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Recent Developments in the Generalized Finite Element Method and Applications in 3-D Fracture Propagation and Coalescence

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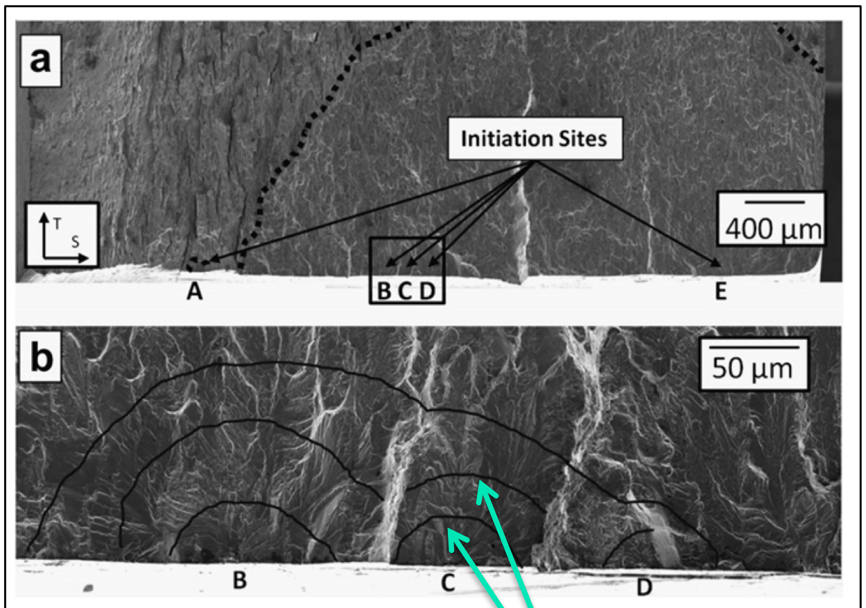
Crack Growth and Coalescence

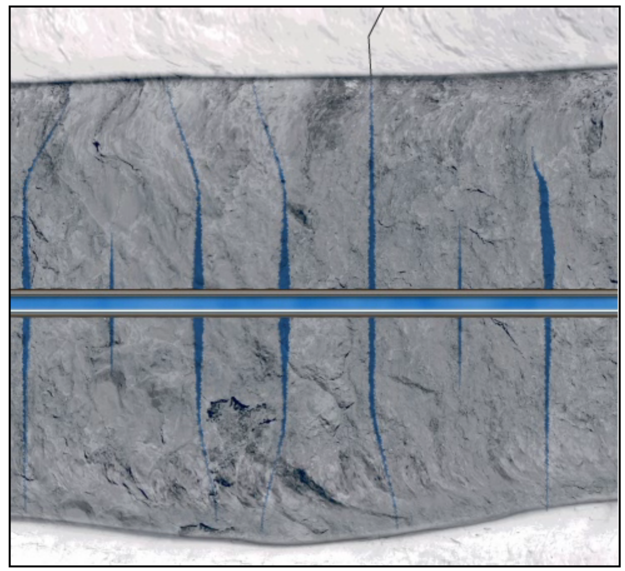
- Understanding crack coalescence is of great importance in many applications

Coalescence of fatigue micro-cracks



Crack fronts

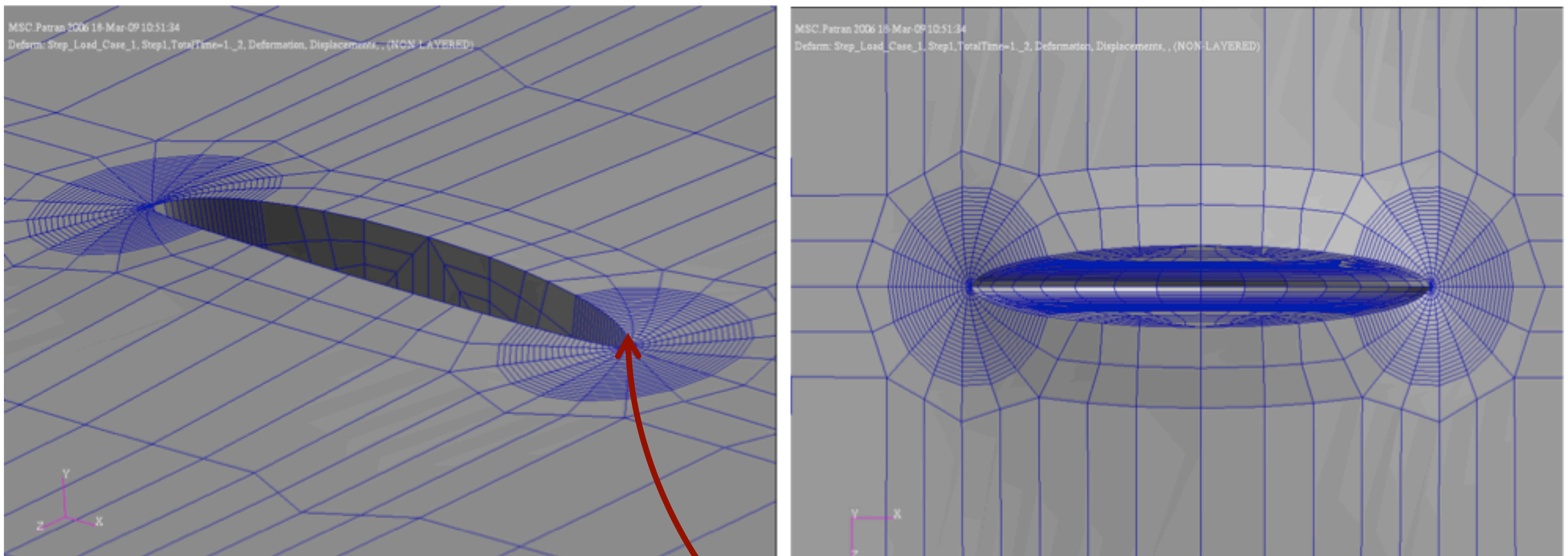
Cluster of hydraulic fractures propagating from a horizontal well



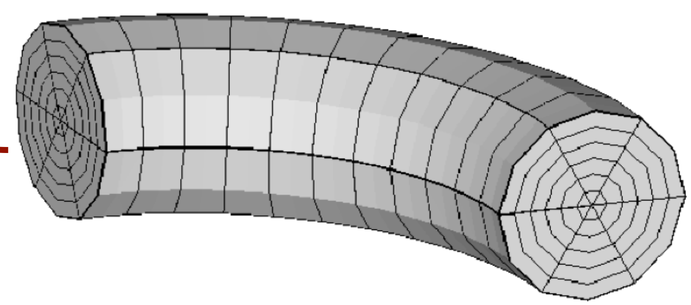
Reflective crack in asphalt overlay

Modeling 3-D Fractures: Limitations of Standard FEM

- It is not "just" fitting the 3-D <u>evolving</u> fracture
- FEM meshes must satisfy special requirements for acceptable accuracy



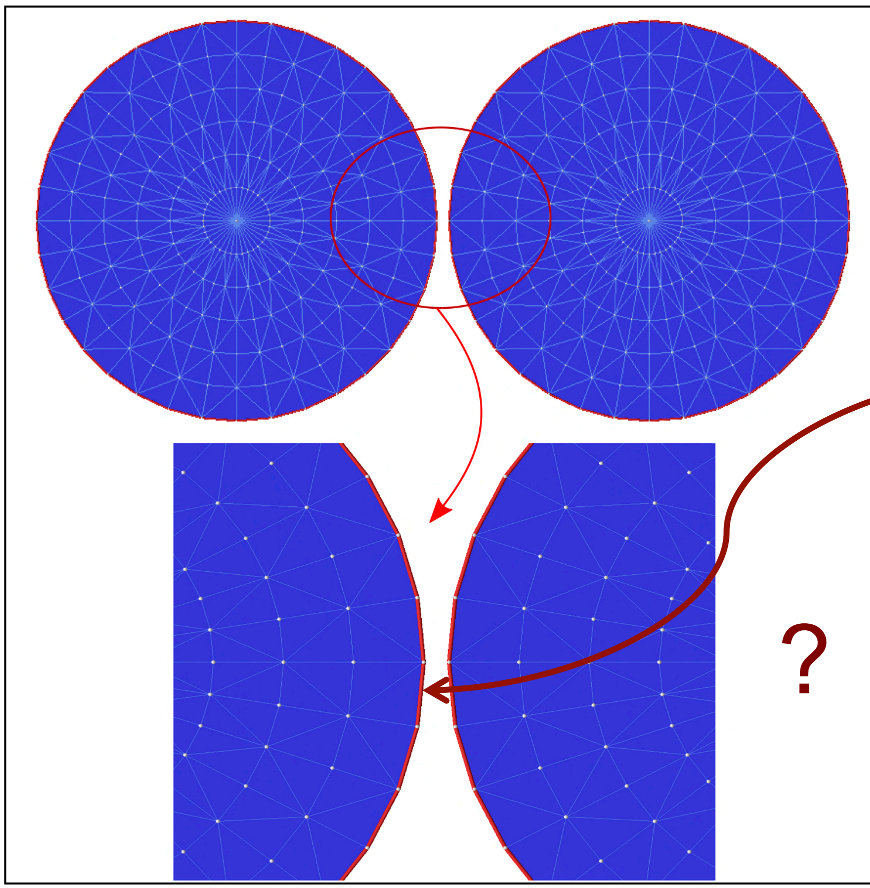
FEM mesh for a surface fracture



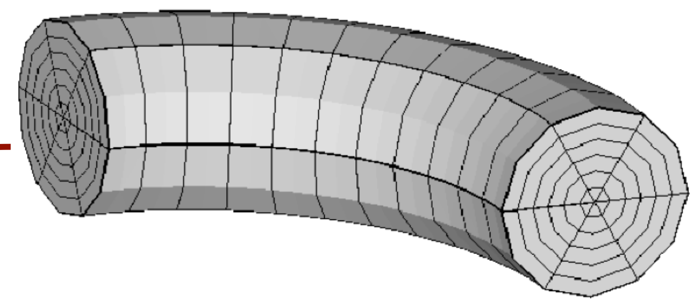
Mesh with quarter-point elements

Limitations of Standard FEM

- Difficulties arise if fracture front is close to complex geometrical features
- Fracture surfaces with sharp turns
- Coalescence of fractures



- Not possible in general to automatically create structured meshes along both fracture fronts when they are in close proximity



- Even with these crafted meshes and quarter-point elements, convergence rate of std FEM is slow (*controlled by singularity at fracture front*)
- **Strategy**: Generalized FEM

Outline

- Motivation and limitations of standard finite element methods
- Basic ideas of GFEM
- GFEM for 3-D fractures
- Application
  - Propagation and coalescence of hydraulic fractures
- Conclusions and outlook

Early Works on Generalized FEMs

- Babuska, Caloz and Osborn, 1994 (Special FEM).
- Duarte and Oden, 1995 (Hp Clouds).
- Babuska and Melenk, 1995 (PUFEM).
- Oden, Duarte and Zienkiewicz, 1996 (Hp Clouds/GFEM).
- Duarte, Babuska and Oden, 1998 (GFEM).
- Belytschko et al., 1999 (Extended FEM).
- Strouboulis, Babuska and Copps, 2000 (GFEM).

- Basic idea:
  - Use a partition of unity to build Finite Element shape functions

- Review paper

  Belytschko T., Gracie R. and Ventura G. A review of extended/generalized finite element methods for material modeling, *Mod. Simul. Matl. Sci. Eng.,* 2009

  "The XFEM and GFEM are basically <u>identical</u> methods: the name generalized finite element method was adopted by the Texas school in 1995–1996 and the name extended finite element method was coined by the Northwestern school in 1999."

Generalized Finite Element Method

- GFEM is a Galerkin method with special test/trial space given by

$$\mathbb{S}_{GFEM} = \mathbb{S}_{FEM} + \mathbb{S}_{ENR}$$

Low order FEM space

Enrichment space with functions related to the given problem

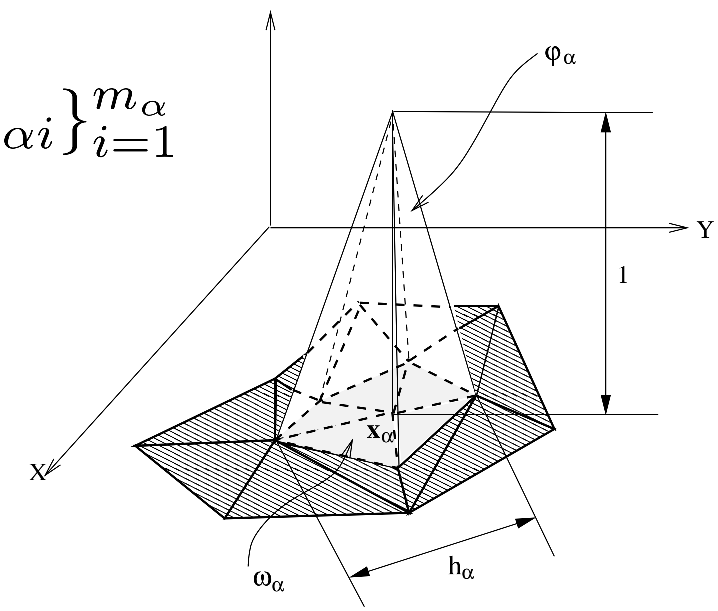
$$\mathbb{S}_{FEM} = \sum_{\alpha \in I_h} c_\alpha \varphi_\alpha, \quad c_\alpha \in \mathbb{R}$$

$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$L_{\alpha i} \in \chi_\alpha(\omega_\alpha)$$

Enrichment function

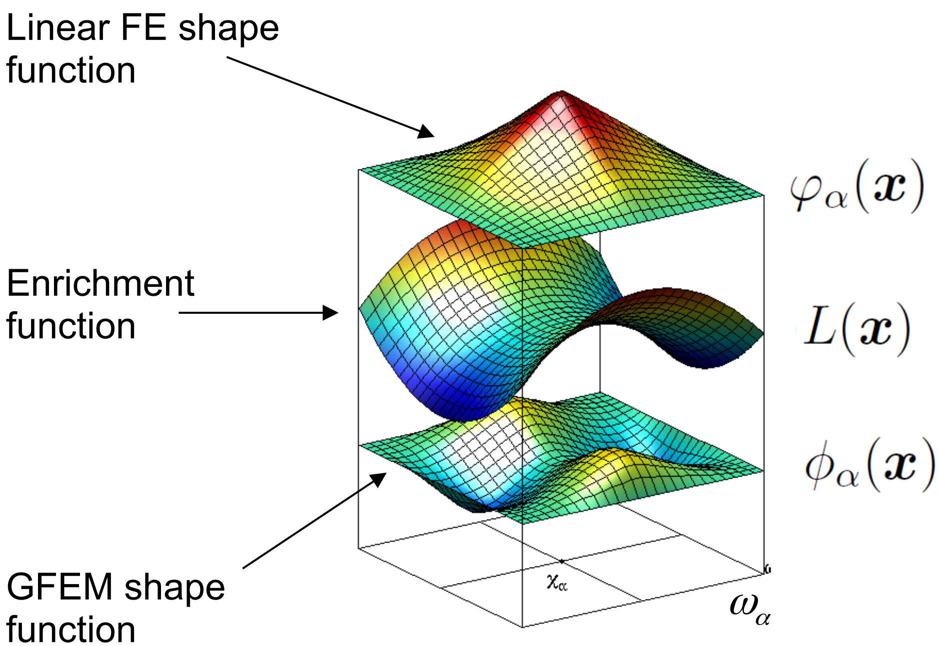
Patch space



Generalized Finite Element Method

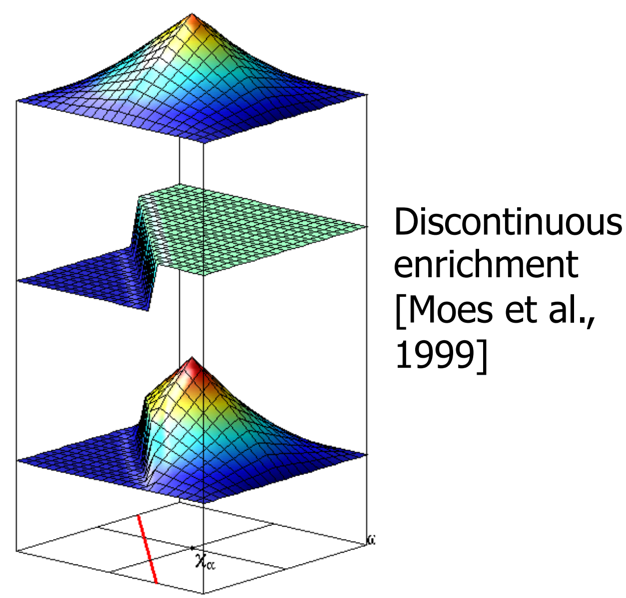
$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$\phi_{\alpha i}(x) = \varphi_\alpha(x) L_{\alpha i}(x) \qquad \sum_\alpha \varphi_\alpha(\boldsymbol{x}) = 1$$



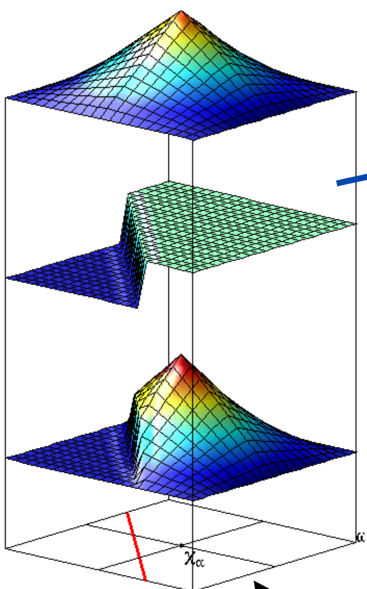
[Oden, Duarte & Zienkiewicz, 1996]

• Allows construction of shape functions incorporating a-priori knowledge about solution



GFEM Approximation for 3-D Fractures

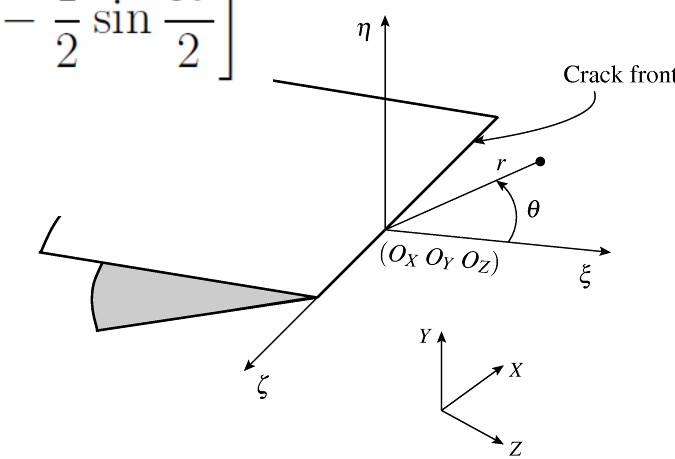
$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



patch ω_α

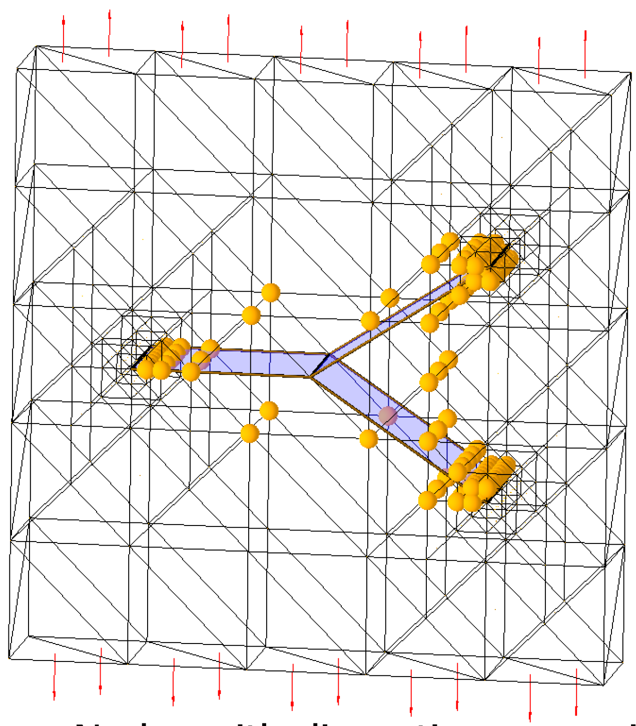
$$\begin{aligned}
\breve{L}^{\xi}_{\alpha 1}(r,\theta) &= \sqrt{r}\left[(\kappa - \frac{1}{2})\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right] \\
\breve{L}^{\eta}_{\alpha 1}(r,\theta) &= \sqrt{r}\left[(\kappa + \frac{1}{2})\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right] \\
\breve{L}^{\zeta}_{\alpha 1}(r,\theta) &= \sqrt{r}\sin\frac{\theta}{2}
\end{aligned}$$

[Duarte & Oden 1996]



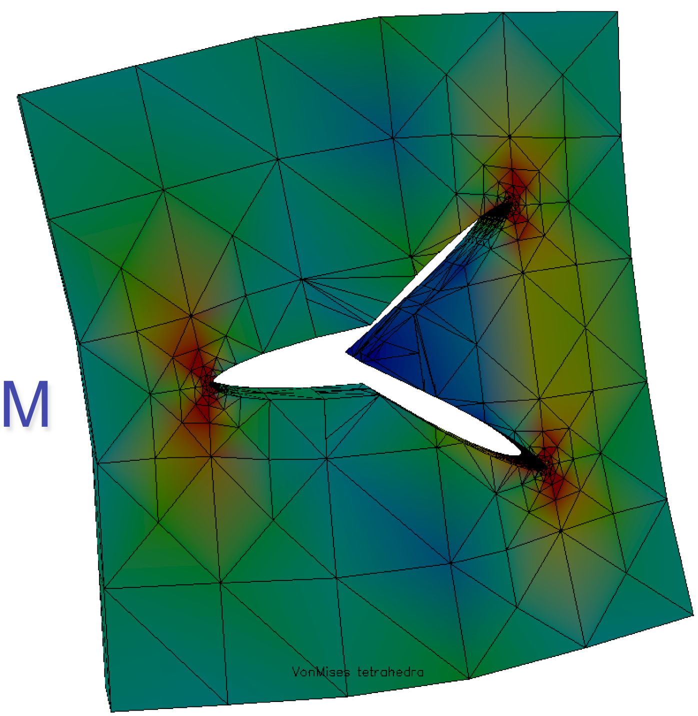
Modeling Fractures with the GFEM

- Fractures are modeled via enrichment functions, *not* the FEM mesh
- Mesh refinement *still required* for acceptable accuracy



= Nodes with discontinuous enrichments

hp-GFEM

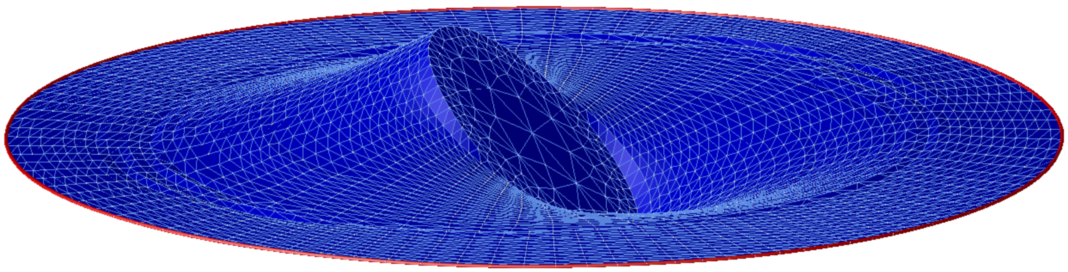


Von Mises stress

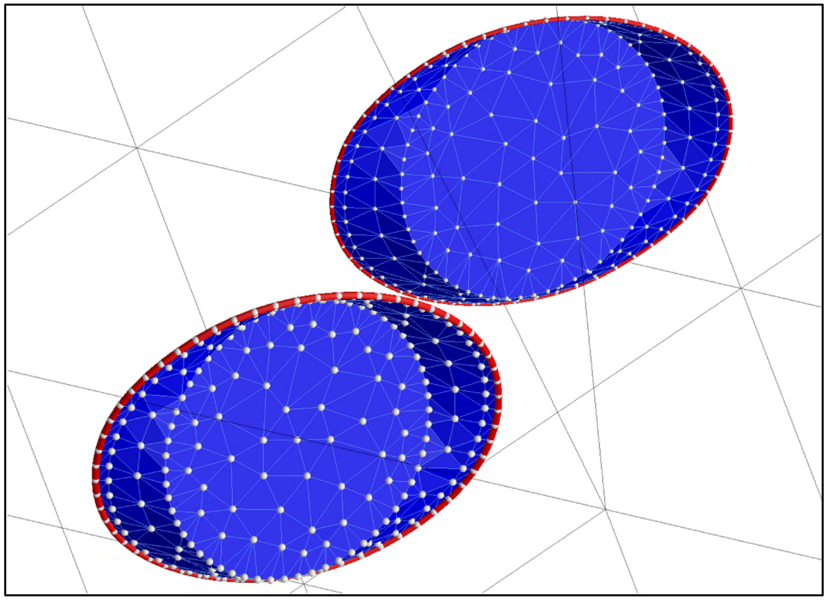
[Duarte et al., International Journal Numerical Methods in Engineering, 2007]

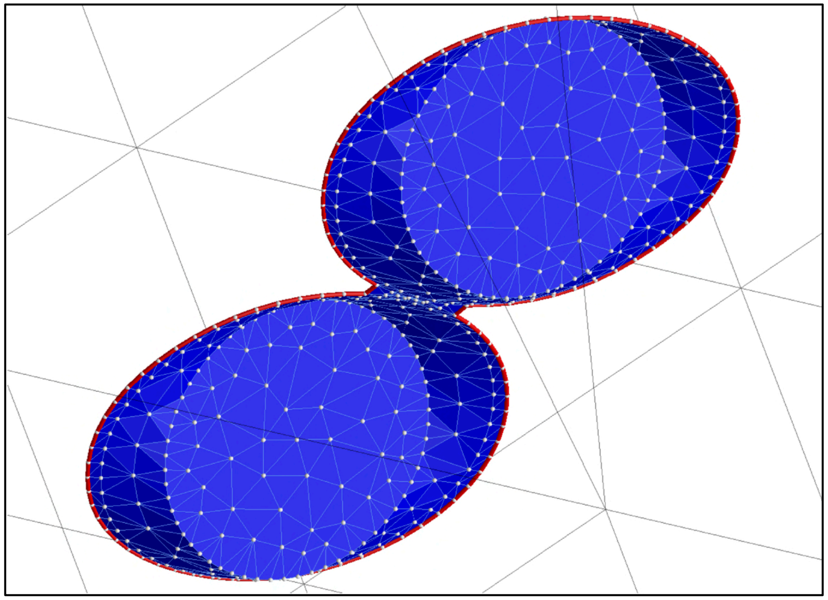
3D Fracture Surface Representation

- High-fidelity explicit representation of fracture surfaces [Duarte et al., 2001, 2009]



- Coalescence of fractures [Garzon et al., 2014]





Conditioning of GFEM Approximations

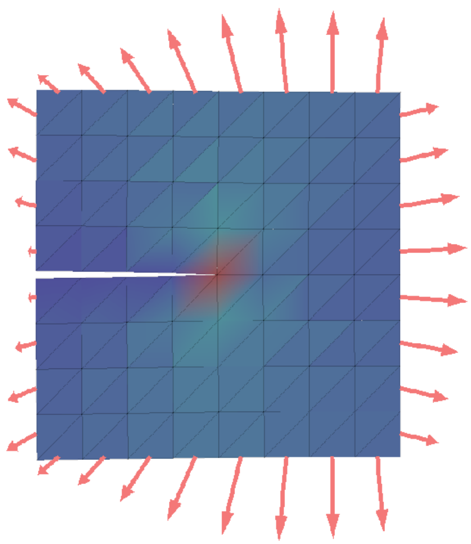
- The conditioning of the G/XFEM stiffness matrix, $\boldsymbol{K}_{GFEM}$, can be much worse than that of the standard FEM, $\boldsymbol{K}_{FEM}$

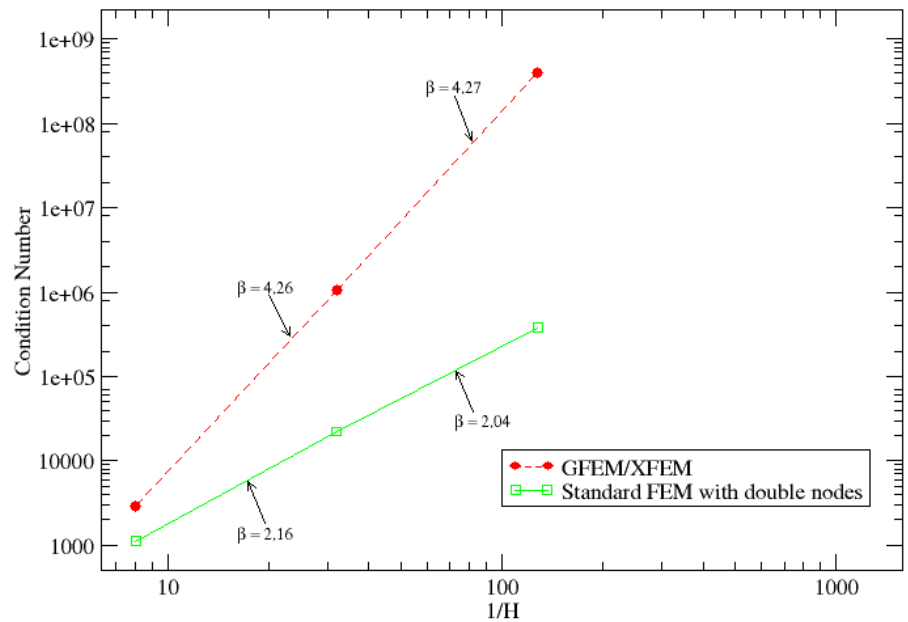
$$\mathfrak{K}(\boldsymbol{K}_{GFEM}) = \mathcal{O}(h^{-4})$$

while

$$\mathfrak{K}(\boldsymbol{K}_{FEM}) = \mathcal{O}(h^{-2})$$

where $\mathfrak{K}(.)$ is the *scaled condition number*.





SGFEM: Stable Generalized FEM

- The SGFEM involves simple local modifications of enrichments used in the GFEM

$$\tilde{L}_{\alpha j}(\boldsymbol{x}) = L_{\alpha j}(\boldsymbol{x}) - \mathrm{I}_{\omega_\alpha}(L_{\alpha j})(\boldsymbol{x})$$

where $\mathrm{I}_{\omega_\alpha}(L_{\alpha j})$ is the piecewise linear FE interpolant of $L_{\alpha j}$ on the patch ω_α

[Babuška & Banerjee CMAME 2012;

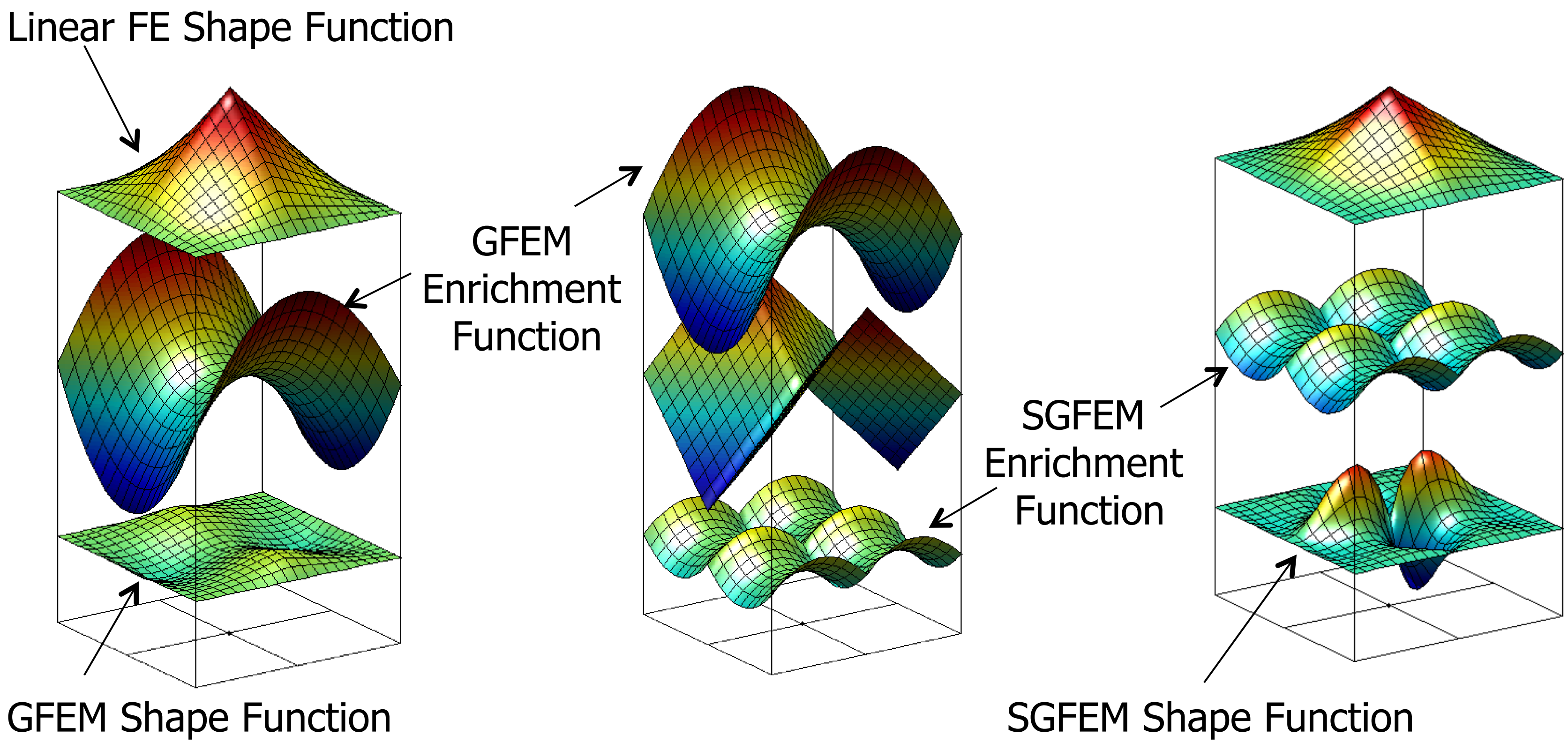
Gupta, Duarte, Babuška & Banerjee CMAME, 2013]

SGFEM: Stable Generalized FEM

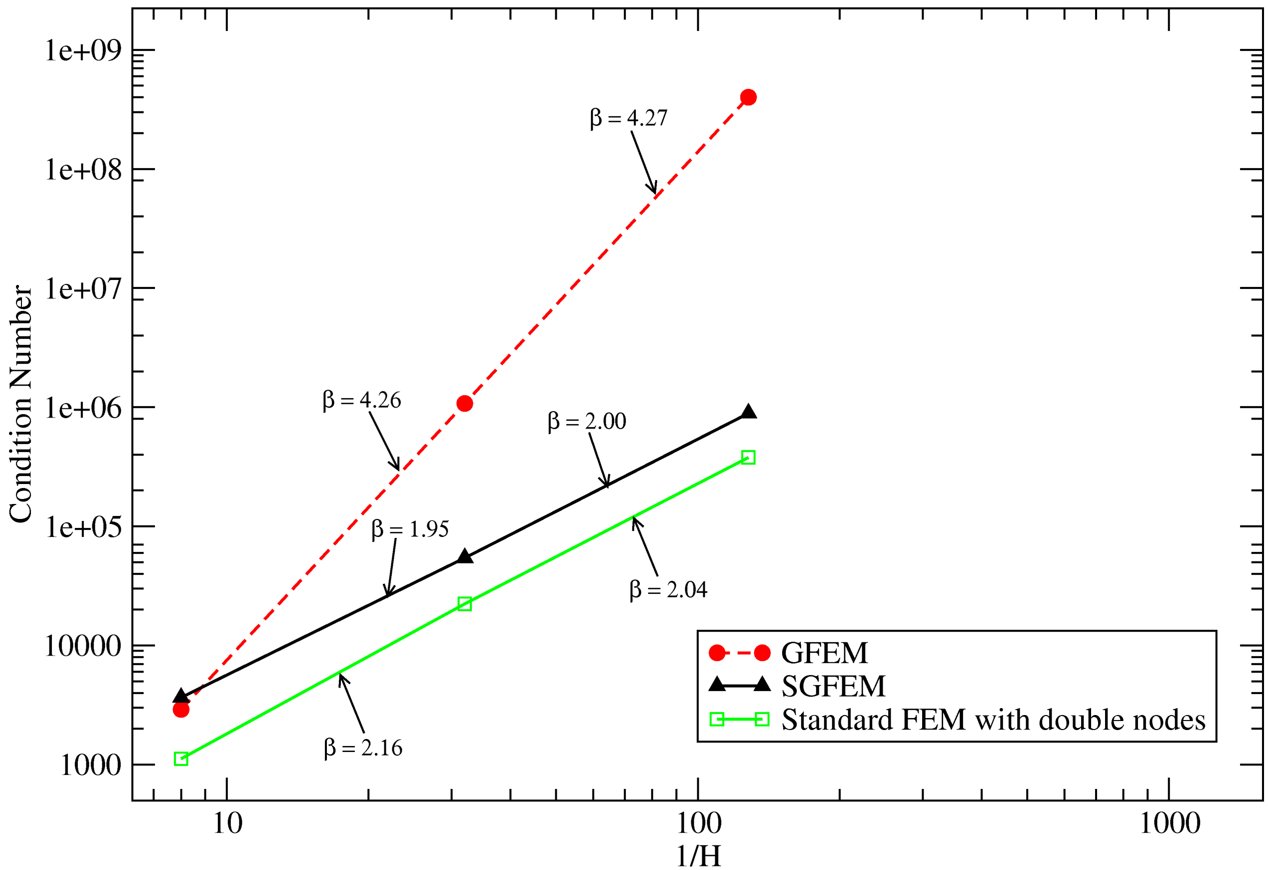
Modification of enrichment functions

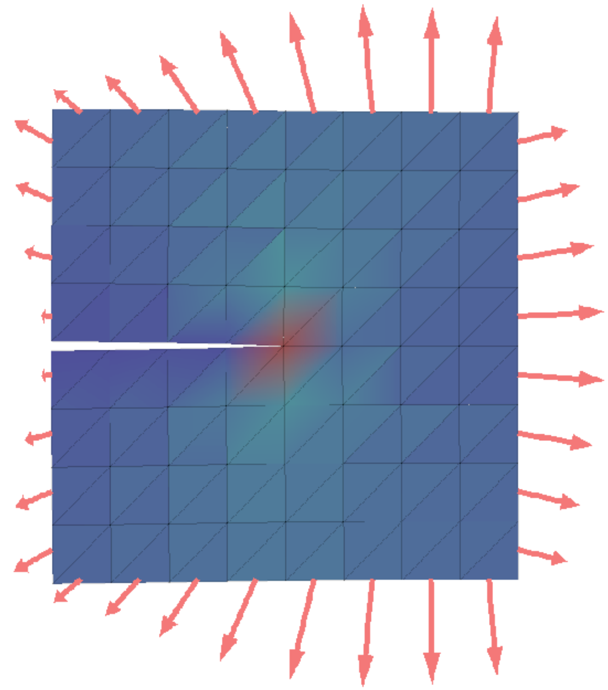
$$\tilde{L}_{\alpha i}(\boldsymbol{x}) = L_{\alpha i}(\boldsymbol{x}) - \mathrm{I}_{\omega_\alpha}(L_{\alpha i})(\boldsymbol{x})$$

$$\tilde{\phi}_{\alpha i}(\boldsymbol{x}) = \varphi_\alpha(\boldsymbol{x})\tilde{L}_{\alpha i}(\boldsymbol{x})$$



SGFEM: Stable Generalized FEM





Conditioning of GFEM/XFEM stiffness matrix $\mathcal{O}(h^{-4})$

Conditioning of SGFEM and FEM stiffness matrix $\mathcal{O}(h^{-2})$

[Gupta, Duarte, Babuska & Banerjee CMAME, 2013]

Outline

- Motivation and limitations of standard finite element methods
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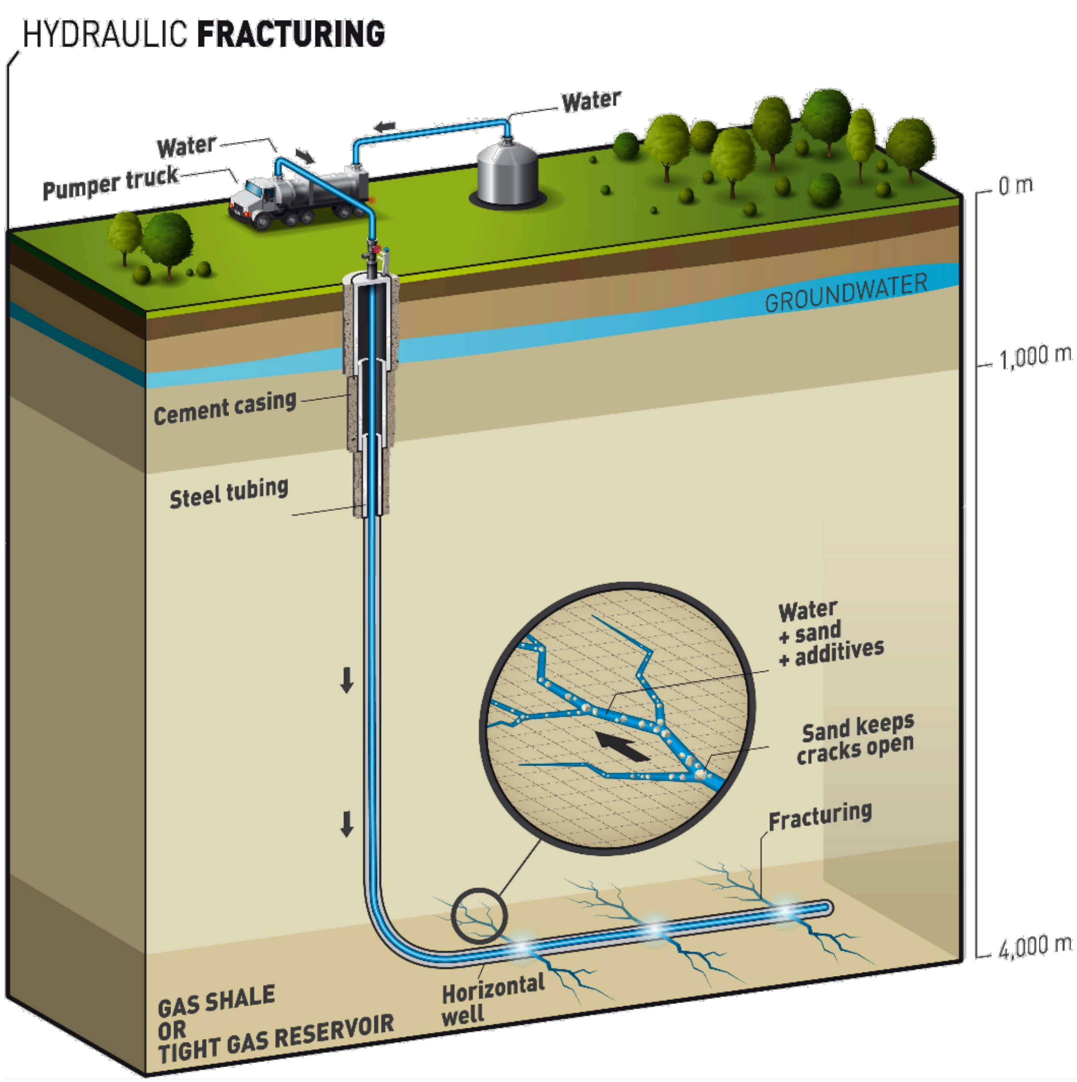
What is Hydraulic Fracturing?



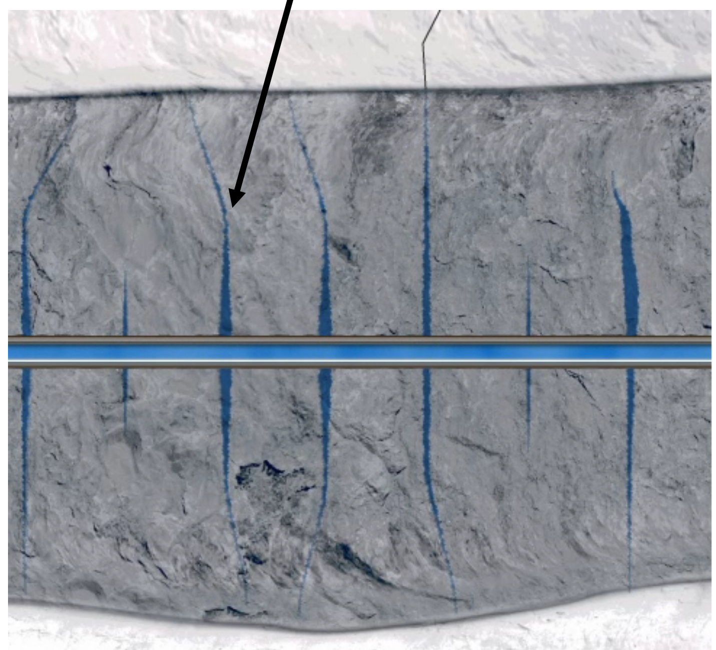
Video

Graham Roberts, New York Times, http://www.nytimes.com/interactive/2011/02/27/us/fracking.html

Hydraulic Fracturing Simulation

Current Focus: 3-D effects not captured by available simulators

- Initial stages of fracture propagation: Fracture re-orientation, interaction and <u>coalescence</u>



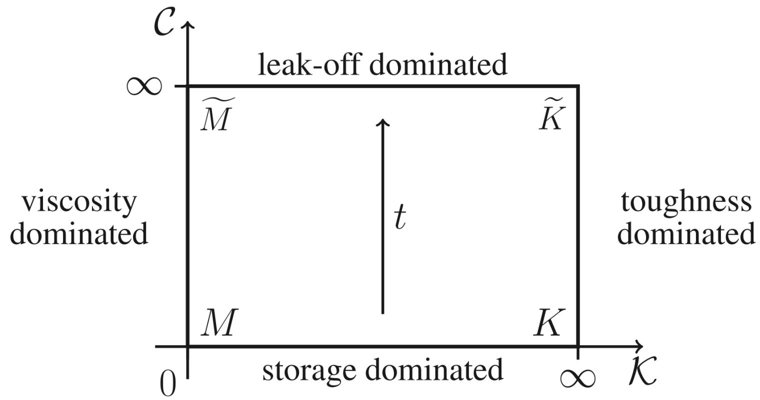
Strategy: Generalized Finite Element Methods

Selection of Enrichment Functions: Hydraulic Fracturing Regimes

- Fracture propagation is governed by
  - two competing energy dissipation mechanisms: Viscous flow and fracturing process;
  - two competing storage mechanisms: In the fracture and in the porous matrix

Dimensionless toughness
$$\mathcal{K} = \frac{4K_{Ic}}{\sqrt{\pi}} \left(\frac{1}{3Q_0 E'^3 \mu} \right)^{1/4}$$

Leak-off coefficient
$$\mathcal{C} = 2C_L \left(\frac{E' t}{12 \mu Q_0^3} \right)^{1/6}$$



Hydraulic fracture parametric space*

Current Focus: Storage-toughness dominated regime

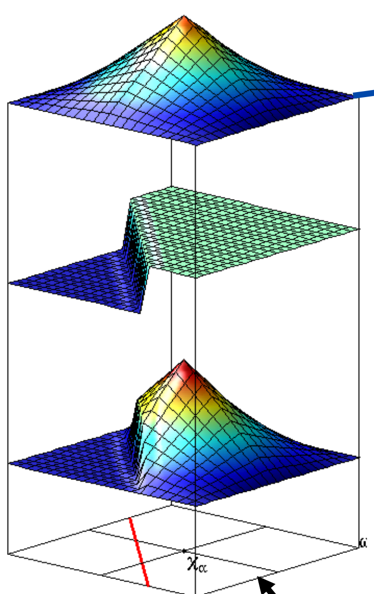
- Low permeability reservoirs: Neglect flow of hydraulic fluid across fracture faces:
  - <u>Storage dominated</u> regime
- High confining stress (no fluid lag) and low viscosity fluid (water):
  - Near constant fluid pressure in fracture; <u>Toughness dominated</u> regime
- Brittle elastic material

*[Carrier & Granet, EFM, 2013]

Selection of Enrichment Functions: Hydraulic Fracturing Regimes

Enrichments for toughness-dominated regime:

$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



patch ω_α

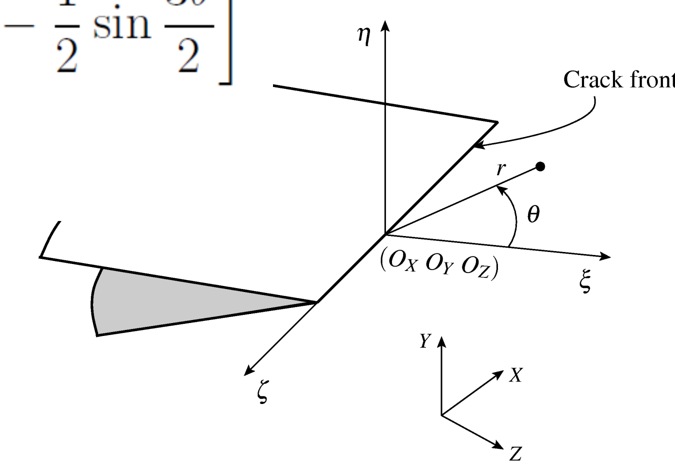
$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

[Duarte & Oden 1996]

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

Valid for toughness-dominated problems



Weak Form at Propagation Step k

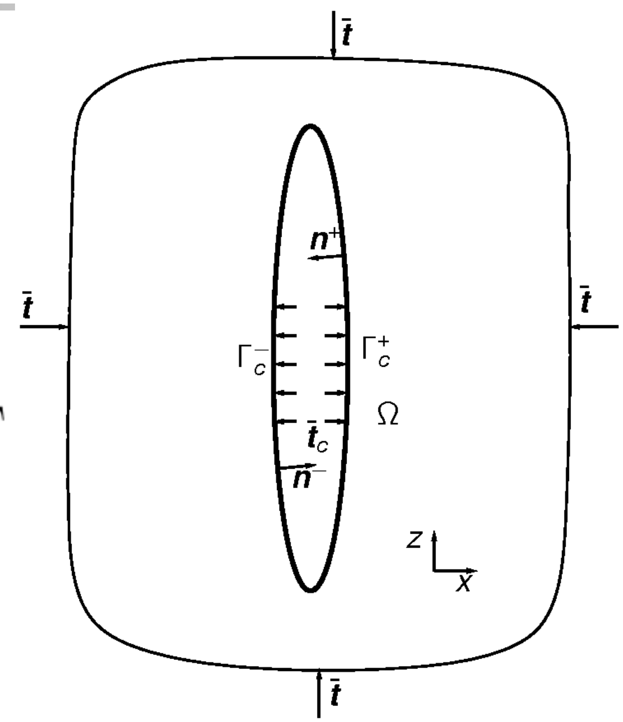
Find $\boldsymbol{u}^k \in H^1(\Omega)$, such that $\forall\ \boldsymbol{v}^k \in H^1(\Omega)$

$$\int_\Omega \boldsymbol{\sigma}(\boldsymbol{u}^k) : \boldsymbol{\varepsilon}(\boldsymbol{v}^k) d\Omega$$

$$= \int_\Omega \boldsymbol{b} \cdot \boldsymbol{v}^k d\Omega + \int_{\partial\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{v}^k d\Gamma + \int_{\Gamma_c^{k+}} \bar{\boldsymbol{t}}_c^{k+} \cdot [\![\boldsymbol{v}^k]\!] d\Gamma$$

where $[\![\boldsymbol{v}^k]\!]$ is the virtual displacement jump across the crack surface Γ^k at propagation step k and

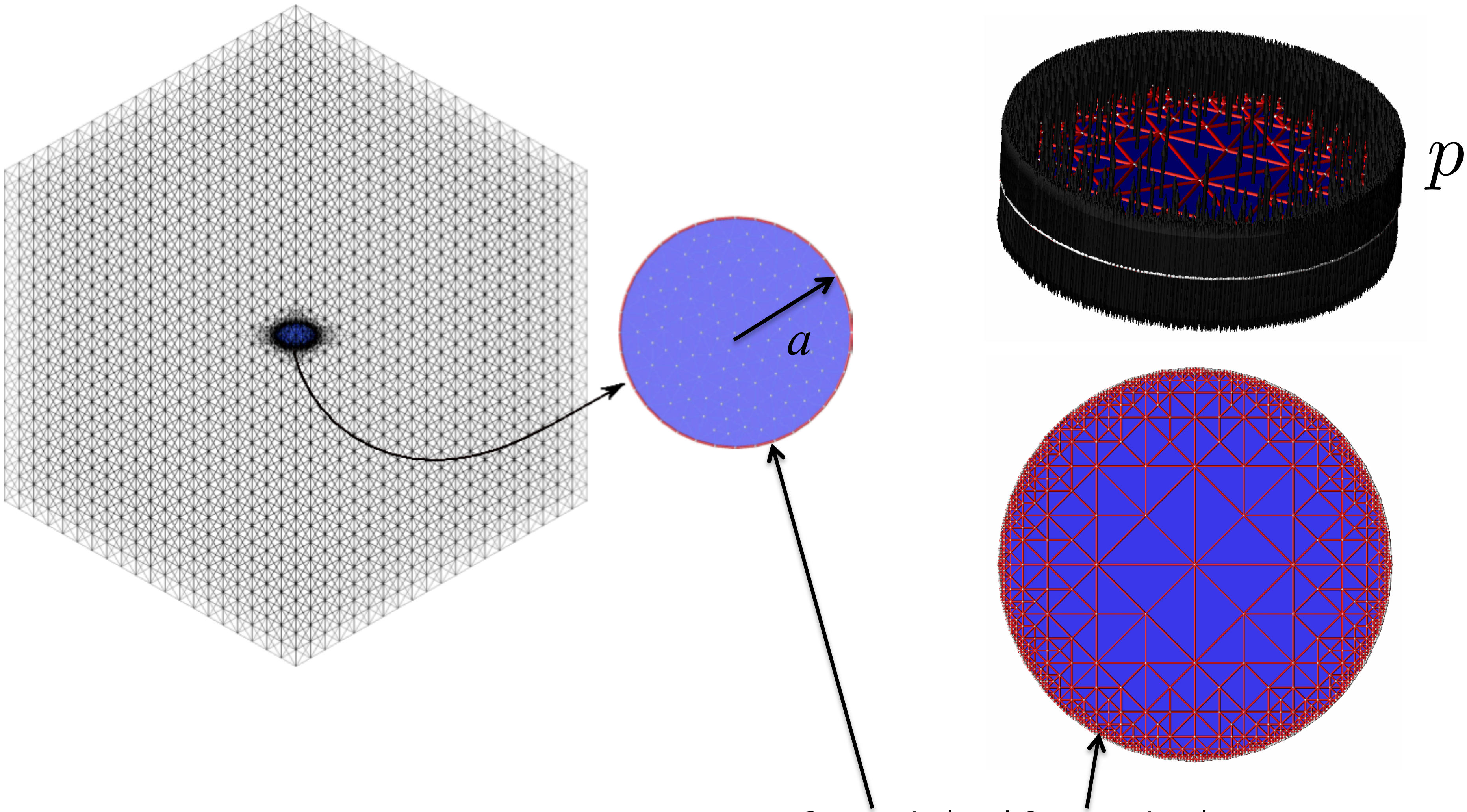
$$\bar{\boldsymbol{t}}_c^{k+} = -p^k \boldsymbol{n}^{k+} = p^k \boldsymbol{n}^{k-}$$



Cross section of fracture

- Pore fluid flow not considered
- Toughness-dominated problem: Pressure on fracture is assumed constant

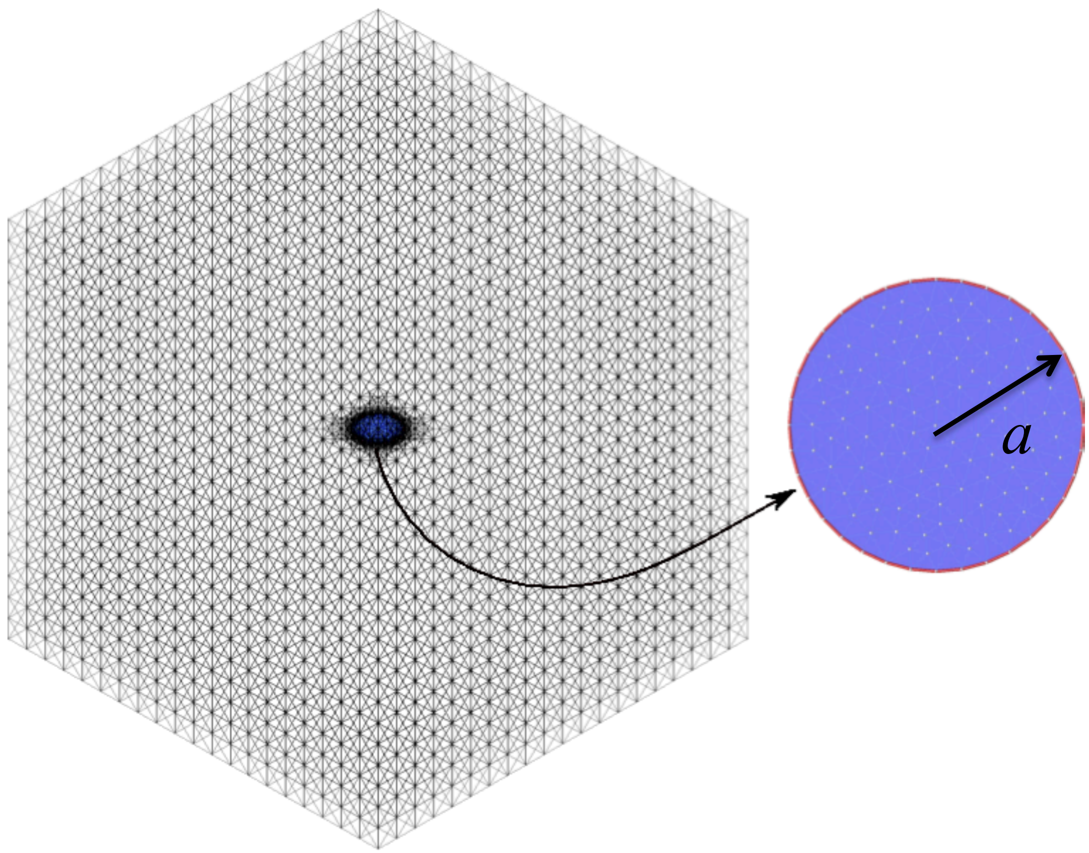
Verification: Propagation of Circular Fracture*



Geometrical and Computational fracture surface loaded with fluid pressure *p*

*[Gupta & Duarte, 2014]

Verification: Propagation of Circular Fracture



Goal: Find critical pressure

$$p_c(a) = \left(\frac{E^* G_c \pi}{4a} \right)^{1/2}$$

Adopt [Bourdin et al. 2012]:

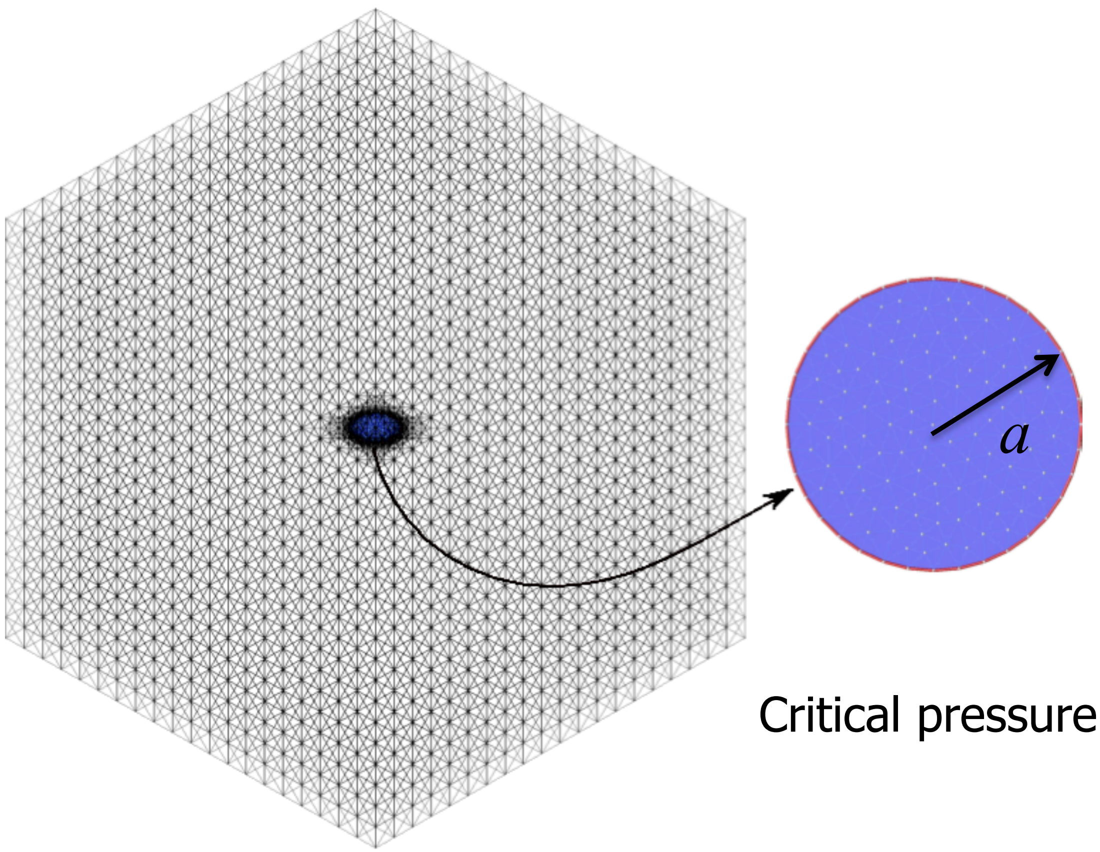
$$E^* = 1$$

$$G_c = 1.91 \times 10^{-9}$$

$$a = 0.5$$

$$p_c(0.5) = 5.477 \times 10^{-5}$$

Propagation of Circular Fracture



GFEM Model

$$h_{\min}/a = 0.016$$

$$h_{\max}/a = 0.027$$

$$p\text{-}order = 2$$

$$N = 215\,376 \;\; dofs$$

$$T = 5.25 \;\; min$$

Critical pressure

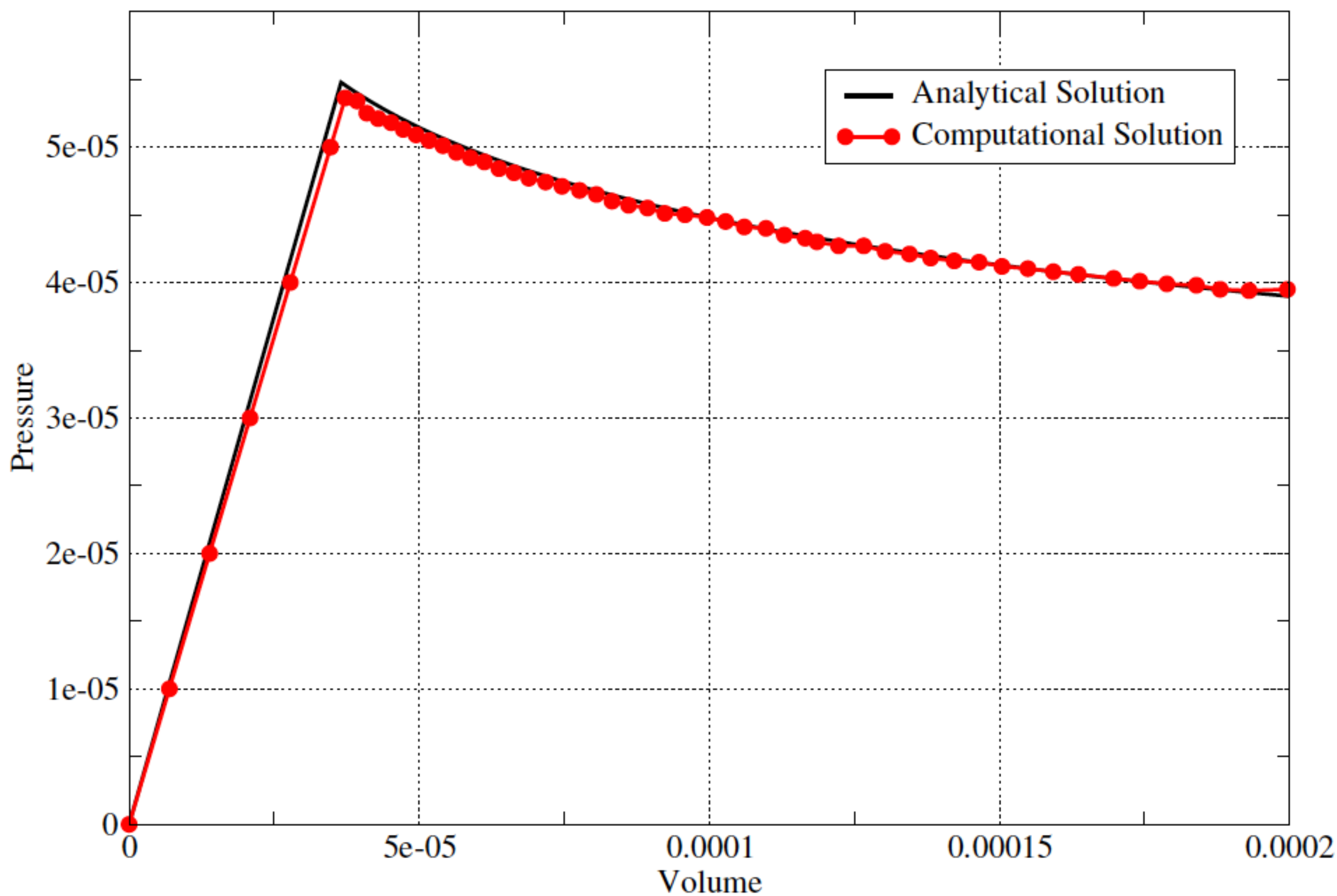
$$p_c^h(a) = \frac{K_c}{K(a)} p$$

$$p_c^h(0.5) = 5.415 \times 10^{-5}$$

$$e_r(p_c) = 1.15\%$$

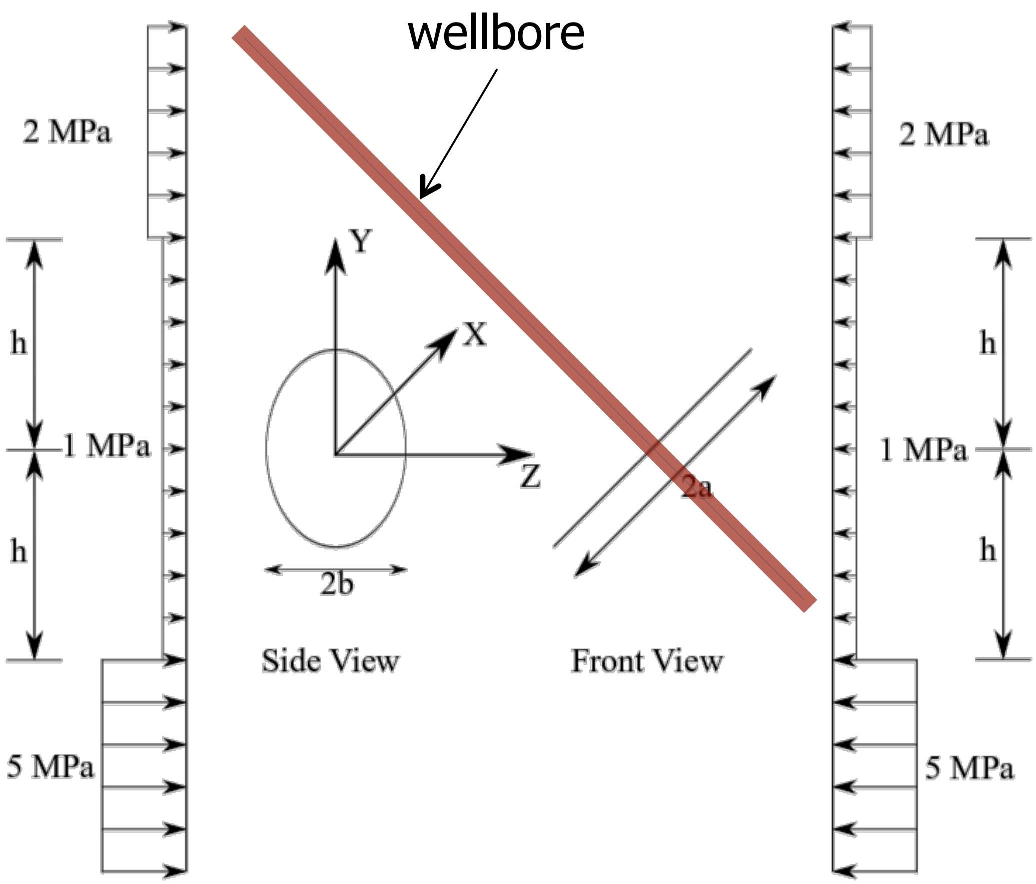
Propagation of Circular Fracture

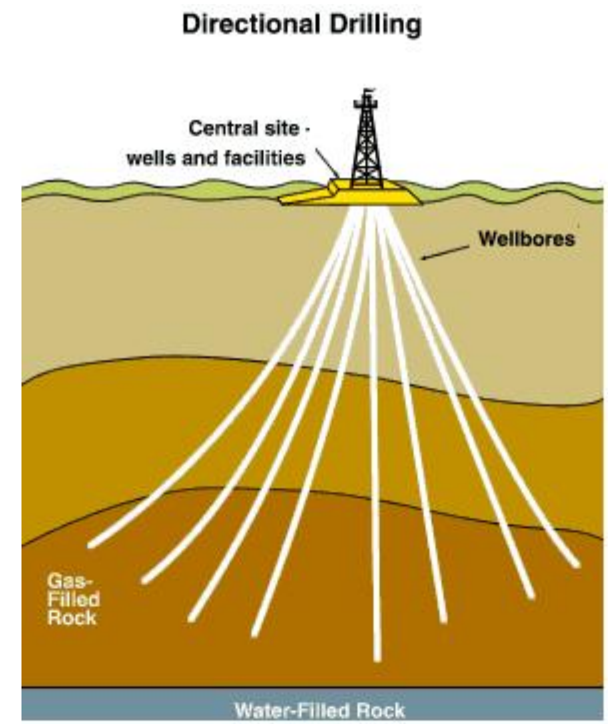
Incrementally advance fracture and repeat previous approach



Application: Fracture Re-Orientation*

- Fracture starts in a direction not perpendicular to minimum *in-situ* stress
- Misalignment of fracture and confining in-situ stresses

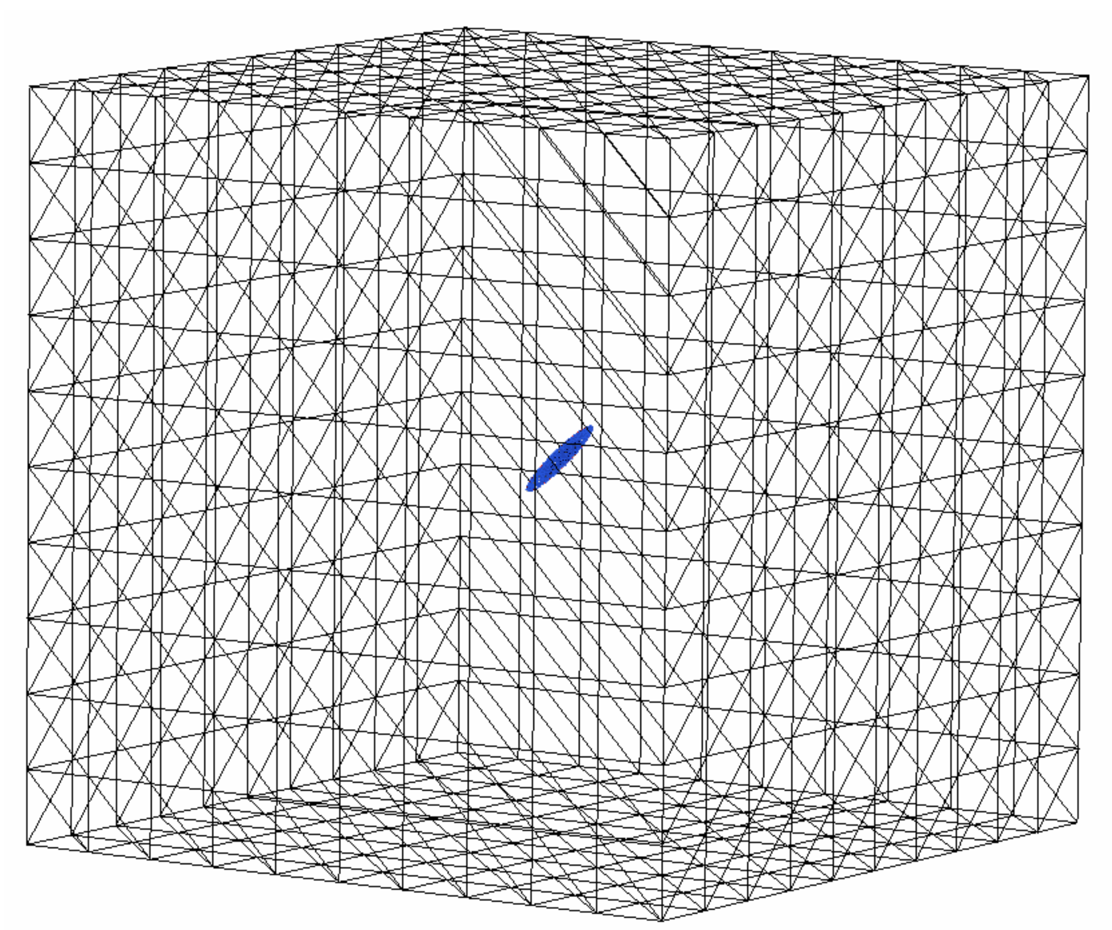


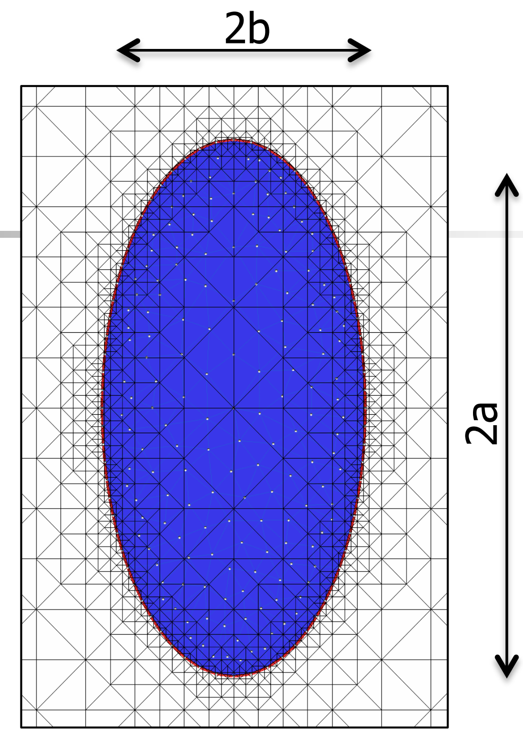


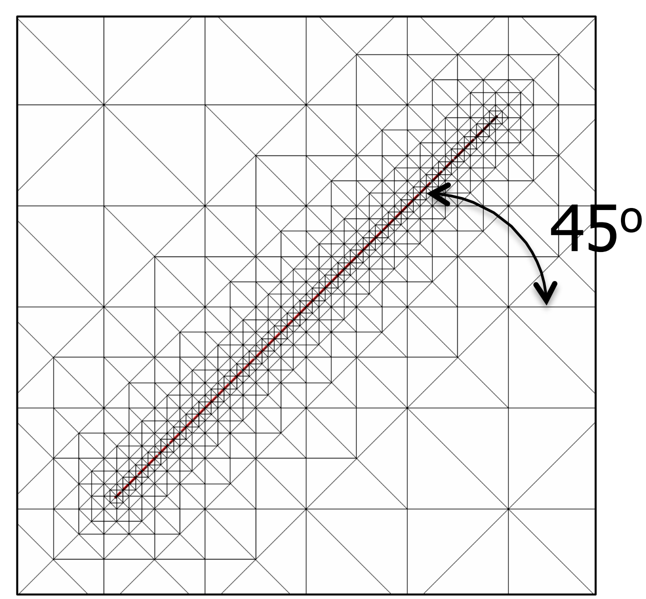
a = 10m
b = 5m
h = 15m
p = 3.5 MPa

*[Rungamornrat et al., 2005; Gupta & Duarte, 2014]

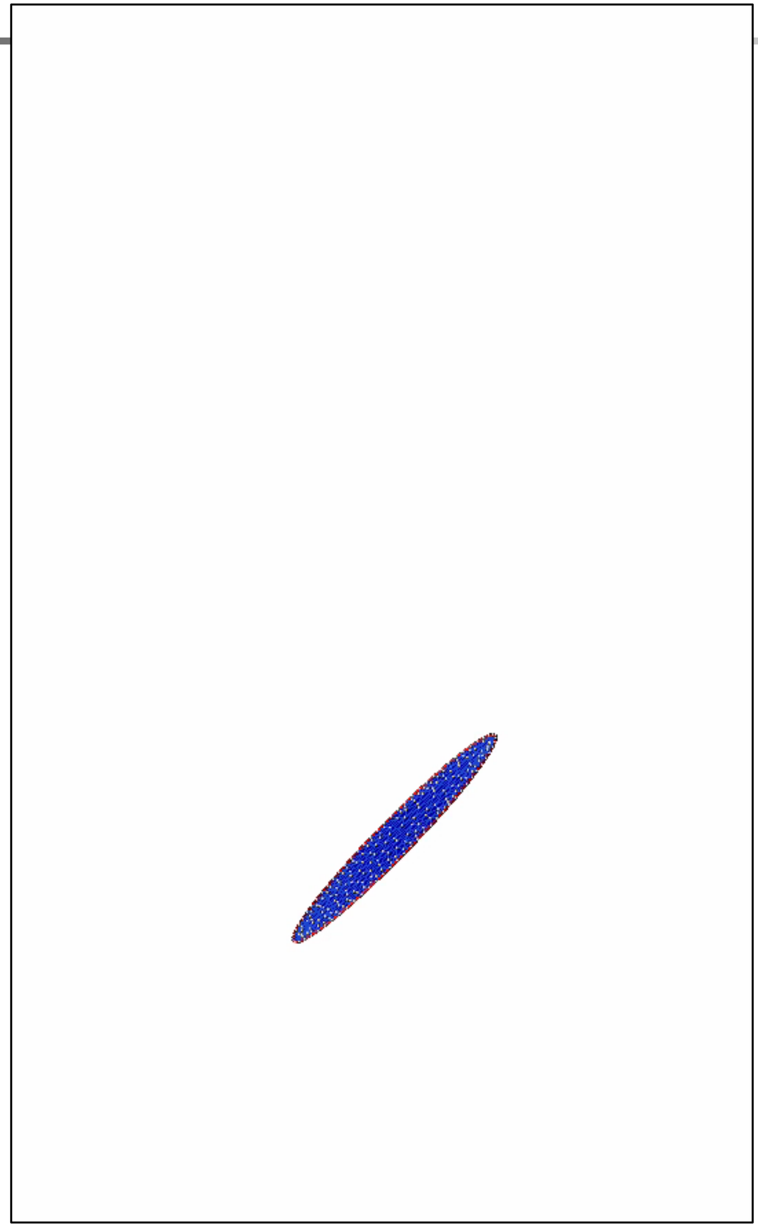
Fracture Re-Orientation



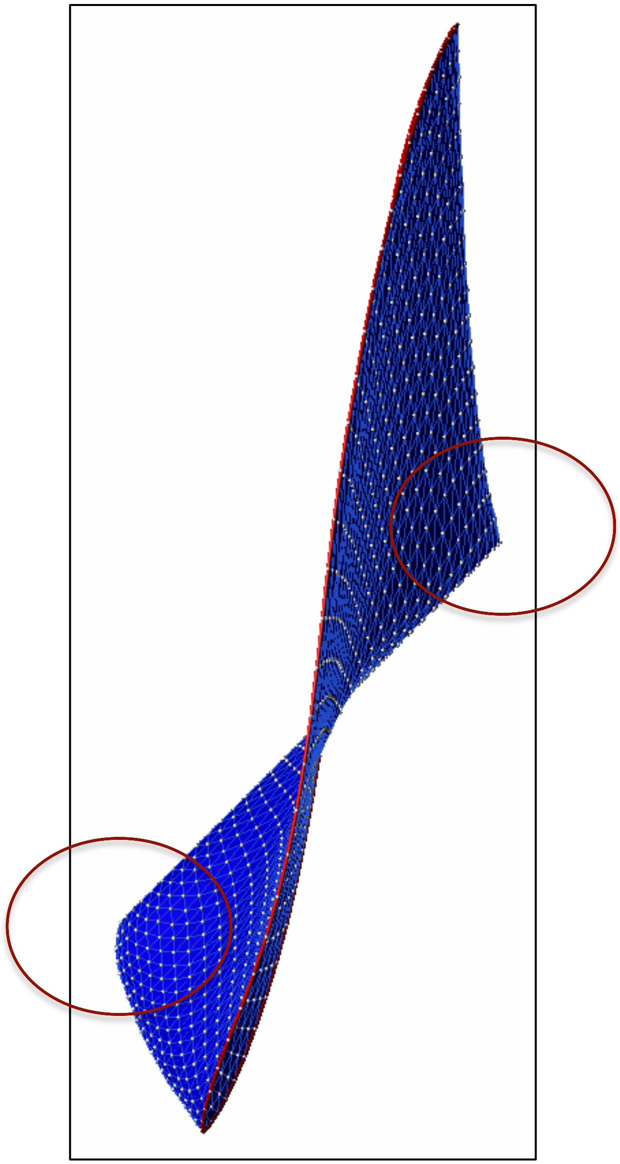


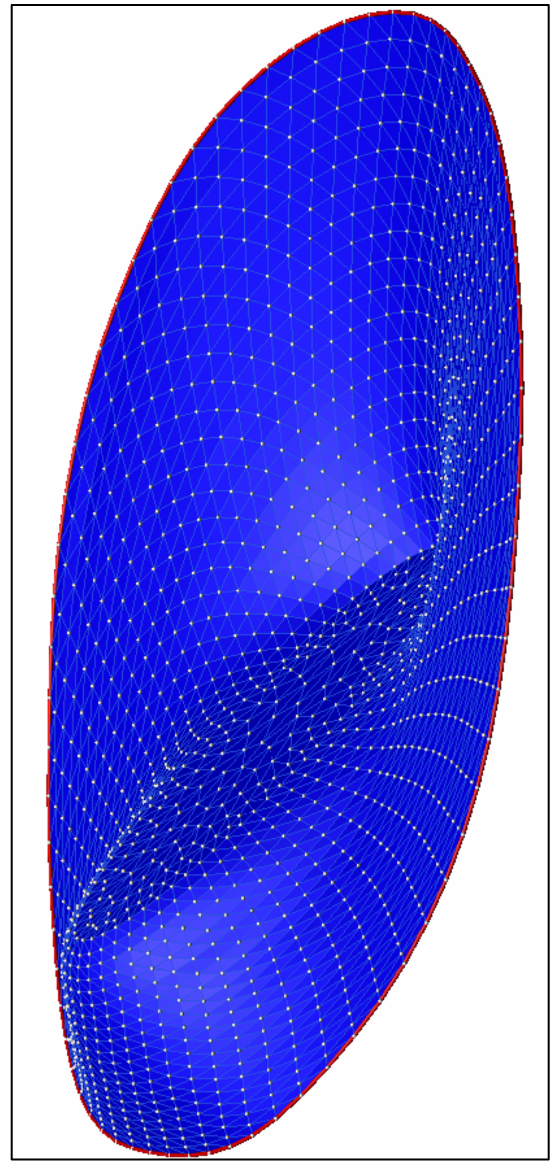


Fracture Re-Orientation

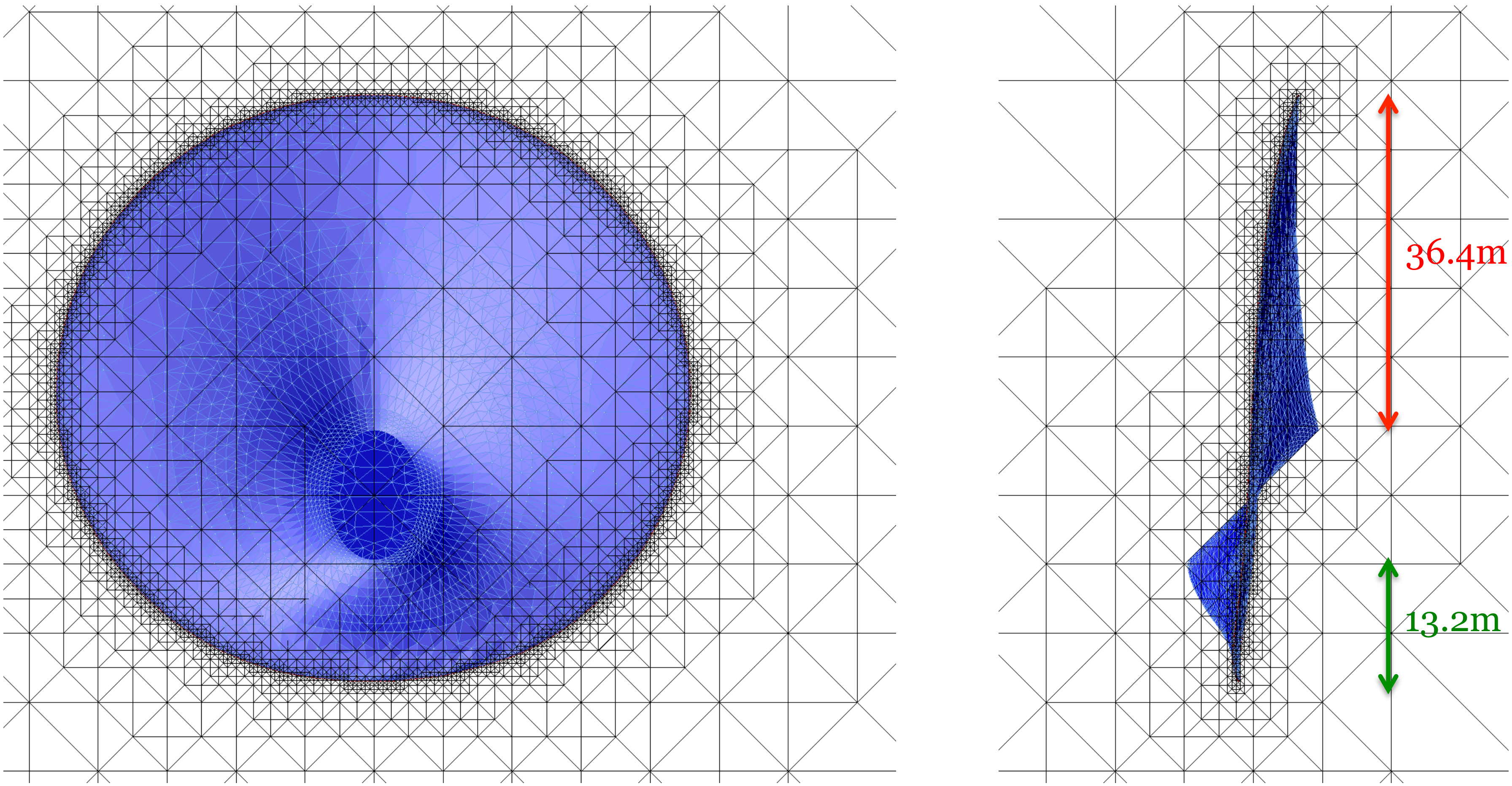


Fracture Re-Orientation: Step 20



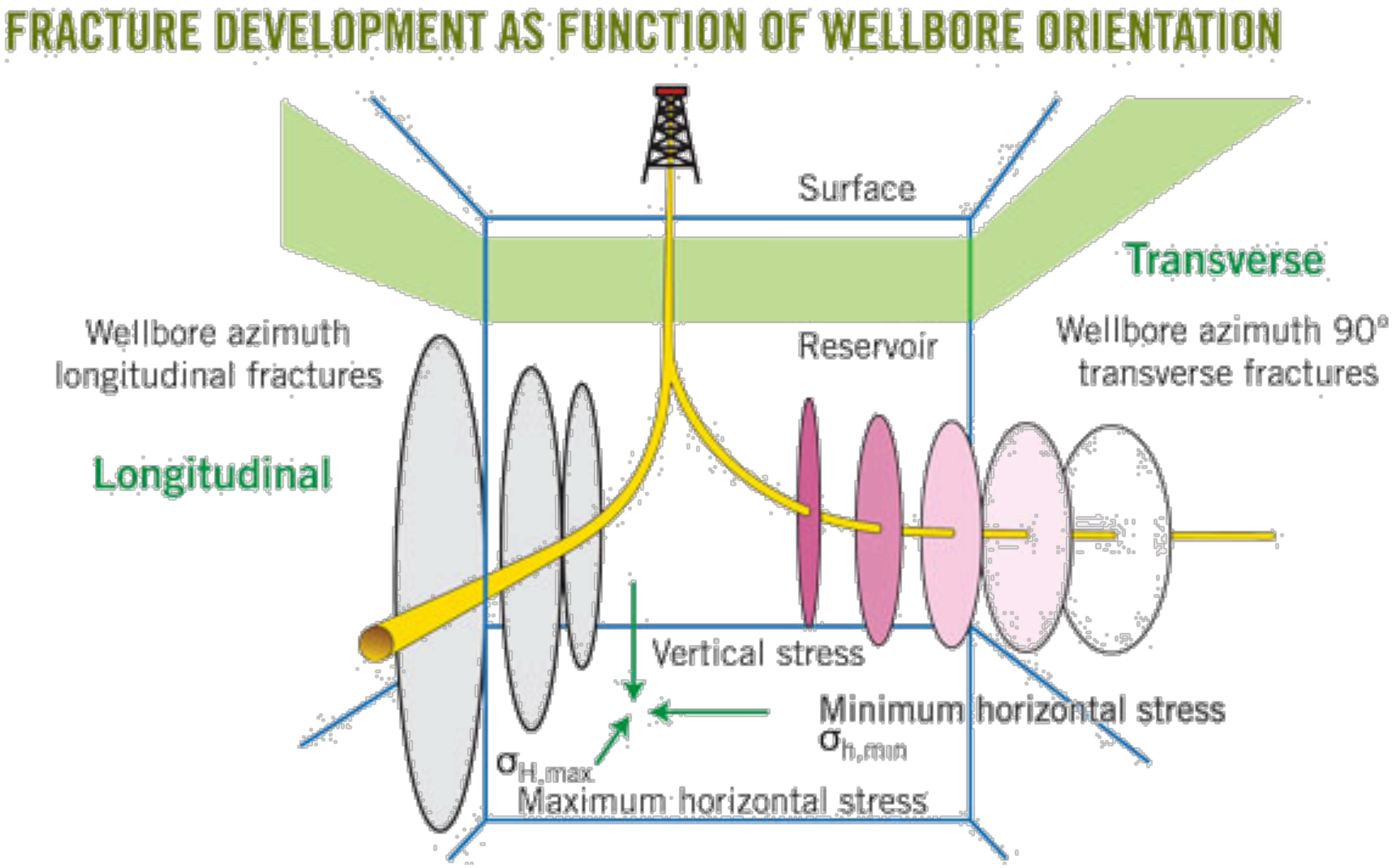


Fracture Re-Orientation: Adaptive Mesh



- Adaptive refinement along fracture front
- Sharp features are preserved
- High fidelity of fracture surface, regardless of computational mesh

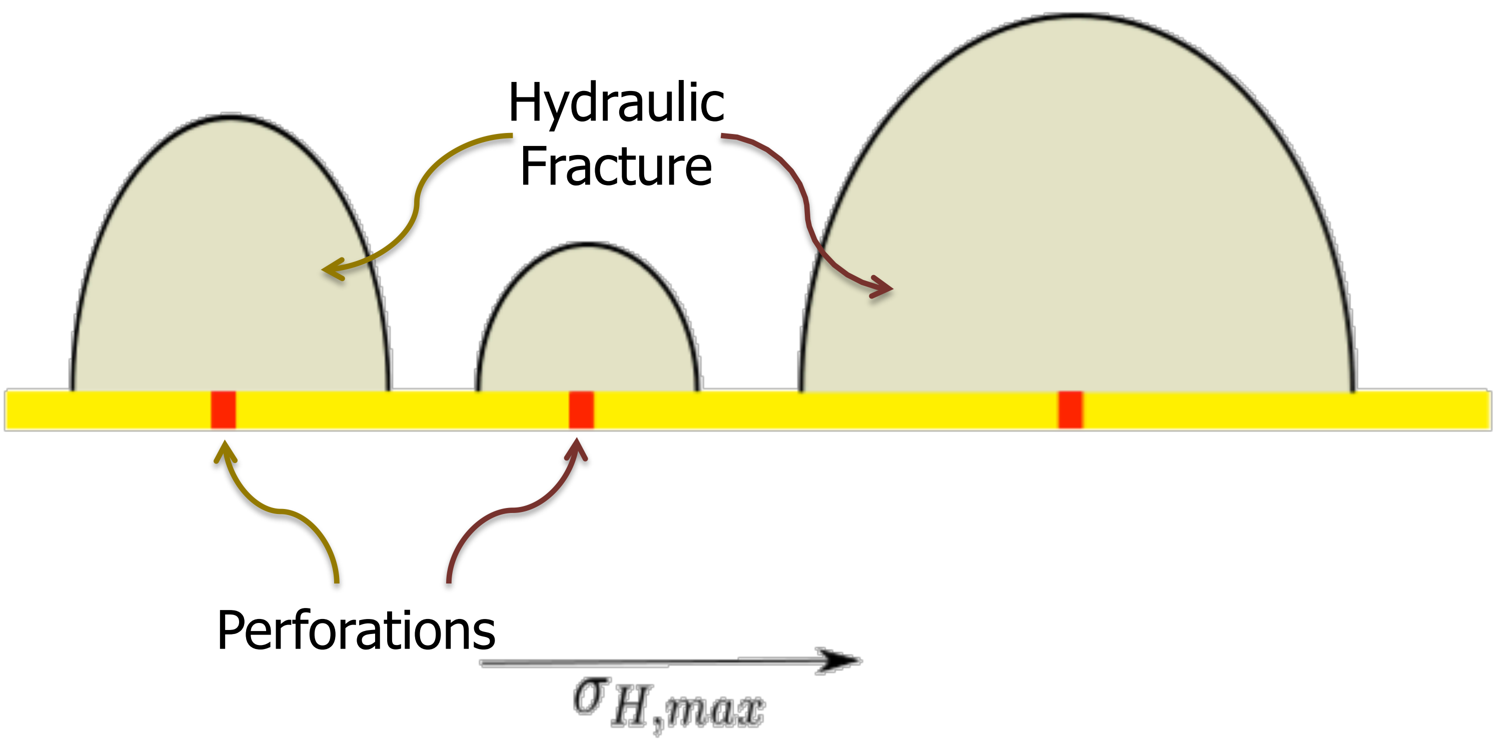
Typical Hydraulic Fracturing



[Z. Rahim et al., 2012]

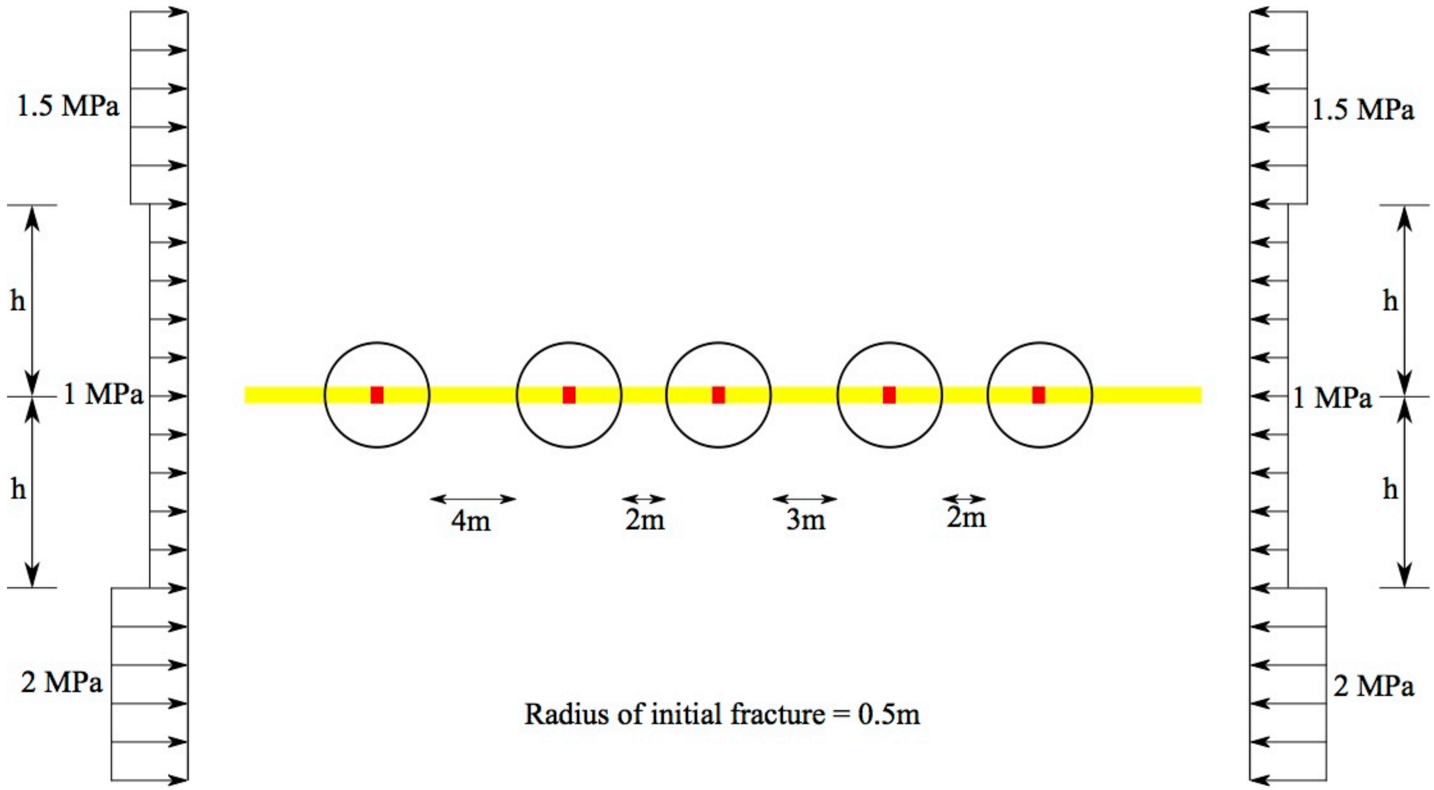
Longitudinal Fractures

- Develop perpendicular to minimum in-situ stress
- Fractures along the length of the wellbore
- Planar fractures from the perforation



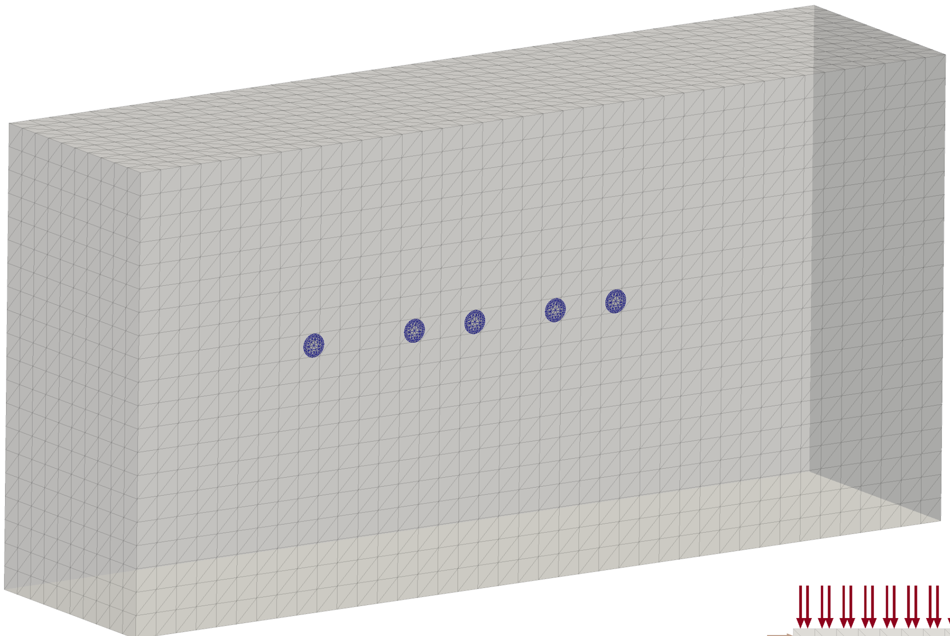
Coalescence of Longitudinal Fractures

- Propagation and coalescence from a horizontal well

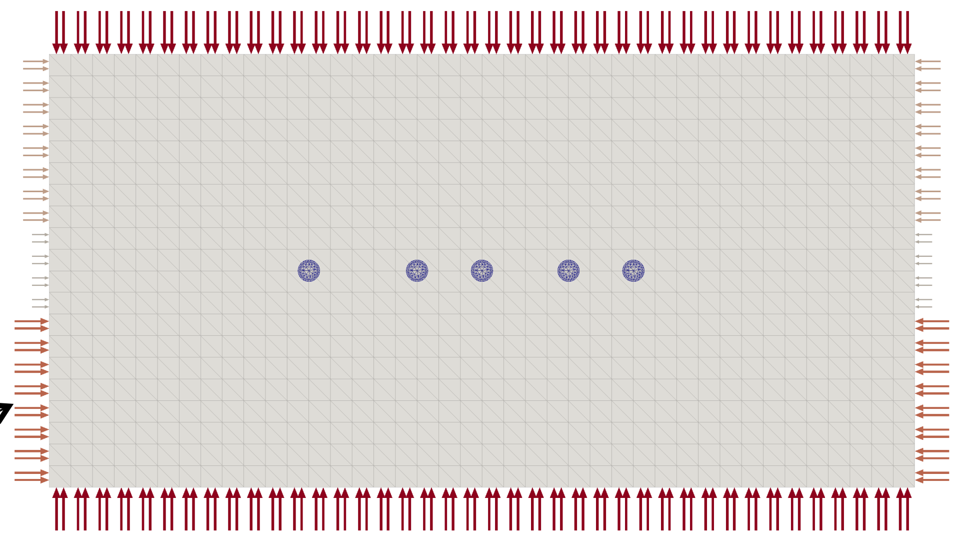


h = 2 m
p = 3.5 MPa

Coalescence of 3-D Fractures: GFEM Model

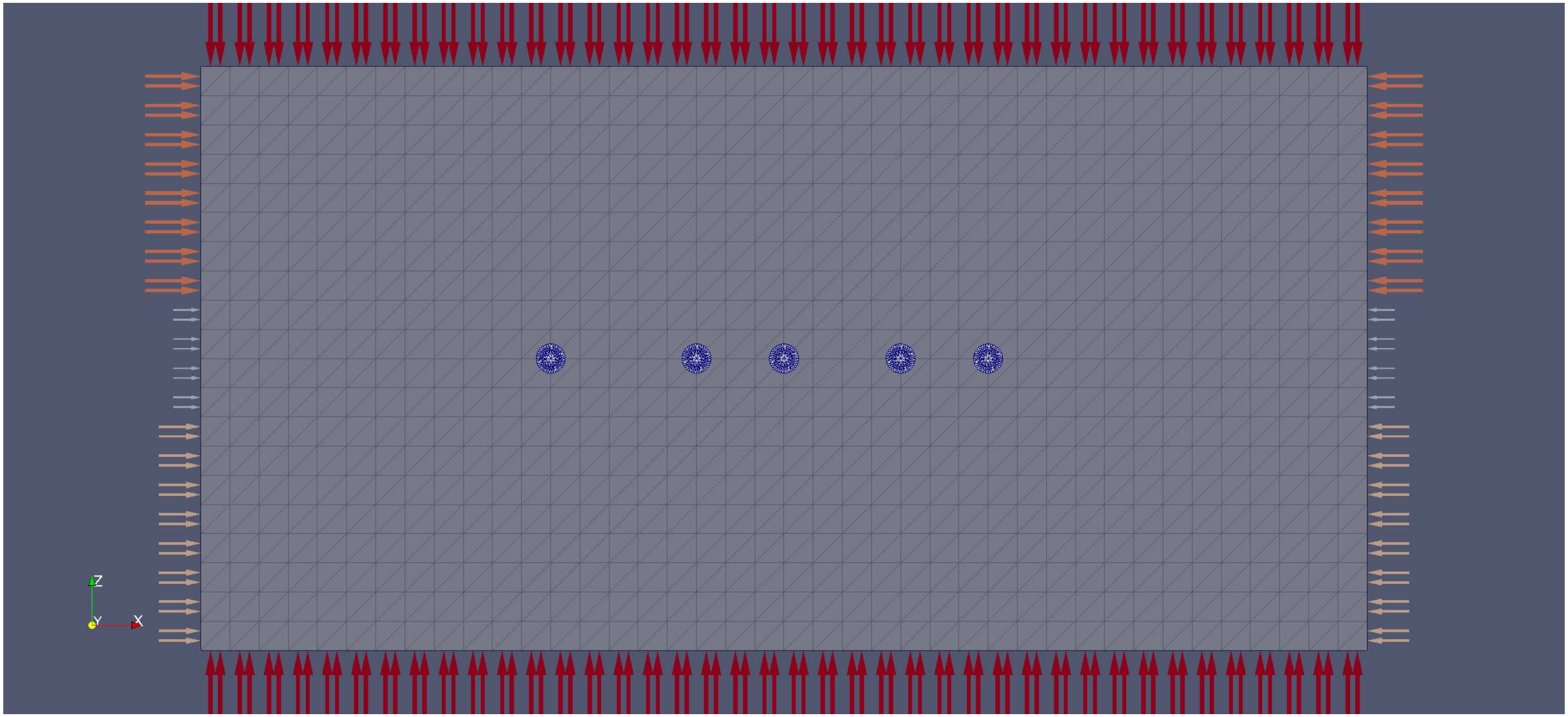


- Input mesh and fracture surfaces for GFEM simulation
- Automatic adaptive mesh refinement performed at each propagation step



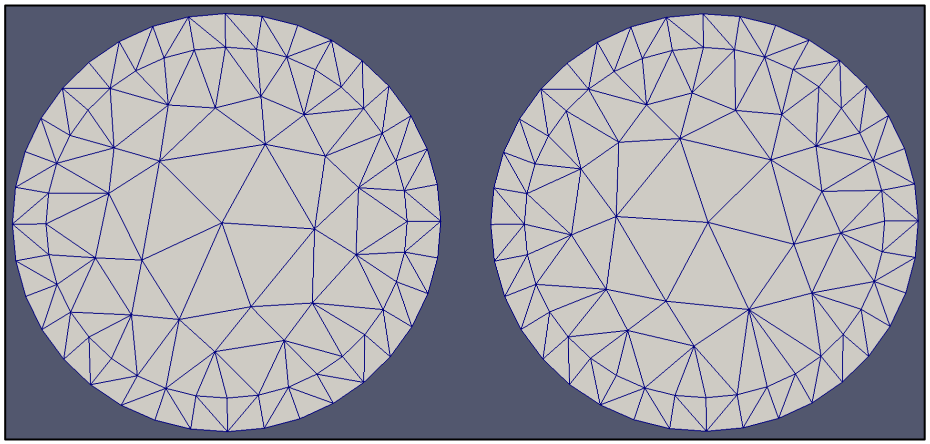
In-situ stress
(all around)

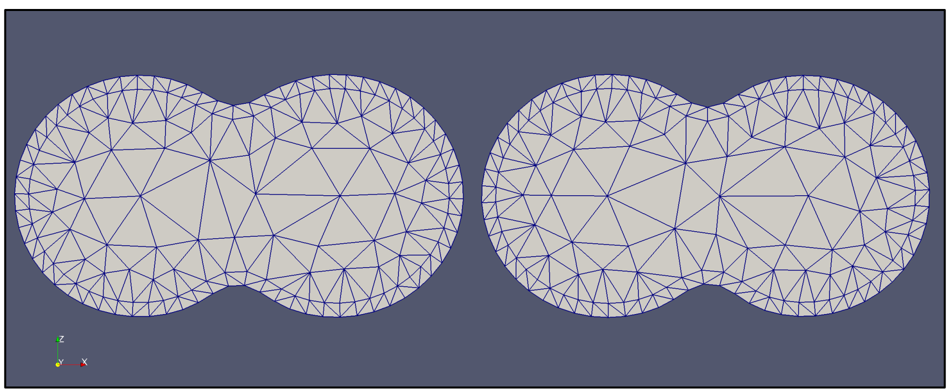
Coalescence of 3-D Fractures



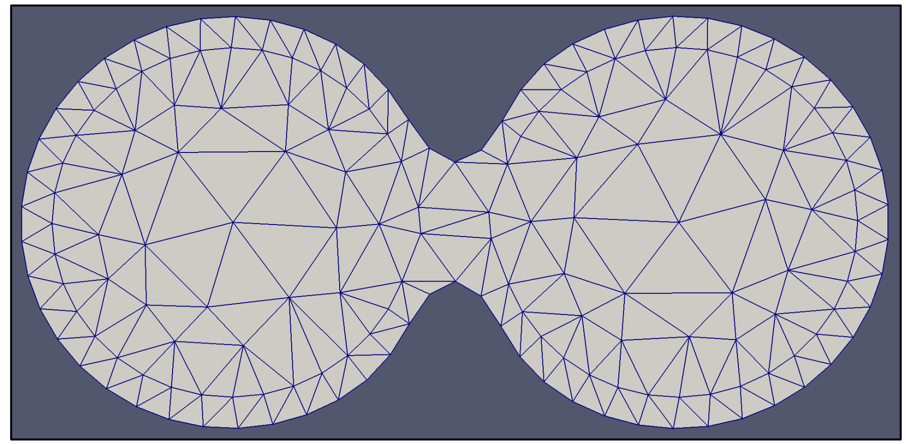
Coalescence of 3-D Fractures

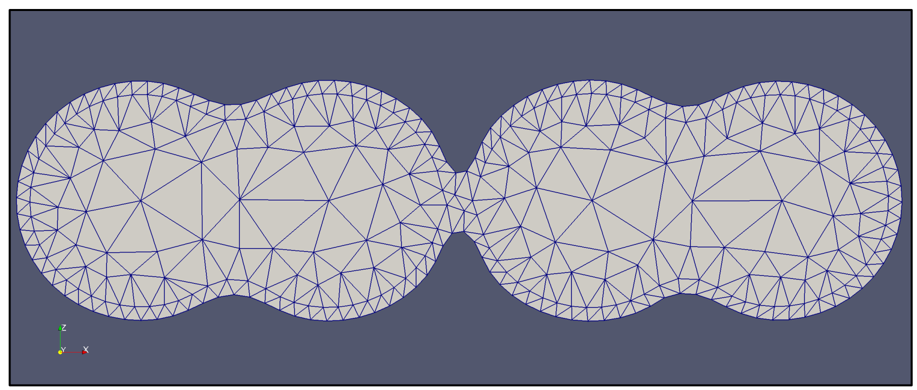
Fractures just prior to coalescence





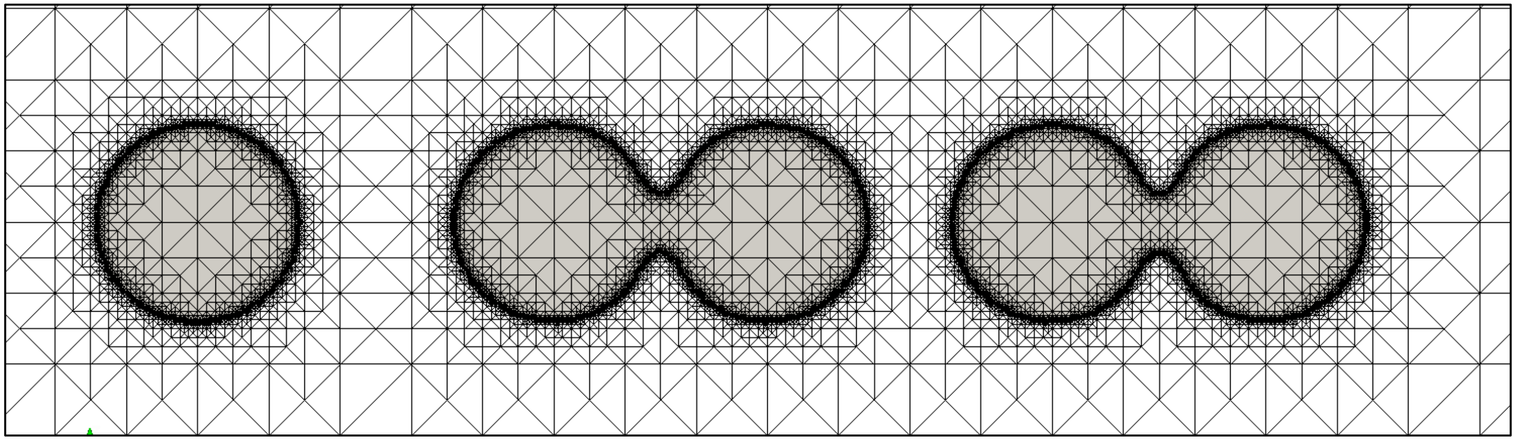
Fractures just after coalescence

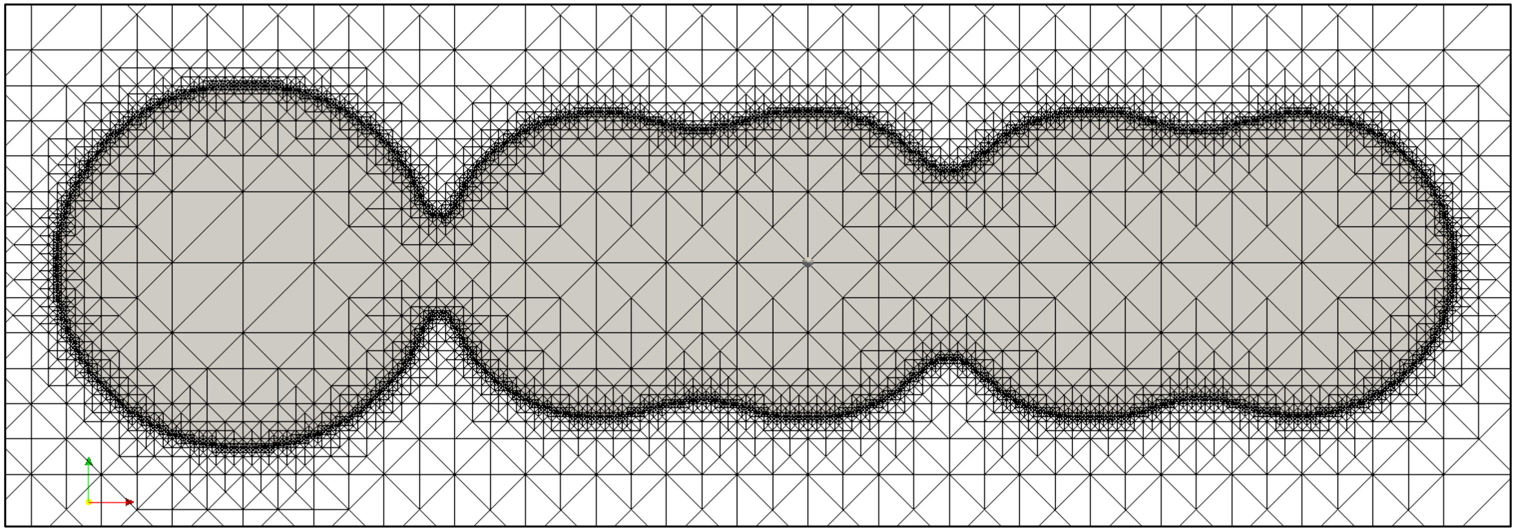




Coalescence of 3-D Fractures

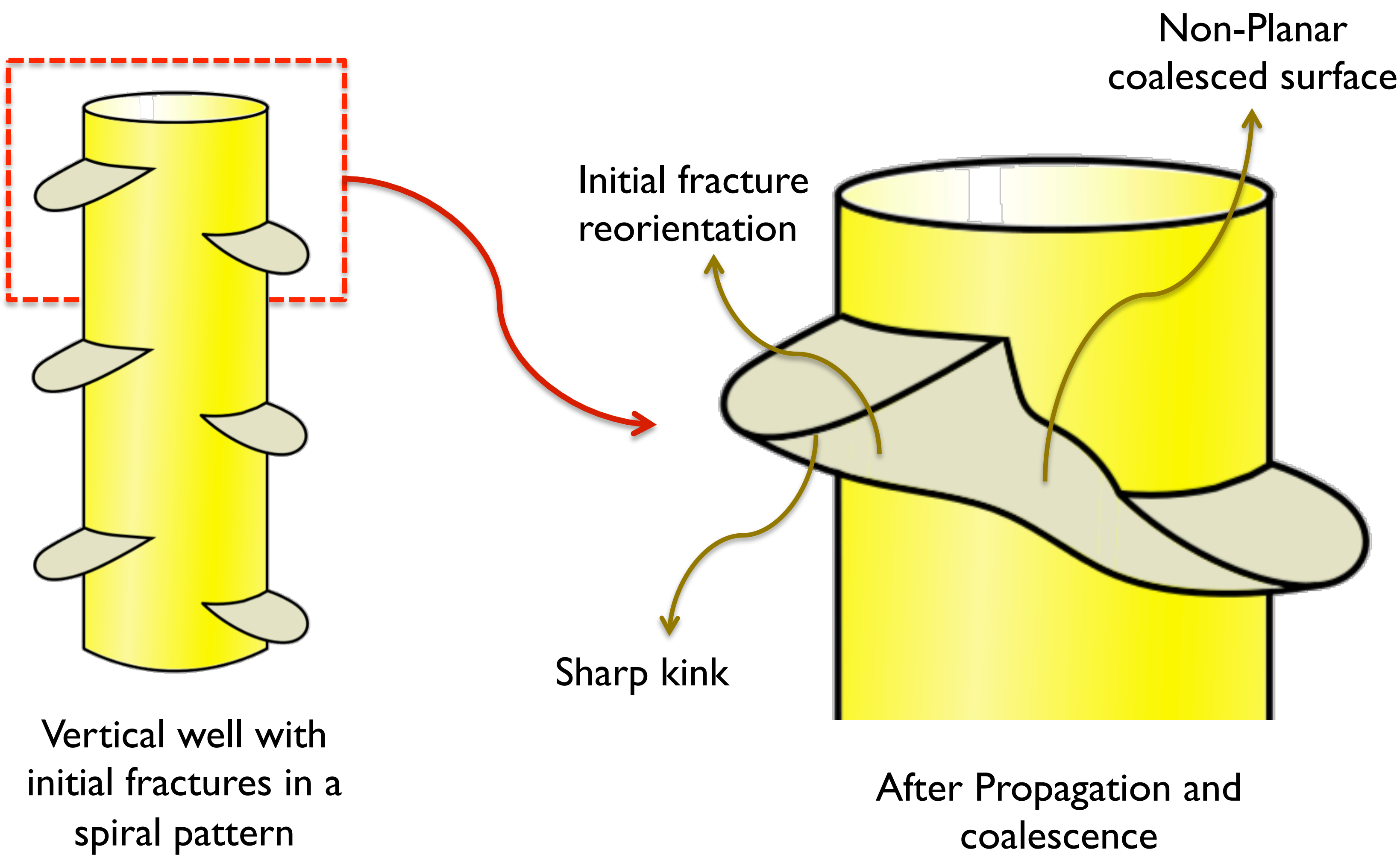
- Adaptive refinement along fracture fronts



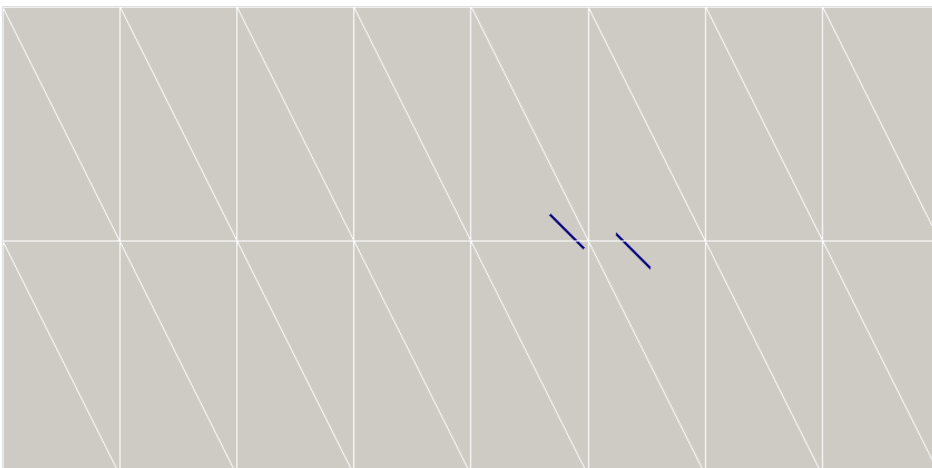


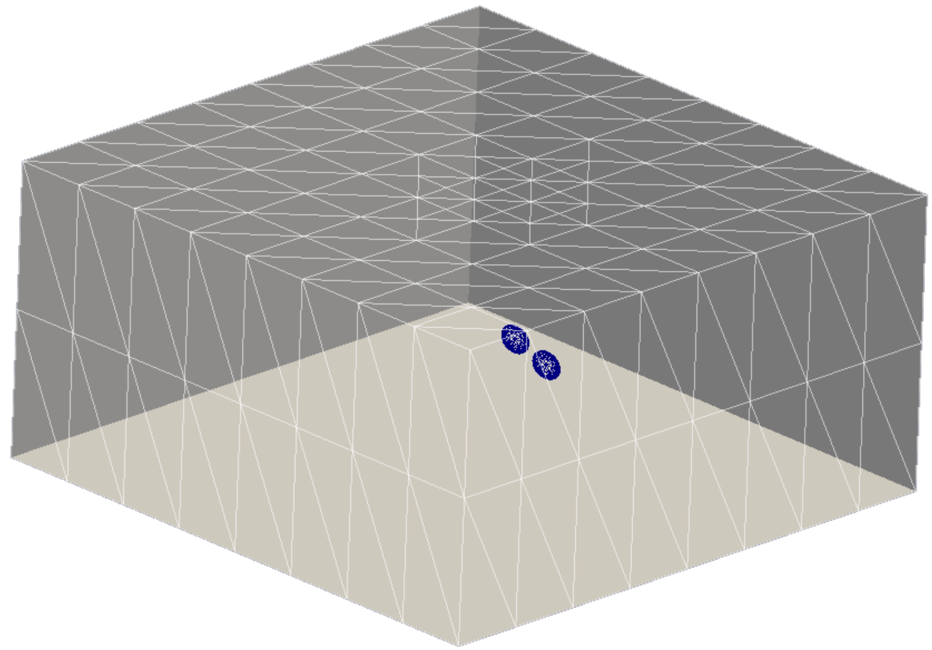
Ongoing Work

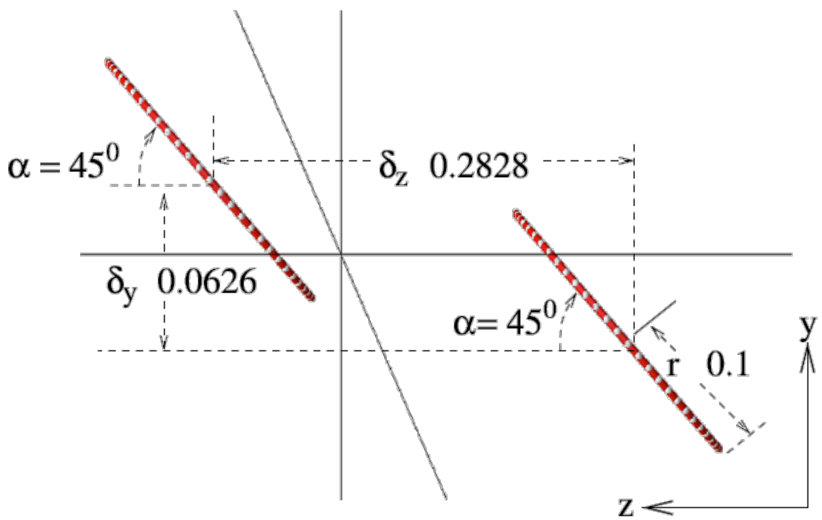
- Coalescence of non-planar fractures near a wellbore



Crack Coalescence – Non-Planar Cracks







Traction BC on top and bottom surface

PropagationMovie1

Conclusions and Outlook

- Generalized FEM removes several limitations of std FEM
- It enables the solution of problems that are difficult or not practical with the FEM
- This is the case of three-dimensional fracture problems involving
  - Complex crack surfaces
  - Fluid-induced fracturing
  - Coalescence of 3-D fractures, etc.
- Ongoing
  - Coalescence of non-planar fractures
  - Coupling porous medium deformation with fluid flow on fracture
  - Interaction between hydraulic and natural fractures

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Obrigado!
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