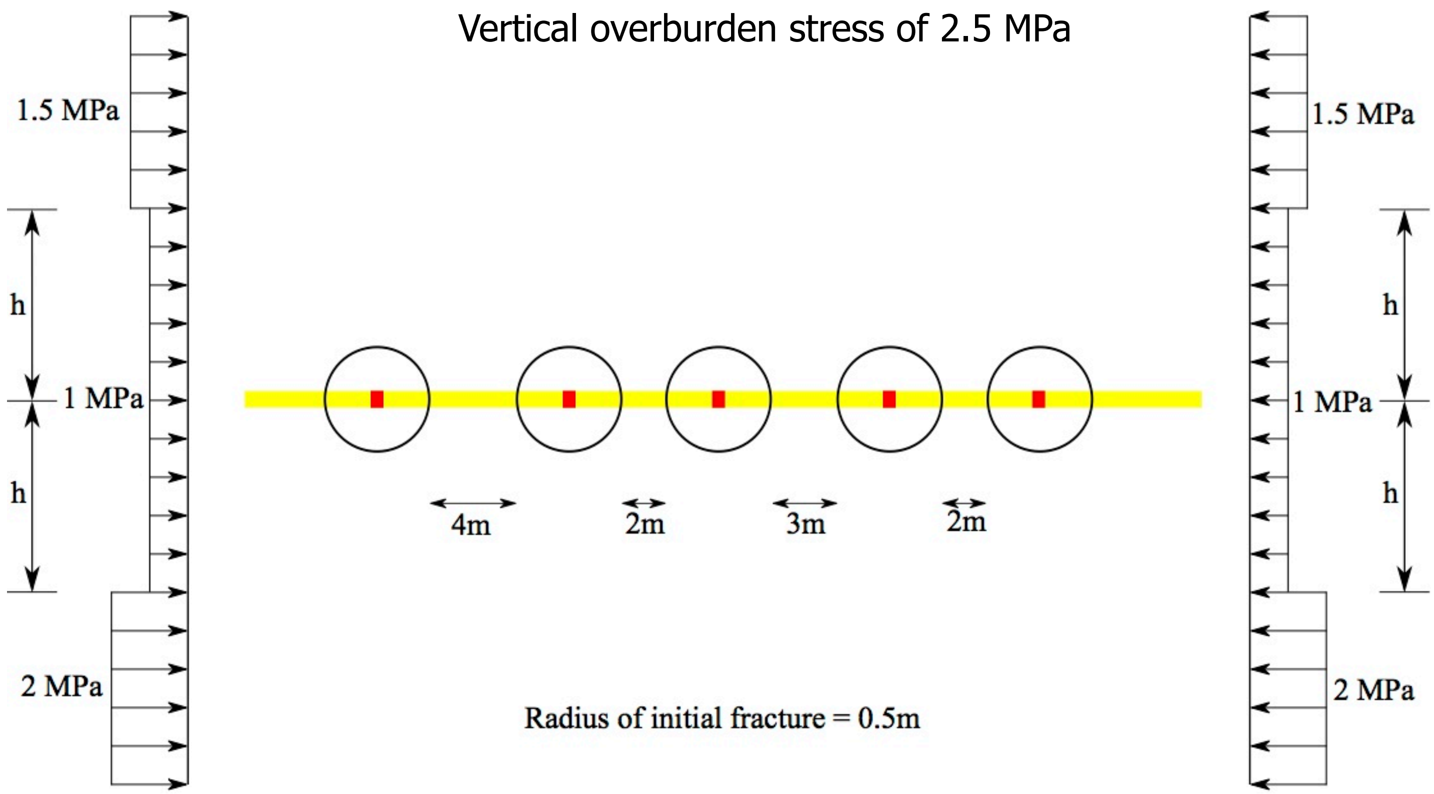
Longitudinal Fractures

- Develop perpendicular to minimum in-situ stress
- Fractures along the length of the wellbore
- Planar fractures from the perforation



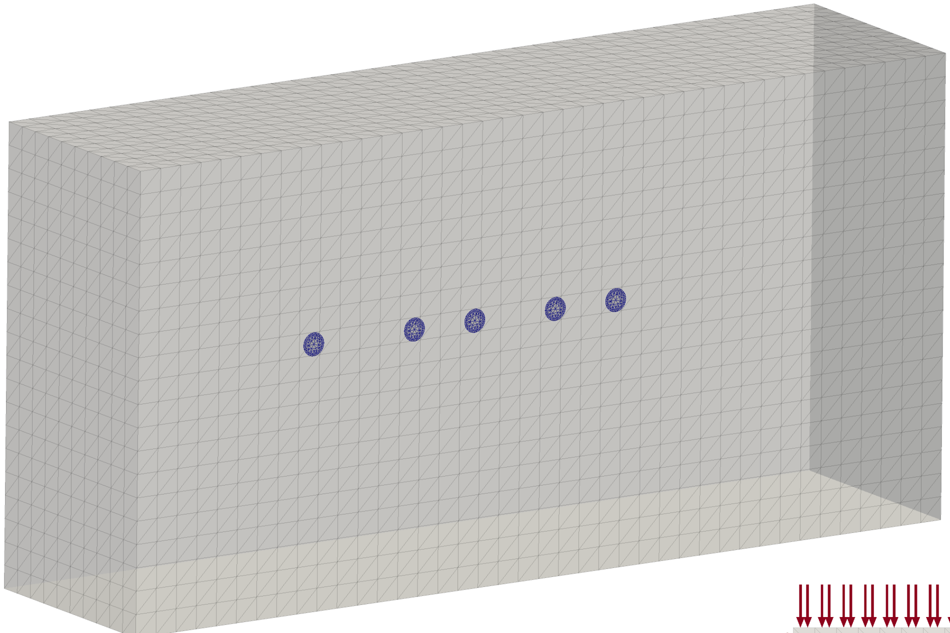
Coalescence of Longitudinal Fractures

- Propagation and coalescence from a horizontal well

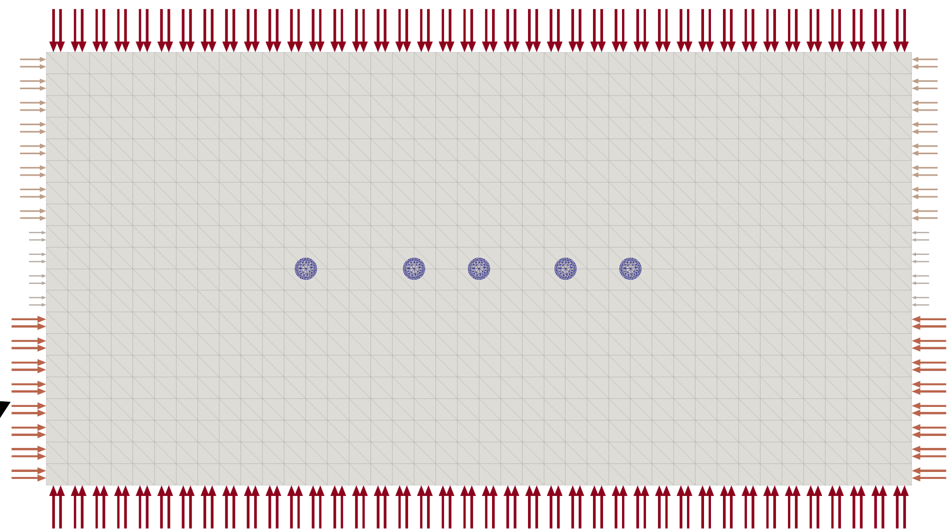


h = 2 m
p = 3.5 MPa

Coalescence of 3-D Fractures: GFEM Model

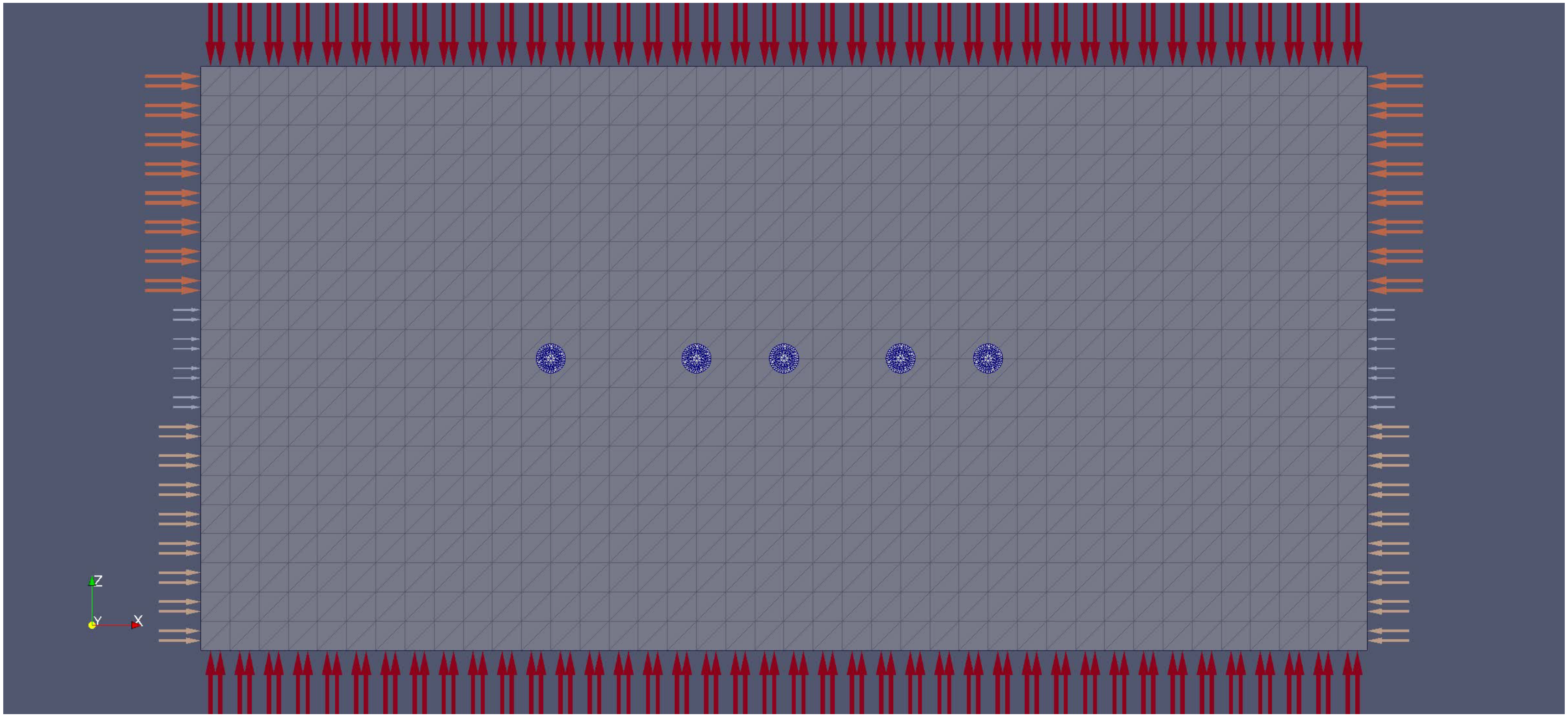


- Input mesh and fracture surfaces for GFEM simulation
- Automatic adaptive mesh refinement performed at each propagation step



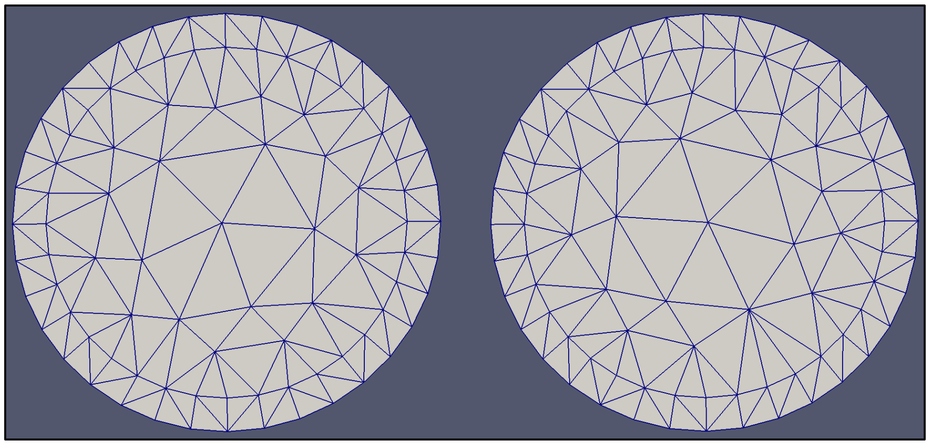
In-situ stress
(all around)

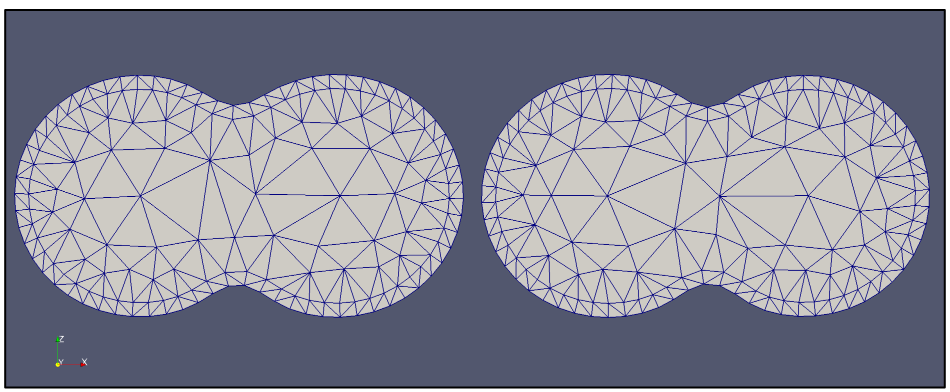
Coalescence of 3-D Fractures



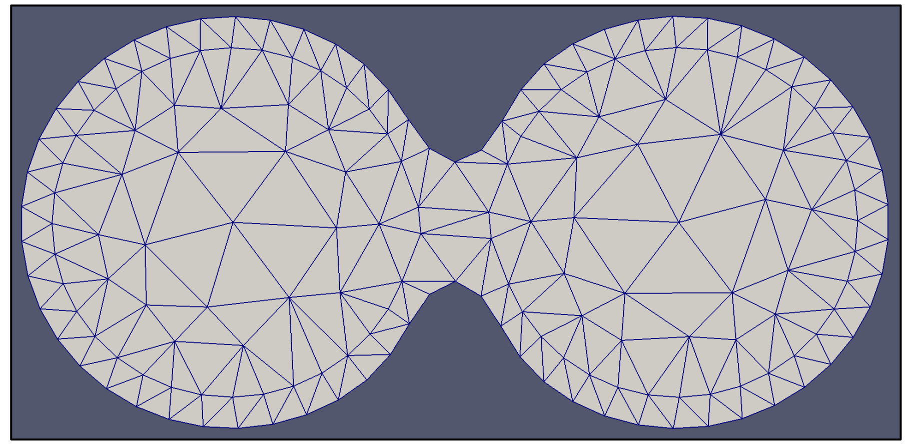
Coalescence of 3-D Fractures

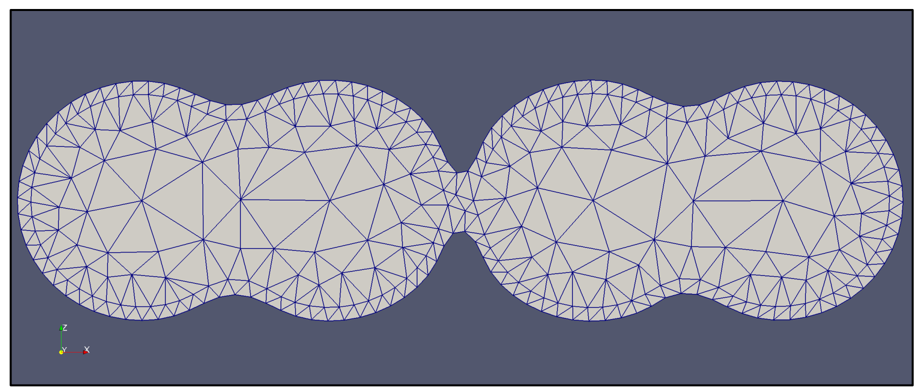
Fractures just prior to coalescence



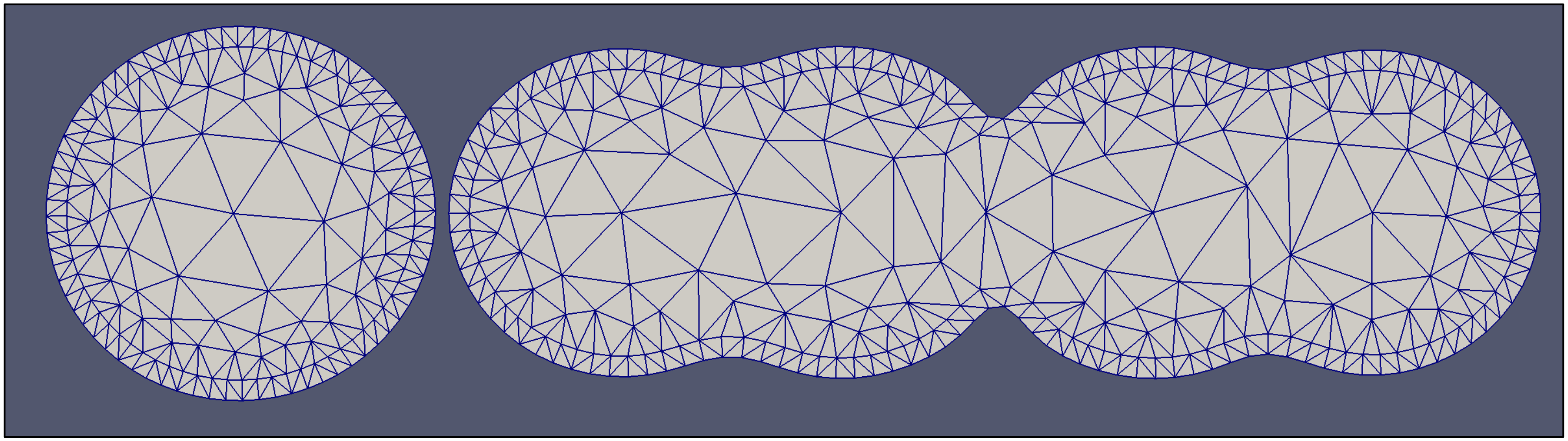


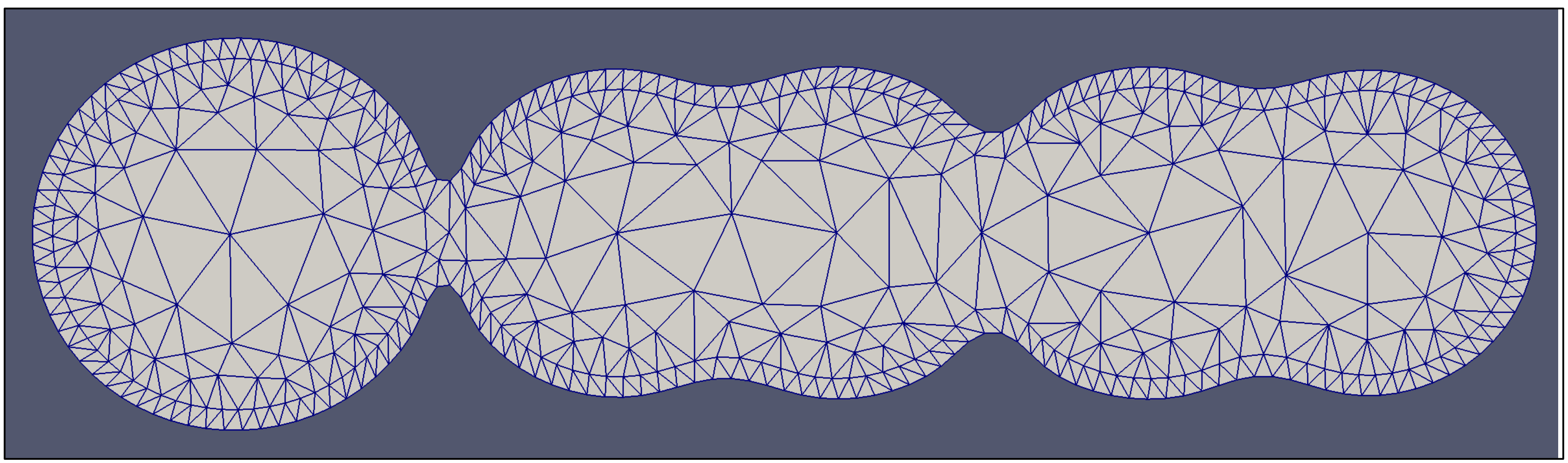
Fractures just after coalescence





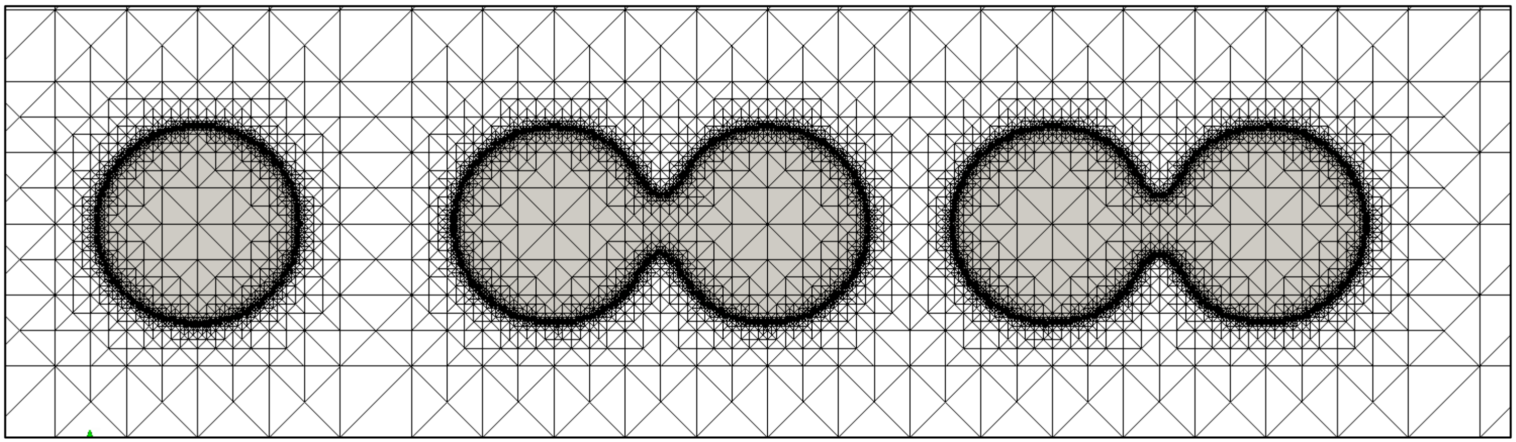
Coalescence of 3-D Fractures

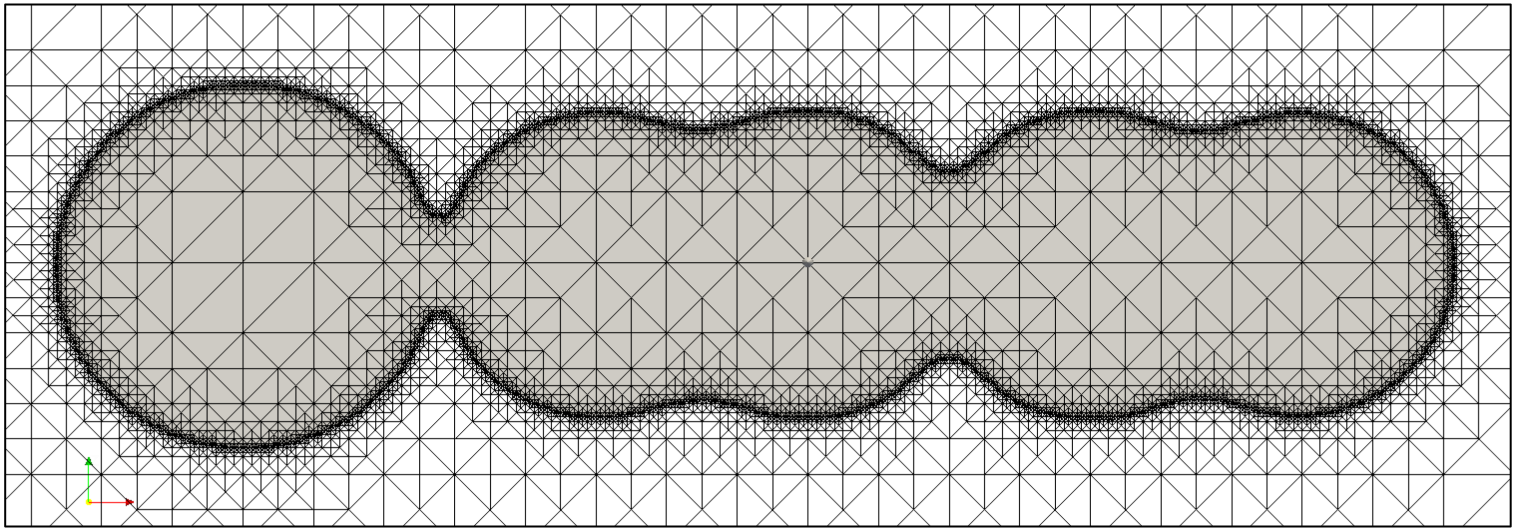




Coalescence of 3-D Fractures

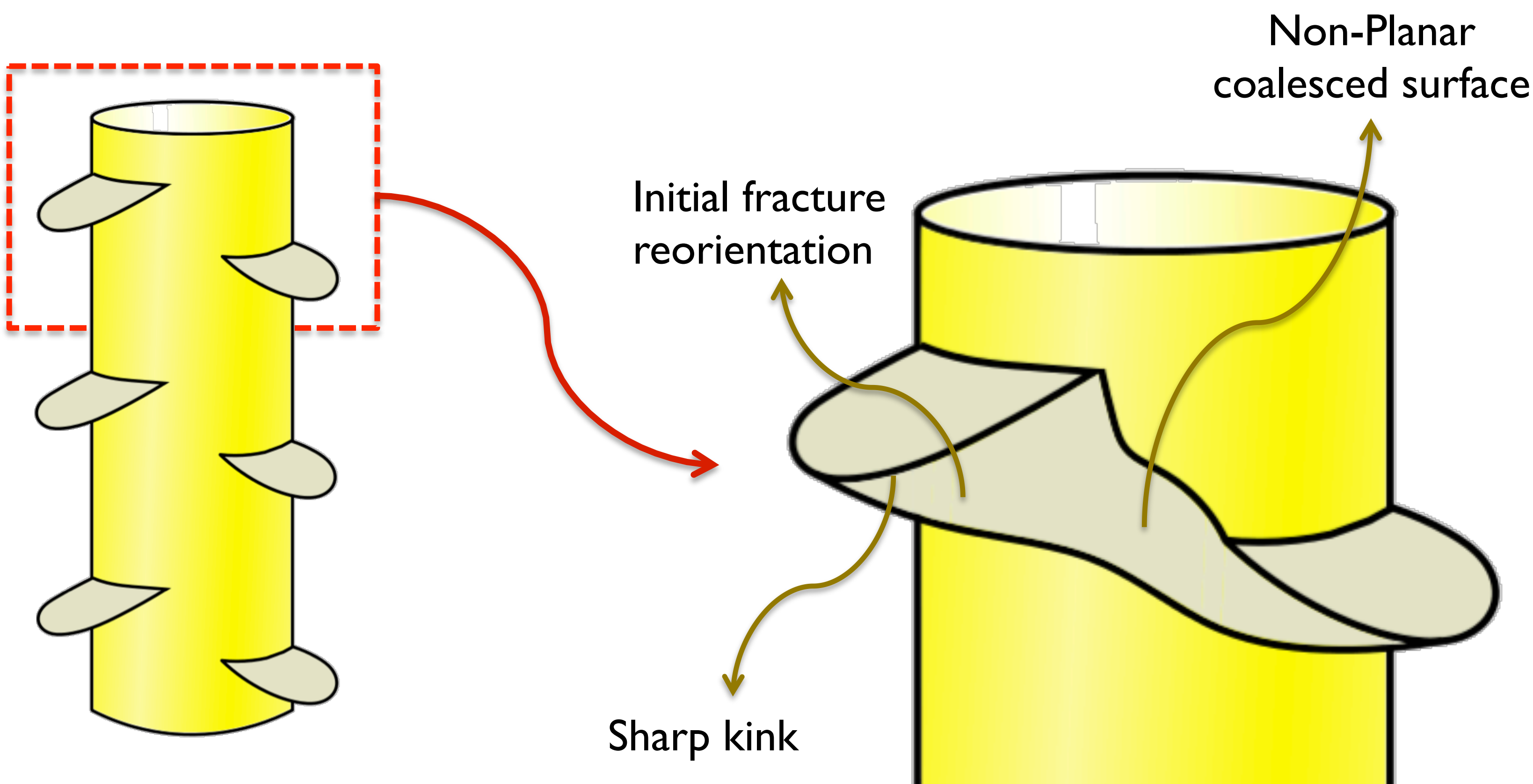
- Adaptive refinement along fracture fronts





Ongoing Work

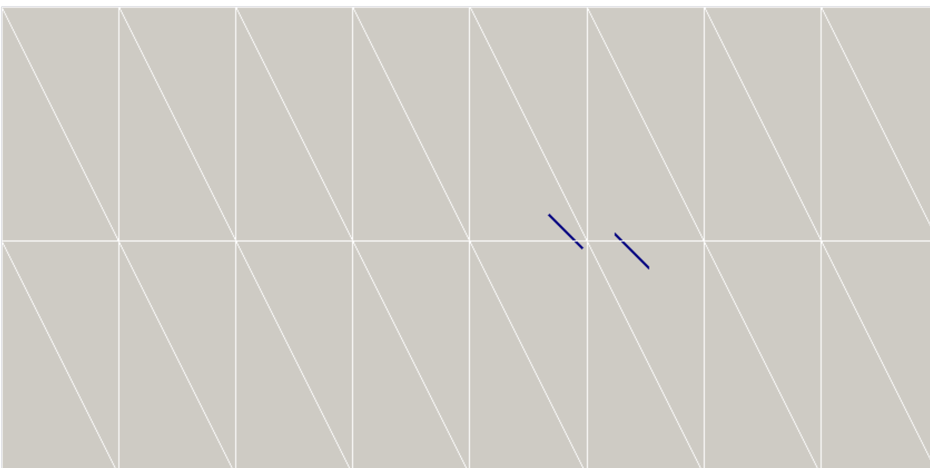
- Coalescence of non-planar fractures near a wellbore

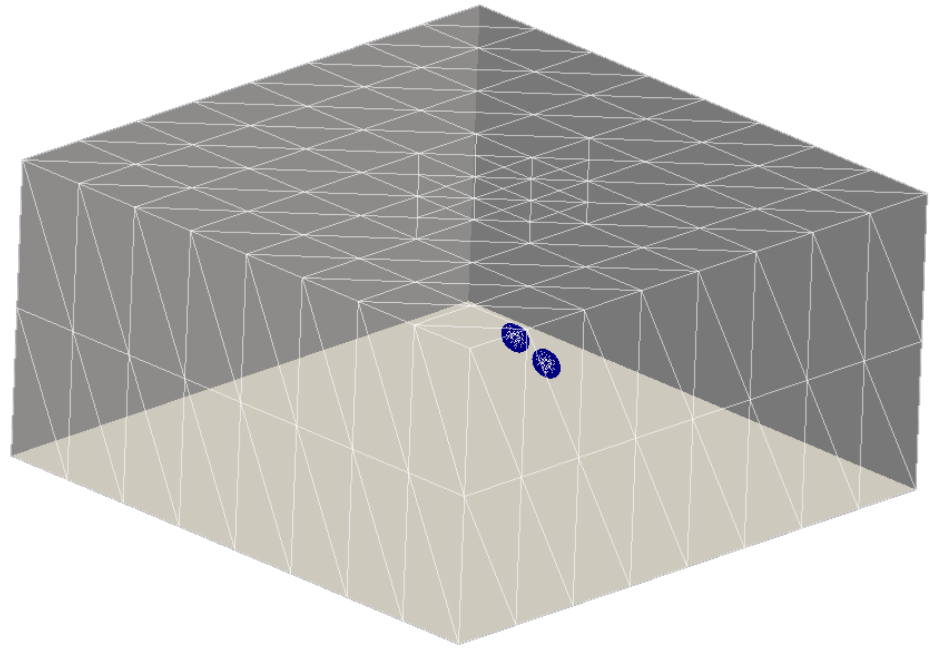


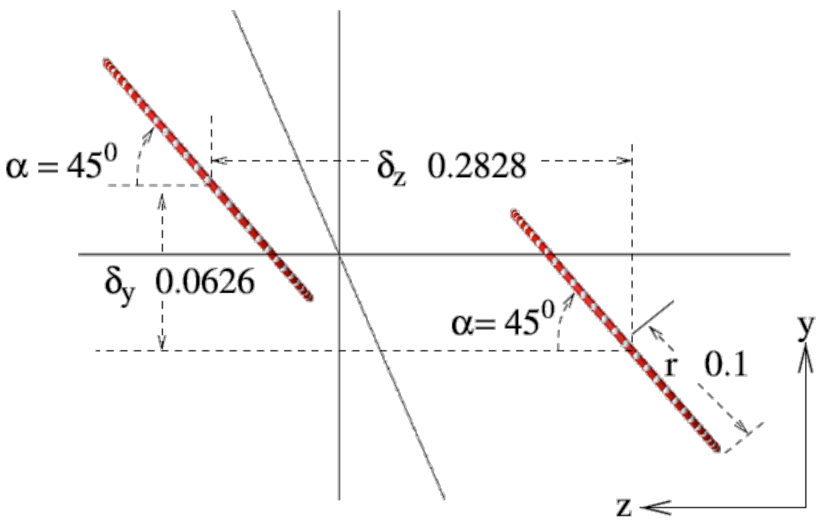
Vertical well with initial fractures in a spiral pattern

After Propagation and coalescence

Crack Coalescence – Non-Planar Cracks







Traction BC on top and bottom surface

PropagationMovie1

Conclusions and Outlook

- Generalized FEM removes several limitations of std FEM
- It enables the solution of problems that are difficult or not practical with the FEM
- This is the case of three-dimensional fracture problems involving
  - Complex crack surfaces
  - Fluid-induced fracturing
  - Coalescence of 3-D fractures, etc.
- Ongoing
  - Coalescence of non-planar fractures
  - Interaction between hydraulic and natural fractures

Acknowledgements

Air Force Research Laboratory - University Collaborative Center in Structural Sciences (C^2S^2)