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Recent Developments in the Generalized Finite Element Method for 3-D Hydraulic Fracture Propagation and Interactions

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Hydraulic Fracturing of Gas Shale Reservoirs

Motivation

- Natural gas production in the US has increased significantly in the past few years thanks to advances in hydraulic fracturing of gas shale reservoirs
- Yet there are concerns about the environmental impact of toxic fluids used in this process

Objectives

- Computational simulations will lead to better designs of hydraulic fracture treatments, thus reducing the amount of toxic fluids used
- Realistic modeling of hydraulic fracturing treatments can, e.g., evaluate the potential impact of interactions between hydraulic fractures and naturally existing fractures in shale reservoirs

Acknowledgements

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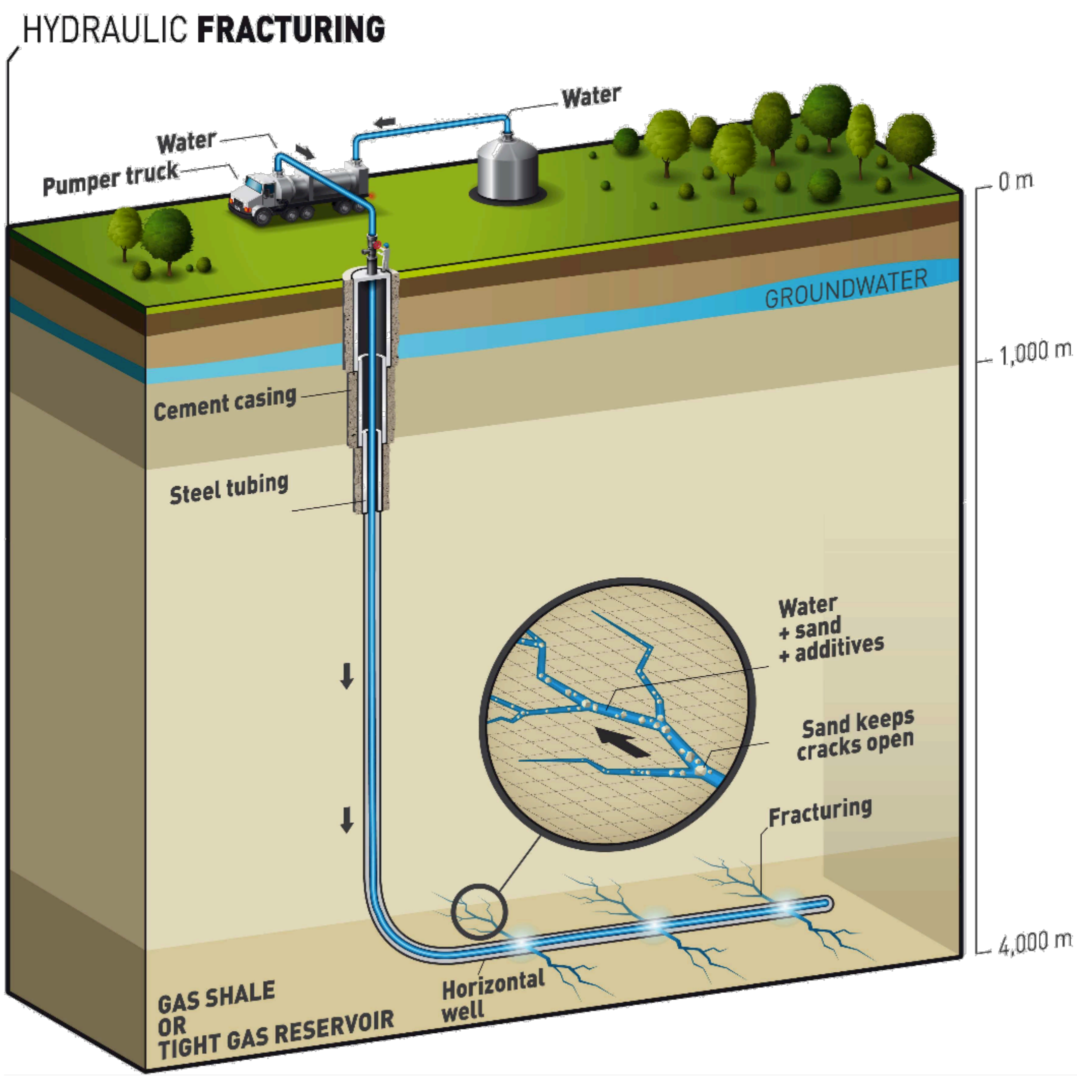
What is Hydraulic Fracturing?



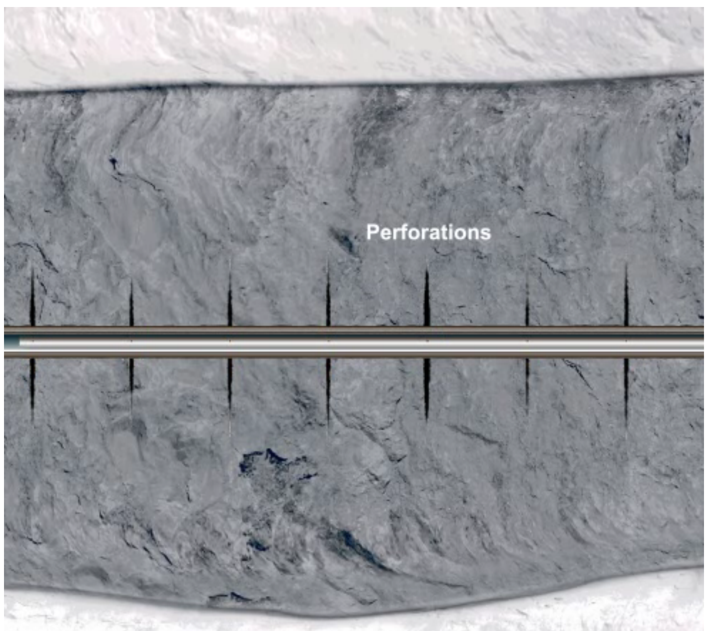
Graham Roberts, New York Times, http://www.nytimes.com/interactive/2011/02/27/us/fracking.html

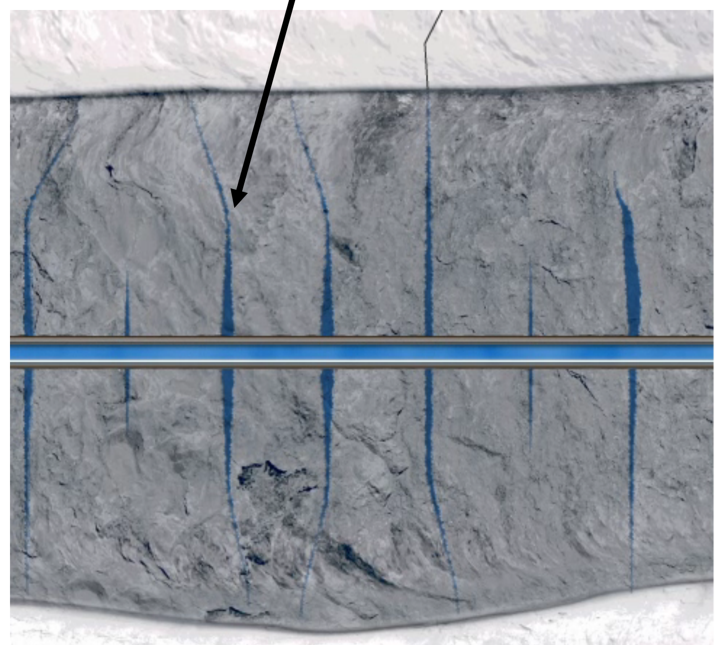
Video

Hydraulic Fracturing Simulation

Current Focus: 3-D effects not captured by available simulators

- Initial stages of fracture propagation: Fracture re-orientation, interaction and <u>coalescence</u>

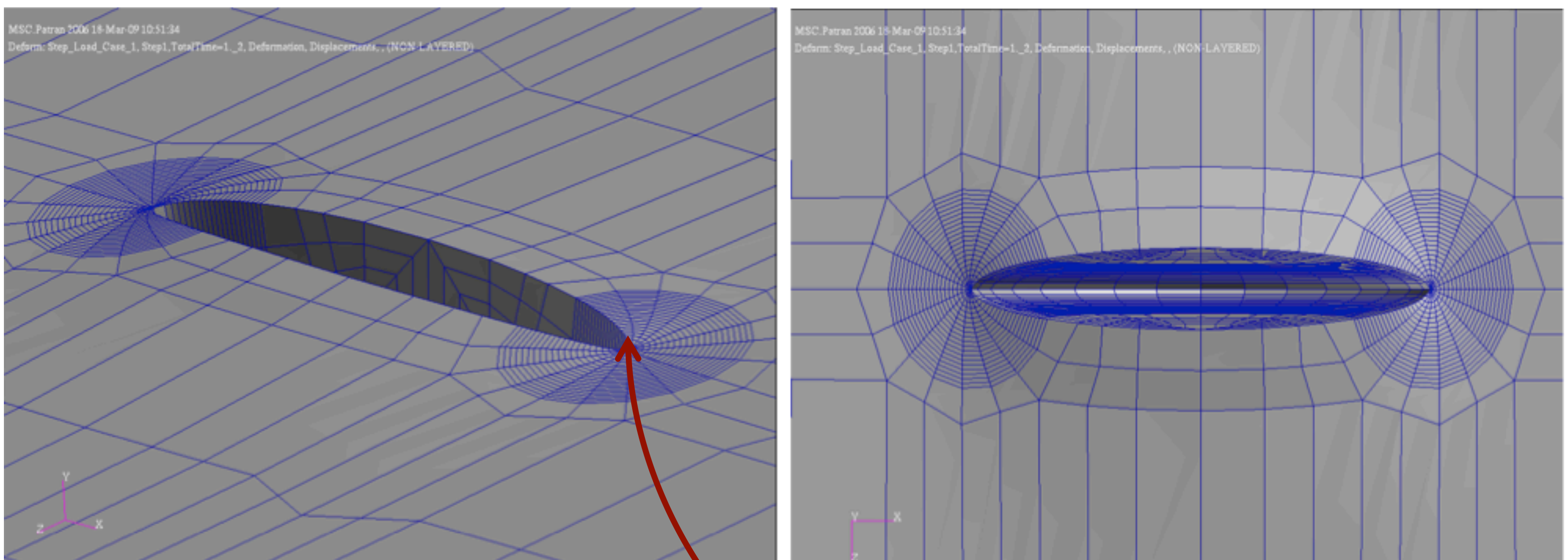




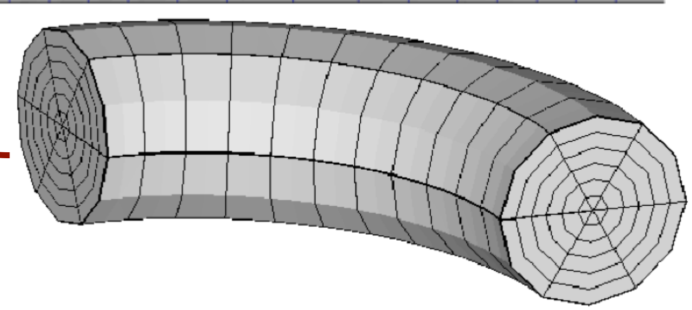
Strategy: Generalized Finite Element Methods

Modeling 3-D Fractures: Limitations of Standard FEM

- It is not "just" fitting the 3-D evolving fracture
- FEM meshes must satisfy special requirements for acceptable accuracy



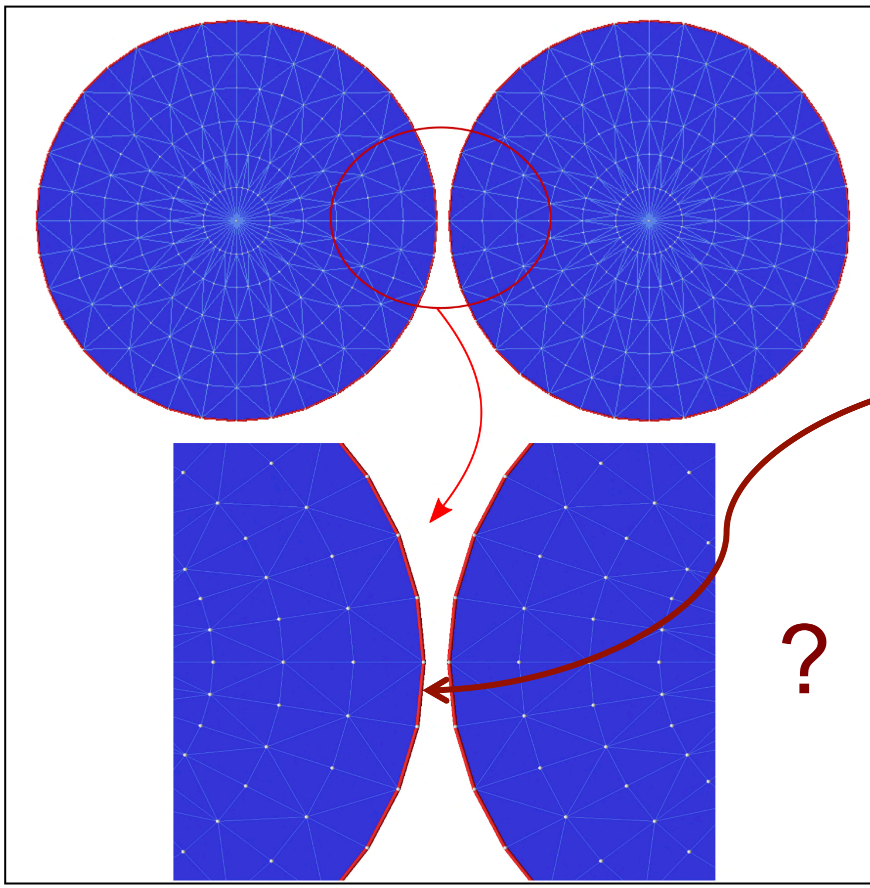
FEM mesh for a surface fracture



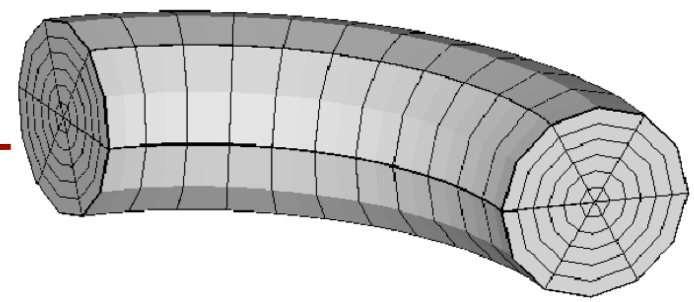
Mesh with quarter-point elements

Limitations of Standard FEM

- Difficulties arise if fracture front is close to complex geometrical features
- Fracture surfaces with sharp turns
- Coalescence of fractures



- Not possible in general to automatically create structured meshes along both fracture fronts when they are in close proximity



- Even with these crafted meshes and quarter-point elements, convergence rate of std FEM is slow (*controlled by singularity at fracture front*)

Outline

- Motivation and limitations of existing methods
- Basic ideas of GFEM
- GFEM for 3D hydraulic fractures
- Applications
  - ✓ Verification
  - ✓ Fracture re-orientation
  - ✓ Coalescence of 3-D fractures
- Conclusions

Early Works on Generalized FEMs

- Babuska, Caloz and Osborn, 1994 (Special FEM).
- Duarte and Oden, 1995 (Hp Clouds).
- Babuska and Melenk, 1995 (PUFEM).
- Oden, Duarte and Zienkiewicz, 1996 (Hp Clouds/GFEM).
- Duarte, Babuska and Oden, 1998 (GFEM).
- Belytschko et al., 1999 (Extended FEM).
- Strouboulis, Babuska and Copps, 2000 (GFEM).

- Basic idea:
  - Use a partition of unity to build Finite Element shape functions
- Review paper

  Belytschko T., Gracie R. and Ventura G. A review of extended/generalized finite element methods for material modeling, *Mod. Simul. Matl. Sci. Eng.,* 2009

  "The XFEM and GFEM are basically <u>identical</u> methods: the name generalized finite element method was adopted by the Texas school in 1995–1996 and the name extended finite element method was coined by the Northwestern school in 1999."

Generalized Finite Element Method

- GFEM is a Galerkin method with special test/trial space given by

$$\mathbb{S}_{GFEM} = \mathbb{S}_{FEM} + \mathbb{S}_{ENR}$$

Low order FEM space

Enrichment space with functions related to the given problem

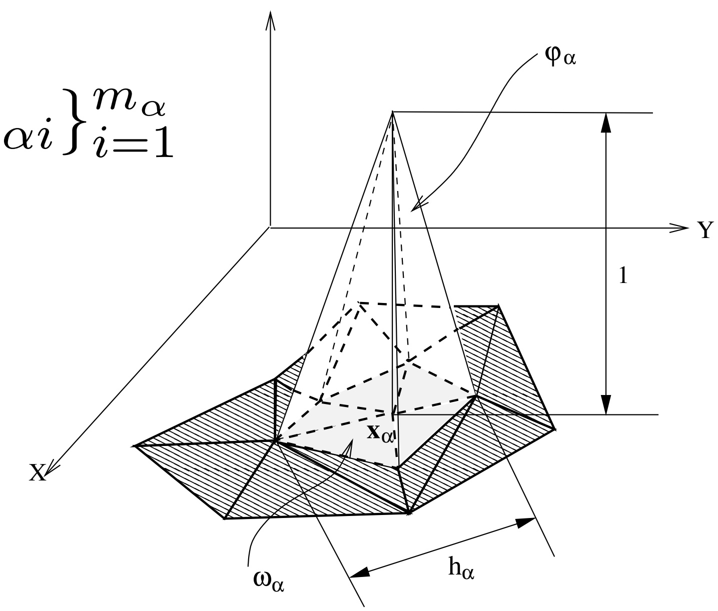
$$\mathbb{S}_{FEM} = \sum_{\alpha \in I_h} c_\alpha \varphi_\alpha, \quad c_\alpha \in \mathbb{R}$$

$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$L_{\alpha i} \in \chi_\alpha(\omega_\alpha)$$

Enrichment function

Patch space



Generalized Finite Element Method

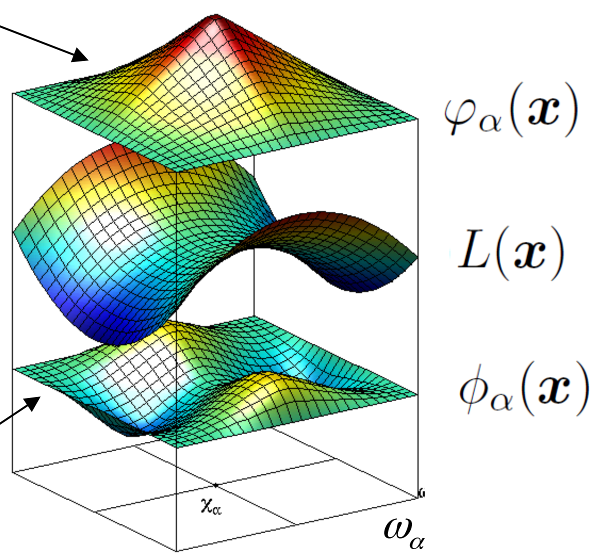
$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$\phi_{\alpha i}(x) = \varphi_\alpha(x) L_{\alpha i}(x) \qquad \sum_\alpha \varphi_\alpha(\boldsymbol{x}) = 1$$

Linear FE shape function

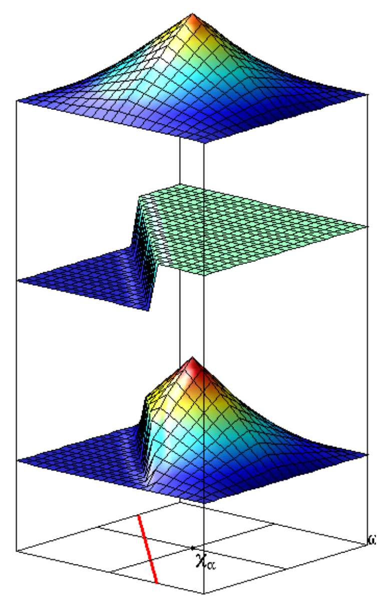
Enrichment function

GFEM shape function



[Oden, Duarte & Zienkiewicz, 1996]

• Allows construction of shape functions incorporating a-priori knowledge about solution



Discontinuous enrichment [Moes et al., 1999]

A-Priori Error Estimate for the GFEM

The error of $\boldsymbol{u}^{hp} \in \mathbb{S}_{GFEM}$ in the energy norm is bounded by

$$\|\boldsymbol{u} - \boldsymbol{u}^{hp}\|_{\varepsilon(\Omega)} \leq C \left(\sum_{\alpha \in I_h} \inf_{\boldsymbol{v}_\alpha \in \chi_\alpha} \|\boldsymbol{u} - \boldsymbol{v}_\alpha\|^2_{\varepsilon(\omega_\alpha)} \right)^{\frac{1}{2}}$$

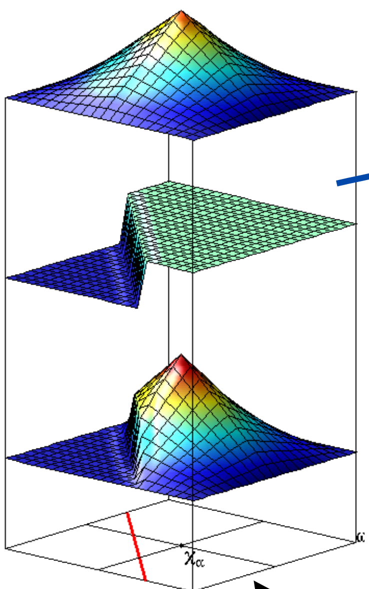
where, C is a constant and $\|.\|_\varepsilon$ denotes the energy norm.

[Babuška & Melenk, IJNME 1997;
Babuška, Banerjee & Osborn, IJCM, 2004]

- The error $\|\boldsymbol{u} - \boldsymbol{u}^{hp}\|_{\varepsilon(\Omega)}$ is bounded by the error of patch approximations $\boldsymbol{u}_\alpha \in \chi_\alpha(\omega_\alpha), \; \alpha \in I_h$.
- Convergence rate of global approximation $\boldsymbol{u}^{hp}$ is not less than that of patch approximations $\boldsymbol{u}_\alpha, \; \alpha \in I_h$.
- Use a-priori information about $\boldsymbol{u}$ to select a basis for $\chi_\alpha(\omega_\alpha), \; \alpha \in I_h$.

GFEM Approximation for 3-D Fractures

$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



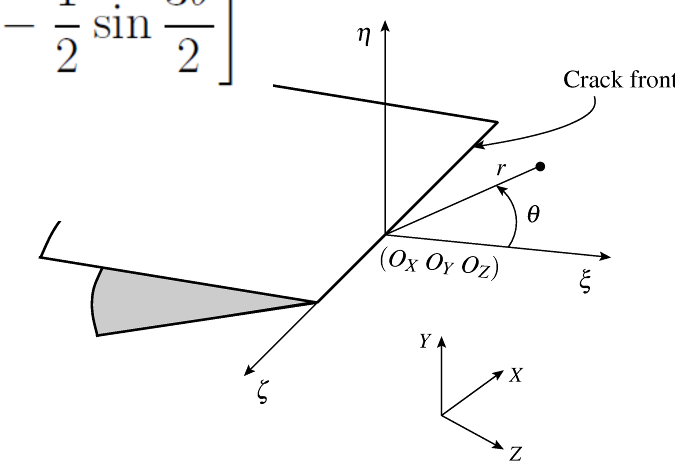
patch ω_α

$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

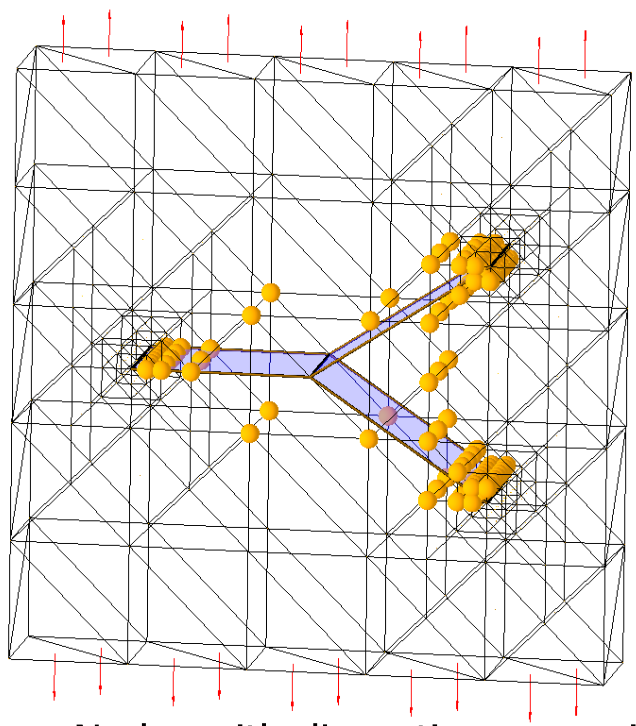
$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

[Duarte & Oden 1996]

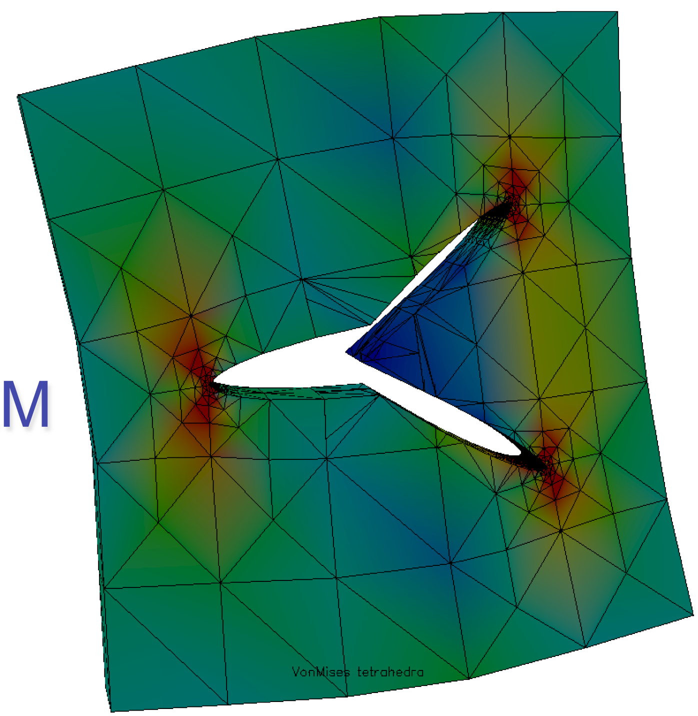


Modeling Fractures with the GFEM

- Discontinuities modeled via enrichment functions, *not* the FEM mesh
- Mesh refinement *still required* for acceptable accuracy



hp-GFEM



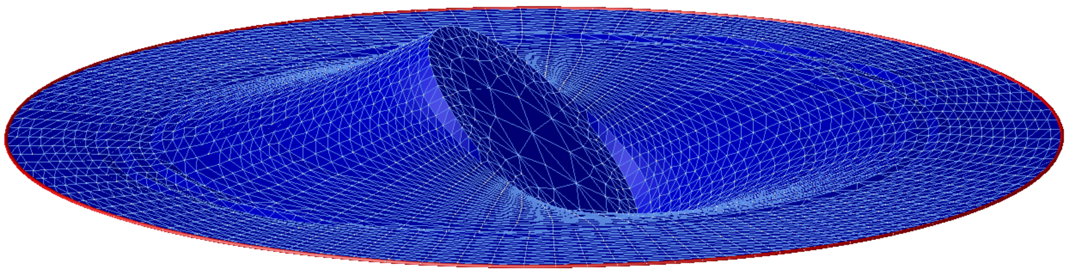
● = Nodes with discontinuous enrichments

Von Mises stress

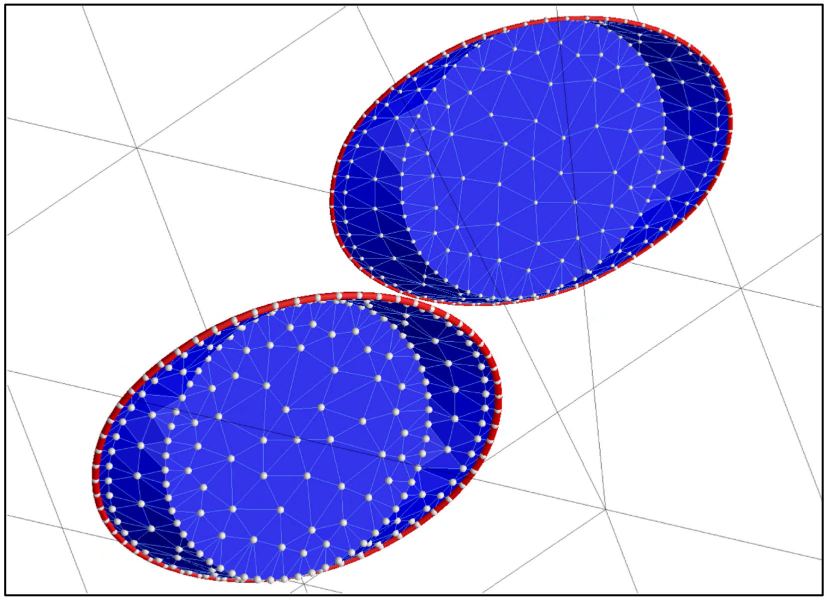
[Duarte et al., International Journal Numerical Methods in Engineering, 2007]

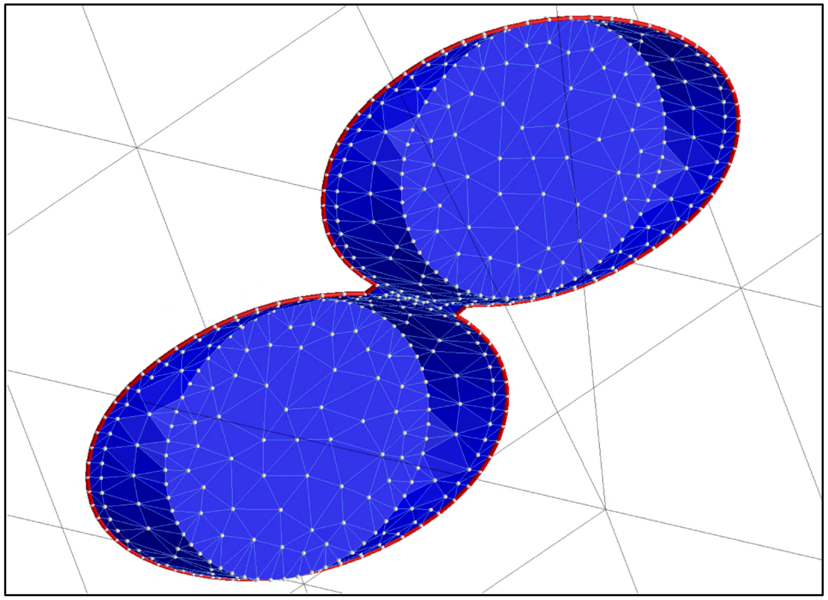
3D Fracture Surface Representation

- High-fidelity explicit representation of fracture surfaces [Duarte et al., 2001, 2009]



- Coalescence of fractures [Garzon et al., 2014]





Conditioning of GFEM Approximations

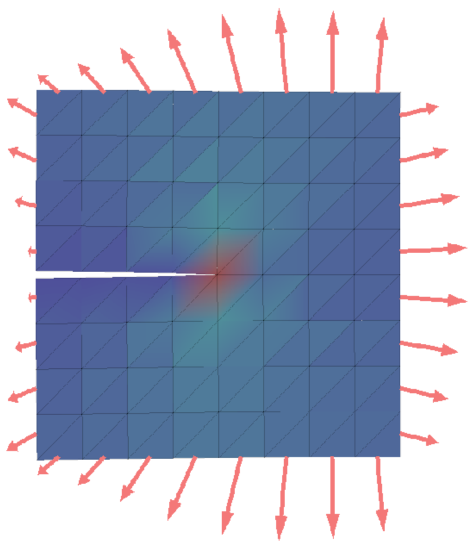
- The conditioning of the G/XFEM stiffness matrix, $\boldsymbol{K}_{GFEM}$, can be much worse than that of the standard FEM, $\boldsymbol{K}_{FEM}$

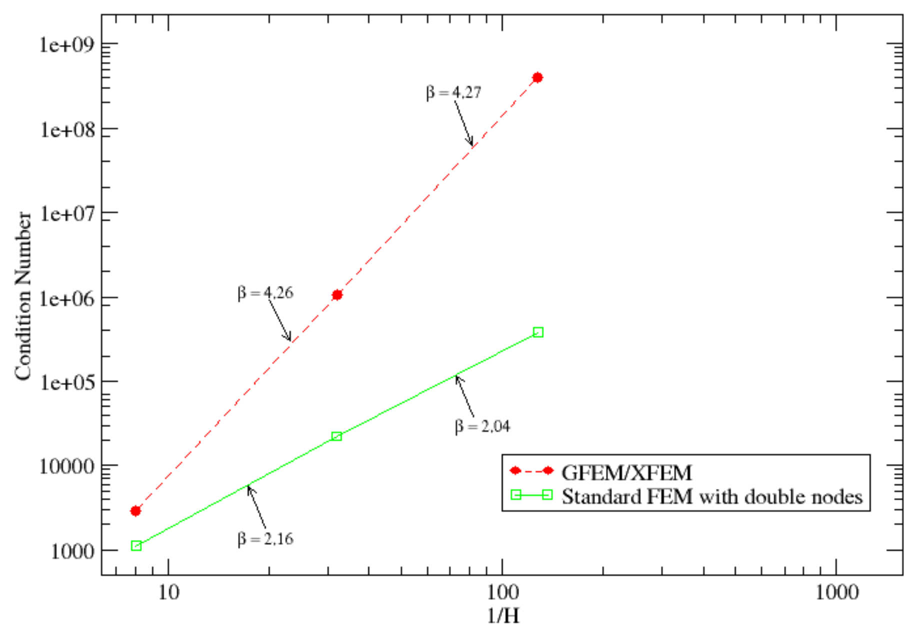
$$\mathfrak{K}(\boldsymbol{K}_{GFEM}) = \mathcal{O}(h^{-4})$$

while

$$\mathfrak{K}(\boldsymbol{K}_{FEM}) = \mathcal{O}(h^{-2})$$

where $\mathfrak{K}(.)$ is the *scaled condition number*.





SGFEM: Stable Generalized FEM

- The SGFEM involves simple local modifications of enrichments used in the GFEM

$$\tilde{L}_{\alpha j}(\boldsymbol{x}) = L_{\alpha j}(\boldsymbol{x}) - \mathrm{I}_{\omega_\alpha}(L_{\alpha j})(\boldsymbol{x})$$

where $\mathrm{I}_{\omega_\alpha}(L_{\alpha j})$ is the piecewise linear FE interpolant of $L_{\alpha j}$ on the patch ω_α

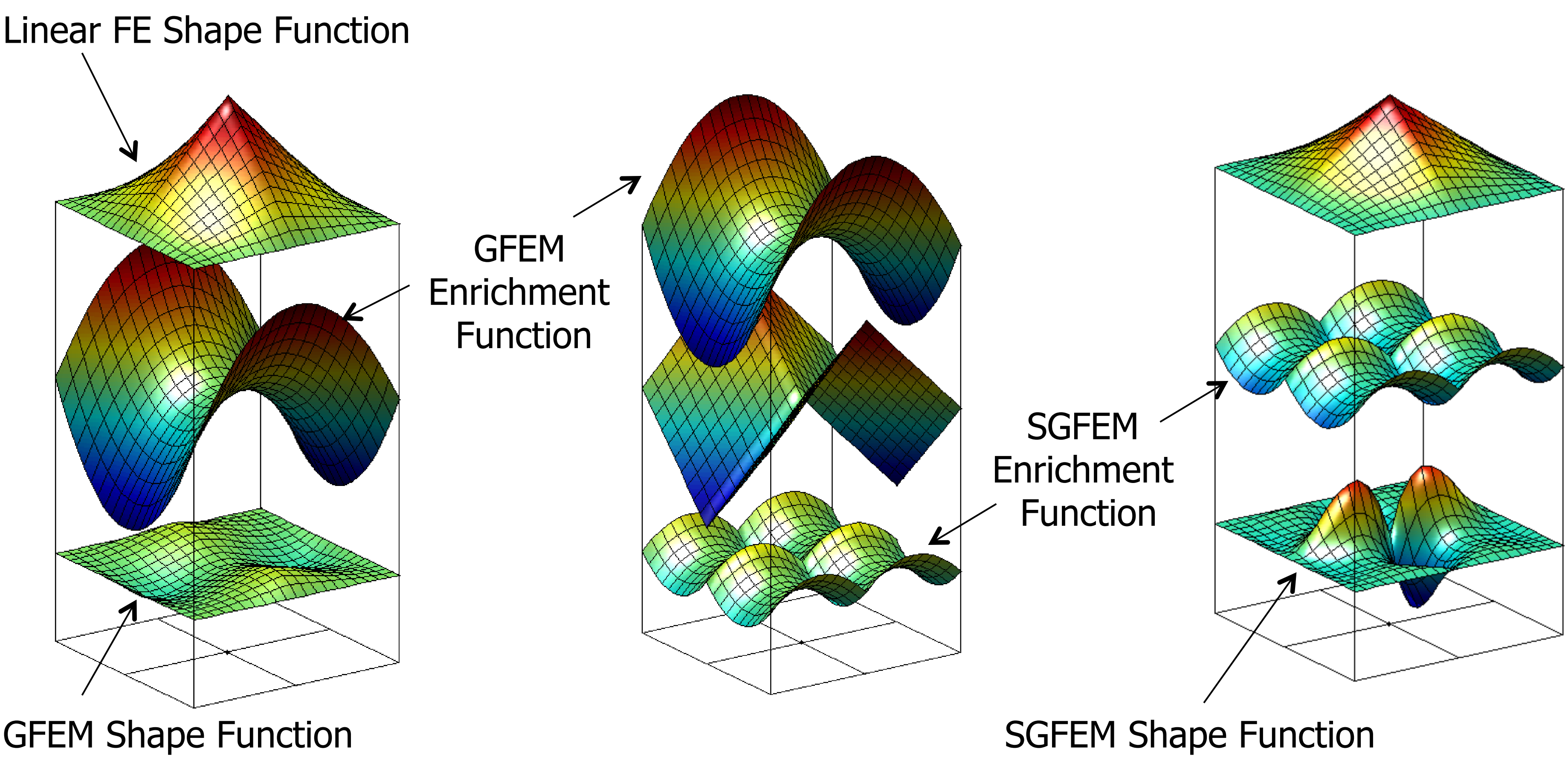
[Babuška & Banerjee CMAME 2012;

Gupta, Duarte, Babuška & Banerjee CMAME, 2013]

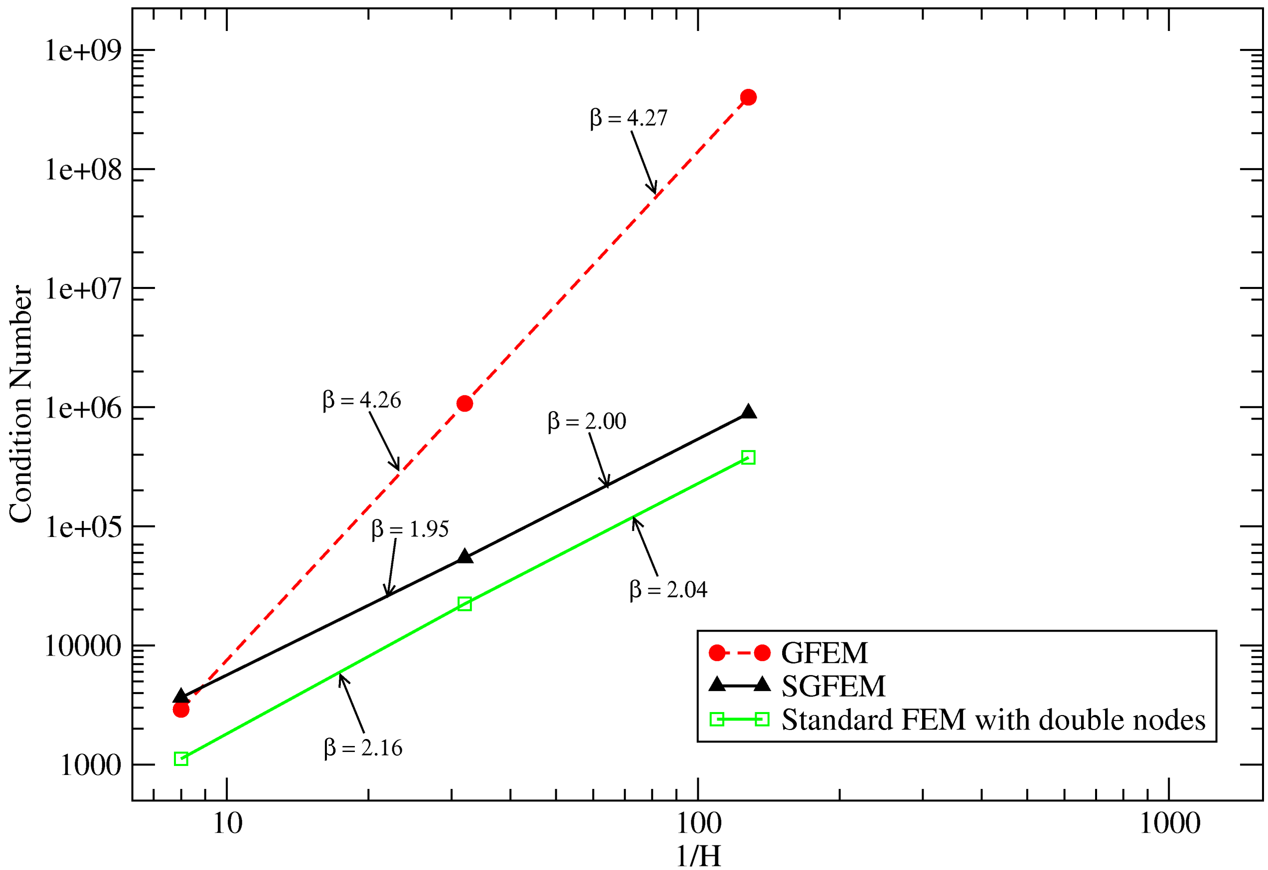
SGFEM: Stable Generalized FEM

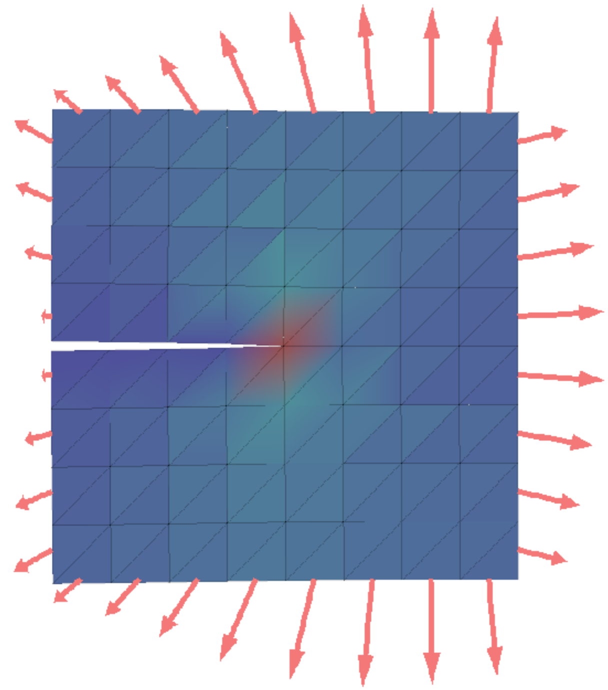
$$\tilde{L}_{\alpha i}(\boldsymbol{x}) = L_{\alpha i}(\boldsymbol{x}) - \mathrm{I}_{\omega_\alpha}(L_{\alpha i})(\boldsymbol{x})$$

$$\tilde{\phi}_{\alpha i}(\boldsymbol{x}) = \varphi_\alpha(\boldsymbol{x})\tilde{L}_{\alpha i}(\boldsymbol{x})$$



SGFEM: Stable Generalized FEM





Conditioning of GFEM/XFEM stiffness matrix $\mathcal{O}(h^{-4})$

Conditioning of SGFEM and FEM stiffness matrix $\mathcal{O}(h^{-2})$

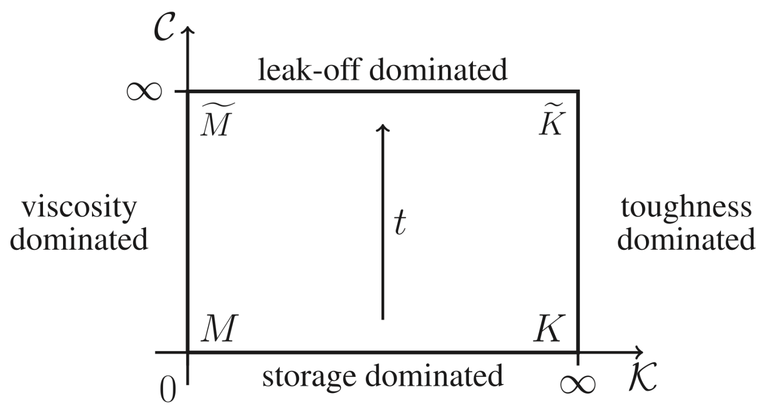
[Gupta, Duarte, Babuska & Banerjee CMAME, 2013]

Selection of Enrichment Functions: Hydraulic Fracturing Regimes

- Fracture propagation is governed by
  - two competing energy dissipation mechanisms: Viscous flow and fracturing process;
  - two competing storage mechanisms: In the fracture and in the porous matrix

Dimensionless toughness
$$\mathcal{K} = \frac{4K_{Ic}}{\sqrt{\pi}}\left(\frac{1}{3Q_0 E'^3 \mu}\right)^{1/4}$$

Leak-off coefficient
$$\mathcal{C} = 2C_L\left(\frac{E't}{12\mu Q_0^3}\right)^{1/6}$$



Hydraulic fracture parametric space*

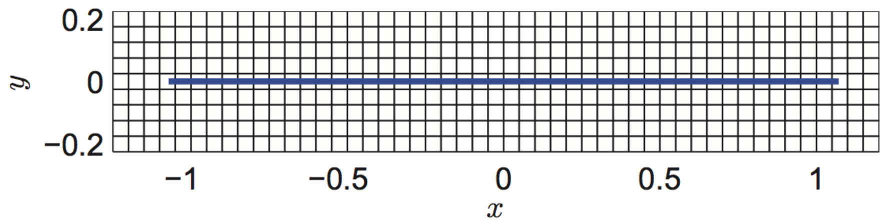
Current Focus: Storage-toughness dominated regime

- Low permeability reservoirs: Neglect flow of hydraulic fluid across fracture faces:
  - Storage dominated regime
- High confining stress and low viscosity fluid (water):
  - Constant pressure distribution in fracture; Toughness dominated regime
- Brittle elastic material

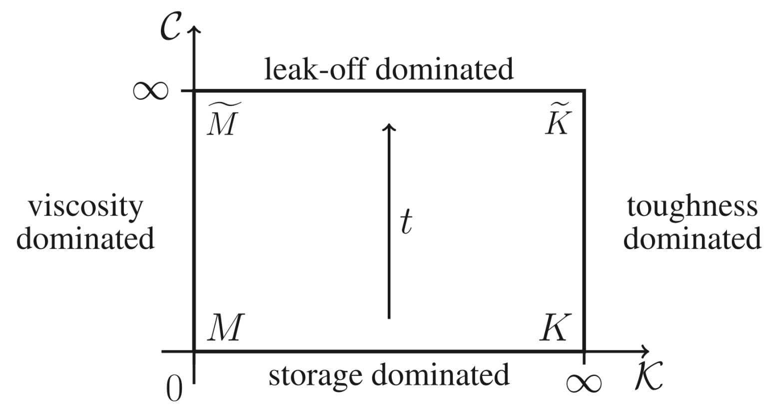
*[Carrier & Granet, EFM, 2013]

Solution for Toughness-Dominated Problem

- Solution of coupled problem [Gordeliy & Peirce, cmame, 2013]

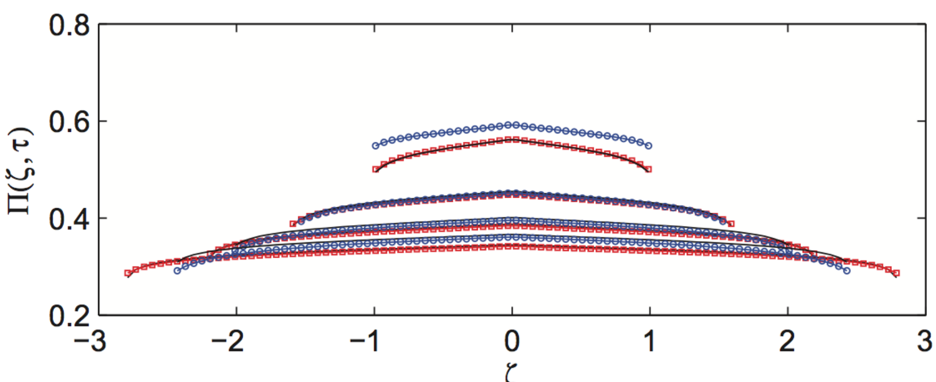


Initial fracture



$$\mathcal{K} = 3$$

$$\mathcal{C} = 0$$



Normalized pressure at several time steps

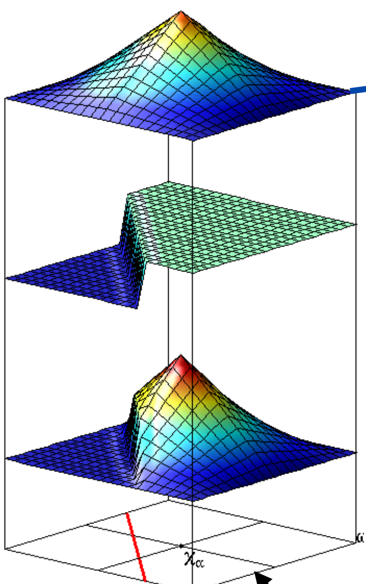
$$\mathcal{K} = \frac{4K_{Ic}}{\sqrt{\pi}} \left(\frac{1}{3Q_0 E'^3 \mu} \right)^{1/4}$$

$$\mathcal{C} = 2C_L \left(\frac{E't}{12\mu Q_0^3} \right)^{1/6}$$

Selection of Enrichment Functions: Hydraulic Fracturing Regimes

Enrichments for toughness-dominated regime:

$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



patch ω_α

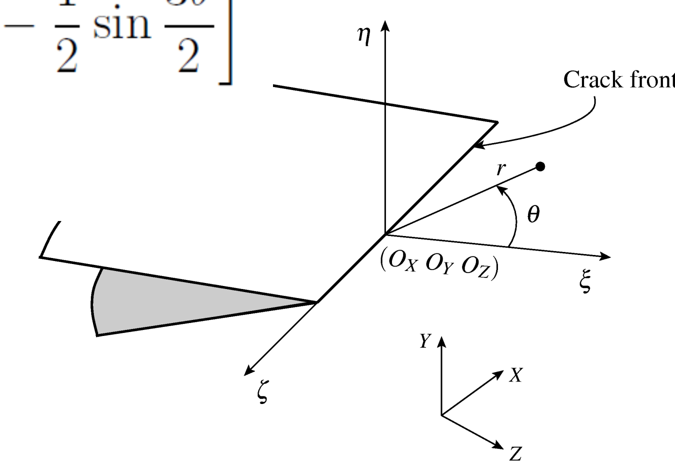
$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

[Duarte & Oden 1996]

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

Valid for toughness-dominated problems



Weak Form at Propagation Step k

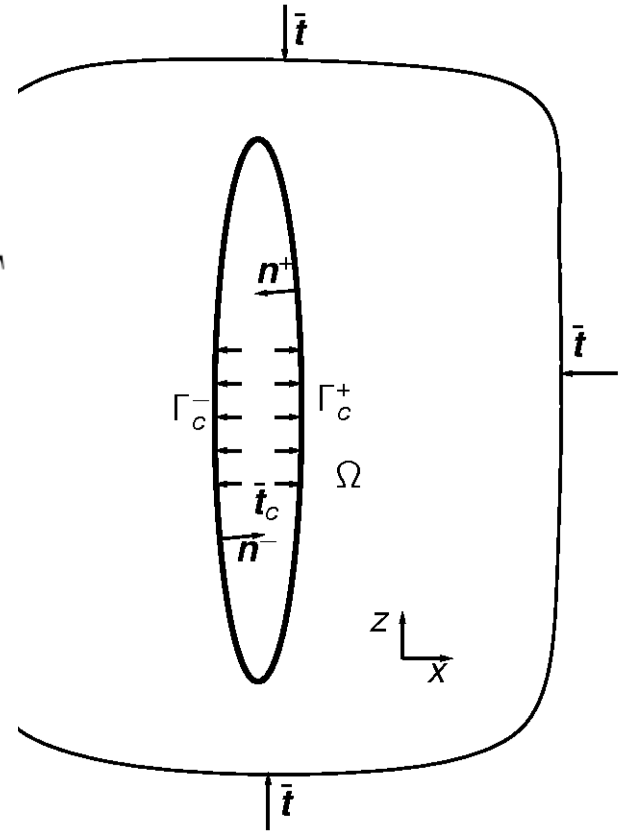
Find $\boldsymbol{u}^k \in H^1(\Omega)$, such that $\forall\ \boldsymbol{v}^k \in H^1(\Omega)$

$$\int_\Omega \boldsymbol{\sigma}(\boldsymbol{u}^k) : \boldsymbol{\varepsilon}(\boldsymbol{v}^k) d\Omega$$

$$= \int_\Omega \boldsymbol{b} \cdot \boldsymbol{v}^k d\Omega + \int_{\partial\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{v}^k d\Gamma + \int_{\Gamma_c^{k+}} \bar{\boldsymbol{t}}_c^{k+} \cdot [\![\boldsymbol{v}^k]\!] d\Gamma$$

where $[\![\boldsymbol{v}^k]\!]$ is the virtual displacement jump across the crack surface Γ^k at propagation step k and

$$\bar{\boldsymbol{t}}_c^{k+} = -p^k \boldsymbol{n}^{k+} = p^k \boldsymbol{n}^{k-}$$



Cross section of fracture

Outline

- Motivation and limitations of existing methods
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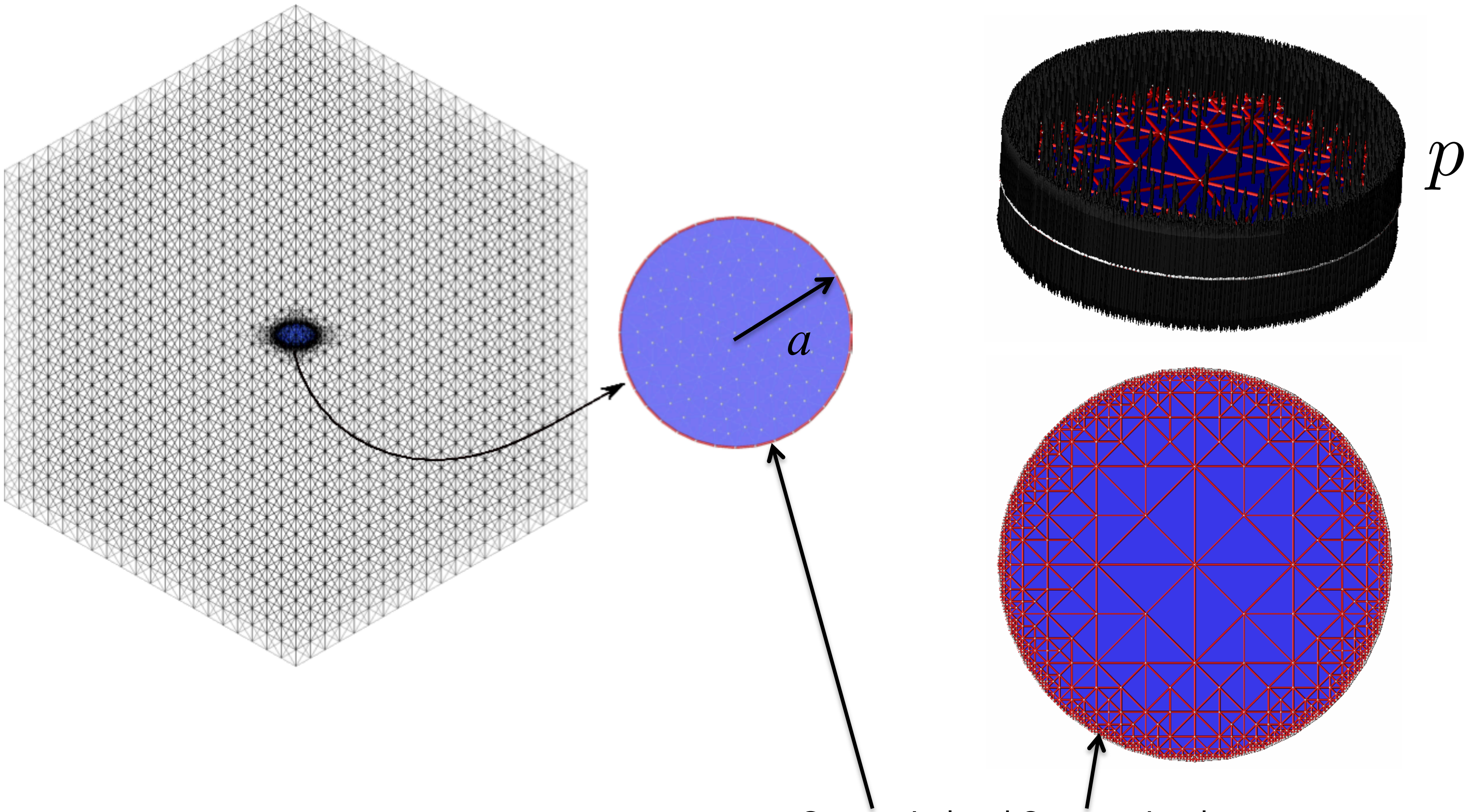
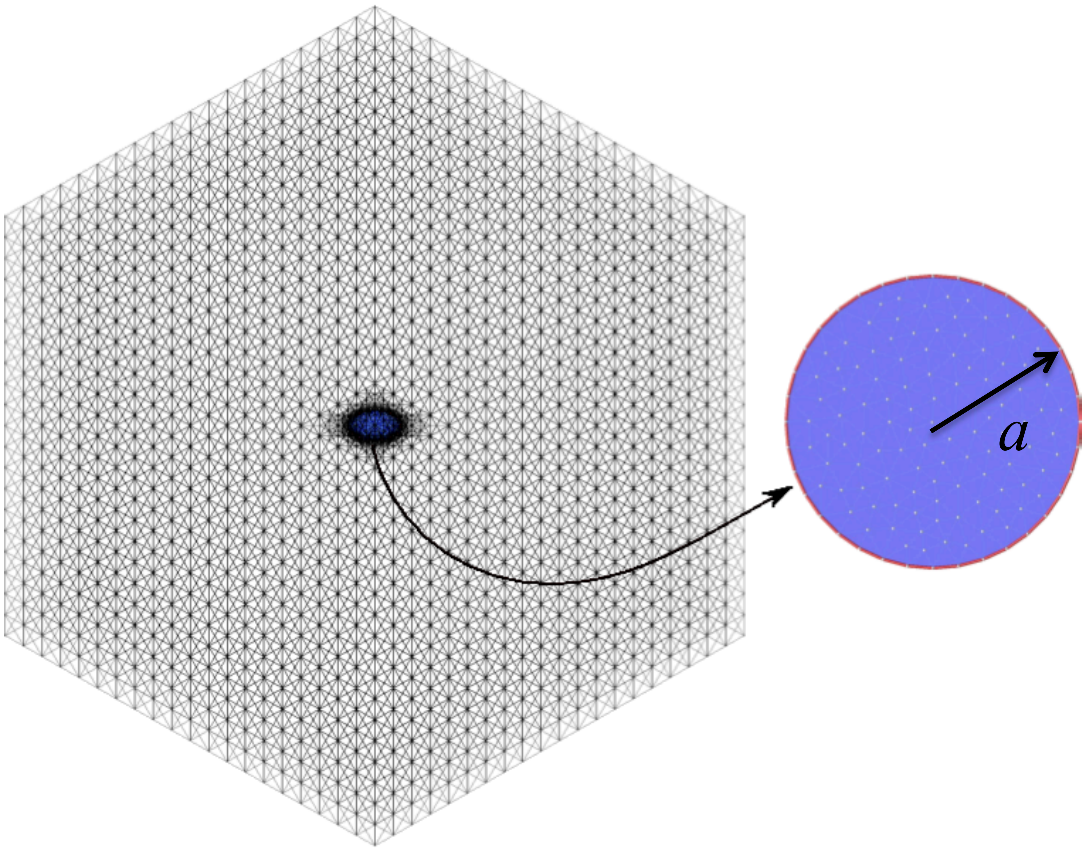
Verification: Propagation of Circular Fracture*



Geometrical and Computational fracture surface loaded with fluid pressure *p*

*[Gupta & Duarte, 2014]

Verification: Propagation of Circular Fracture



Critical pressure

$$p_c(a) = \left(\frac{E^* G_c \pi}{4a}\right)^{1/2}$$

Adopt [Bourdin et al. 2012]:

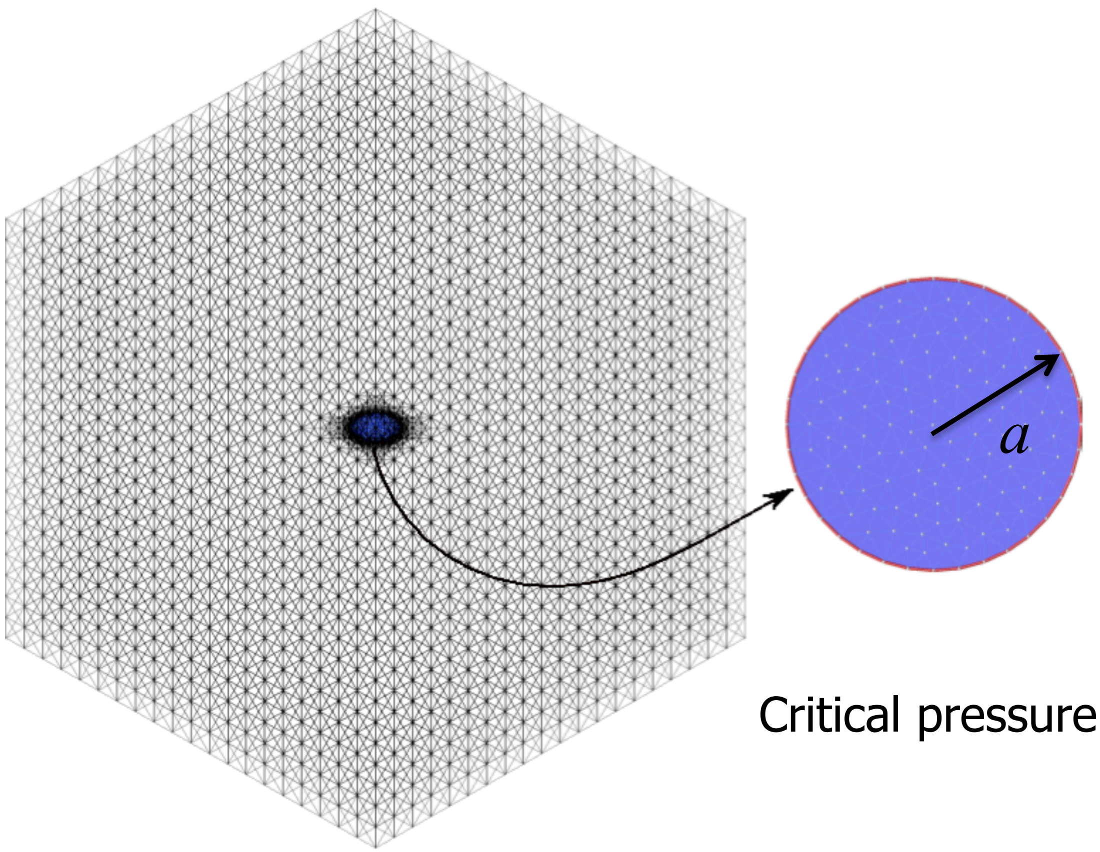
$$E^* = 1$$

$$G_c = 1.91 \times 10^{-9}$$

$$a = 0.5$$

$$p_c(0.5) = 5.477 \times 10^{-5}$$

Propagation of Circular Fracture



GFEM Model

$$h_{\min}/a = 0.016$$

$$h_{\max}/a = 0.027$$

$$p\text{-}order = 2$$

$$N = 215\,376 \;\; dofs$$

$$T = 5.25 \;\; min$$

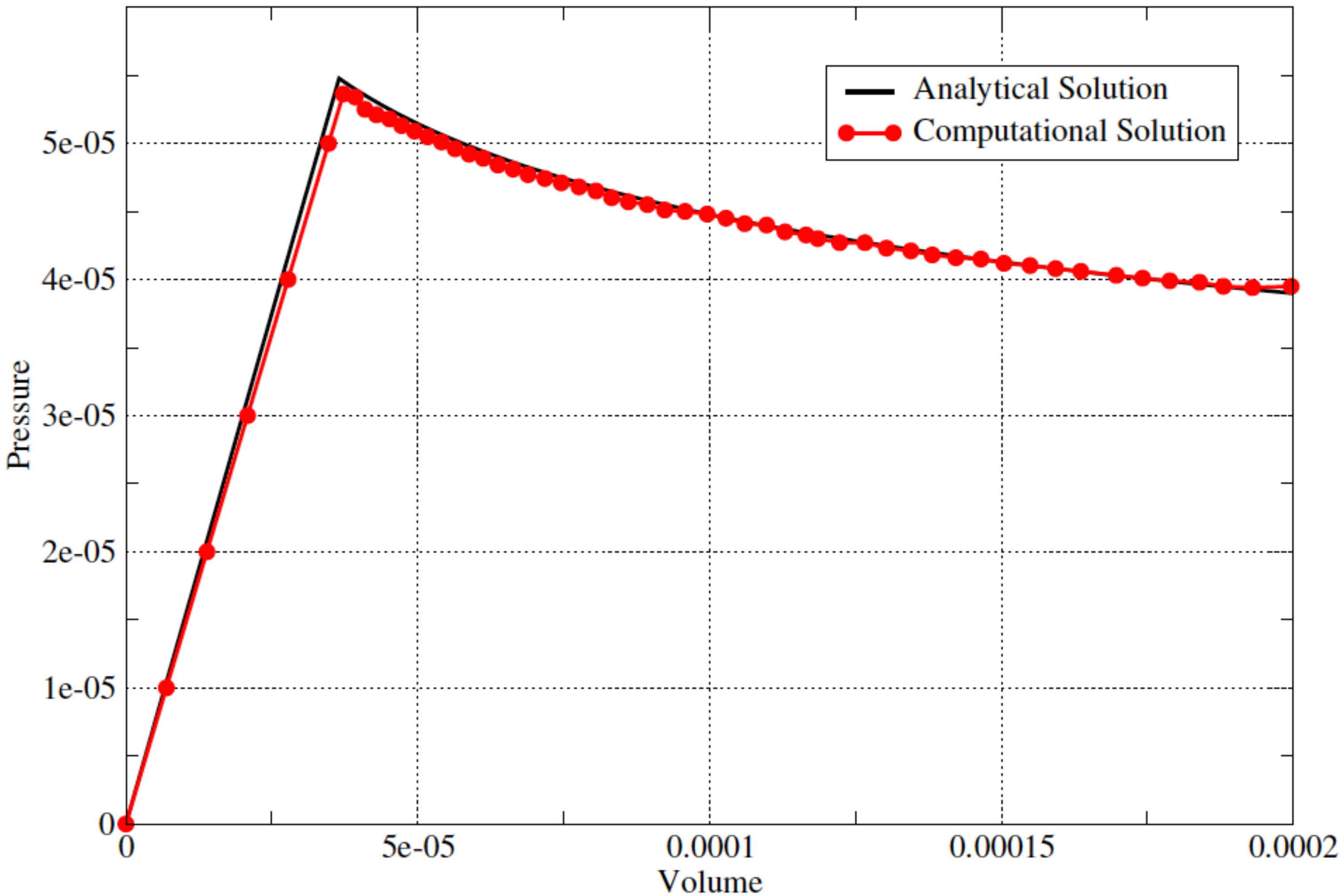
Critical pressure $$p_c^h(a) = \frac{K_c}{K(a)} p$$

$$p_c^h(0.5) = 5.415 \times 10^{-5}$$

$$e_r(p_c) = 1.15\%$$

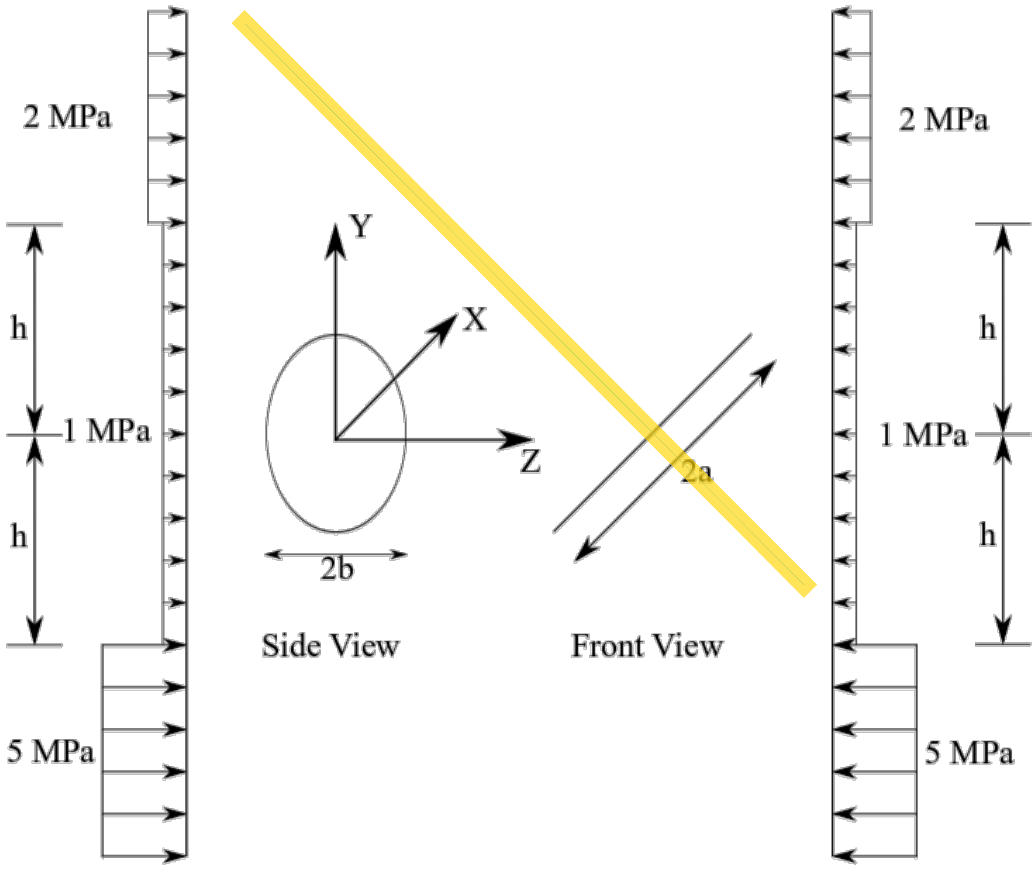
Propagation of Circular Fracture

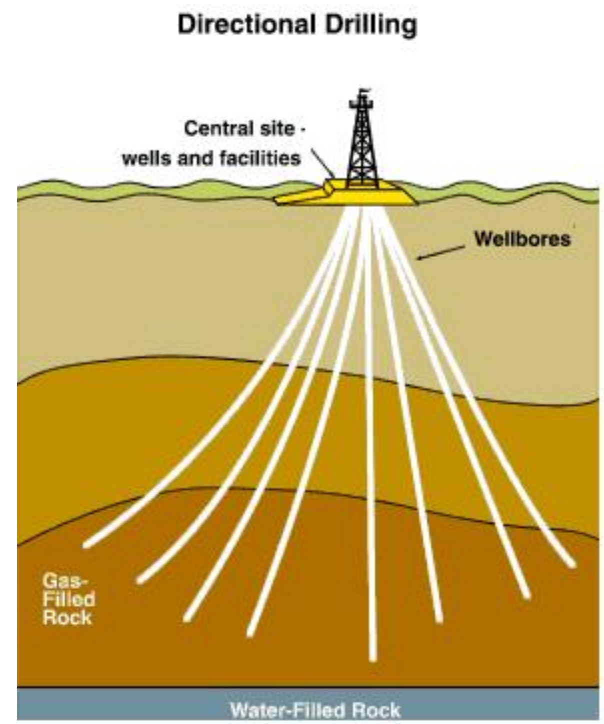
Repeating for each step of fracture propagation



Application: Fracture Re-Orientation*

- Fracture starts in a direction not perpendicular to minimum *in-situ* stress
- Misalignment of fracture and confining in-situ stresses

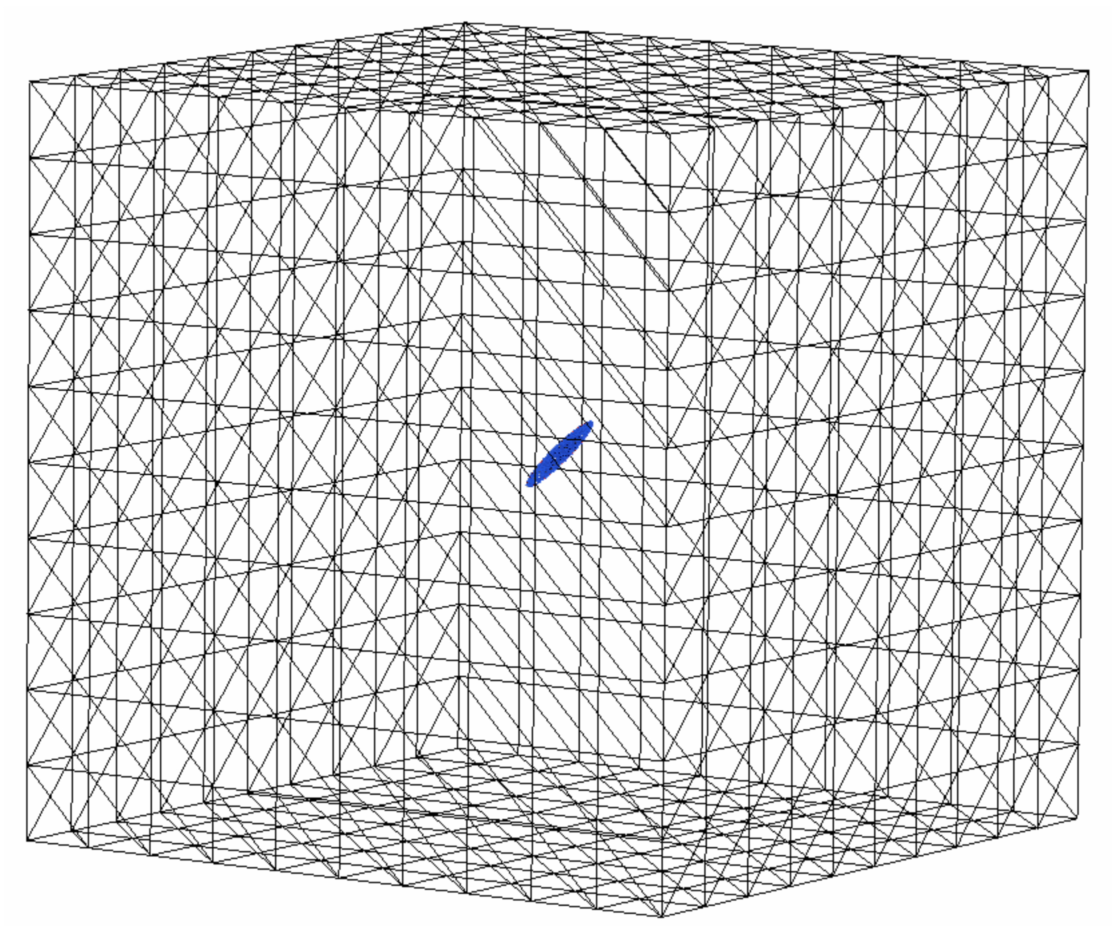


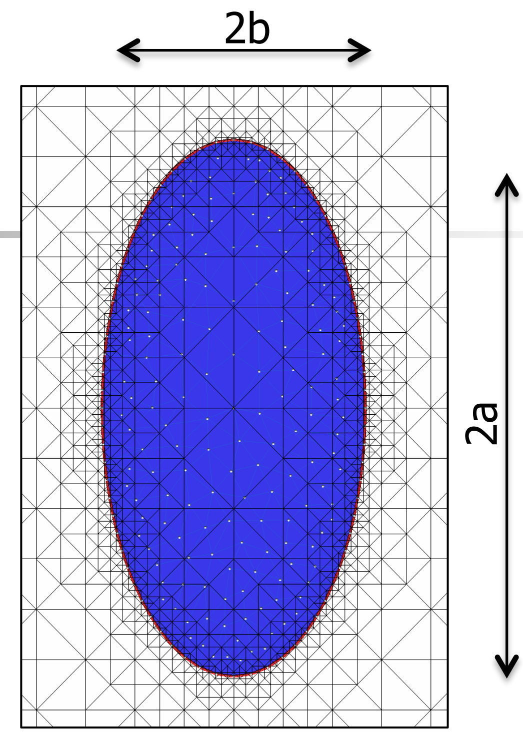


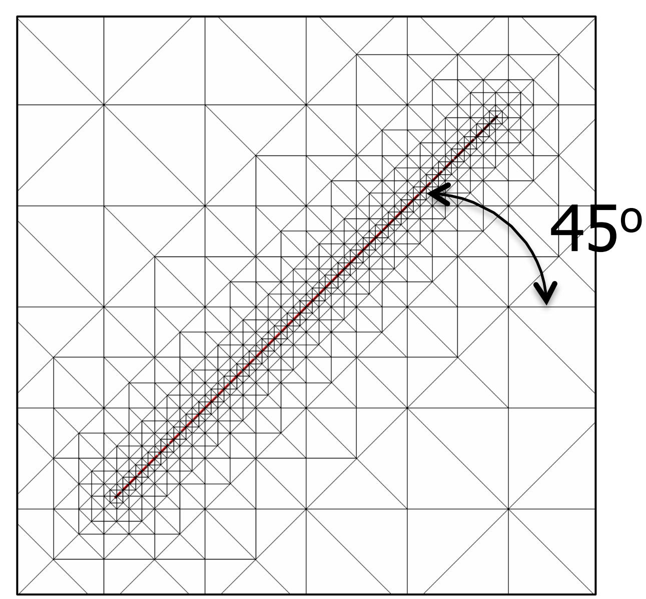
a = 10m
b = 5m
h = 15m
p = 3.5 MPa

*[Rungamornrat et al., 2005; Gupta & Duarte, 2014]

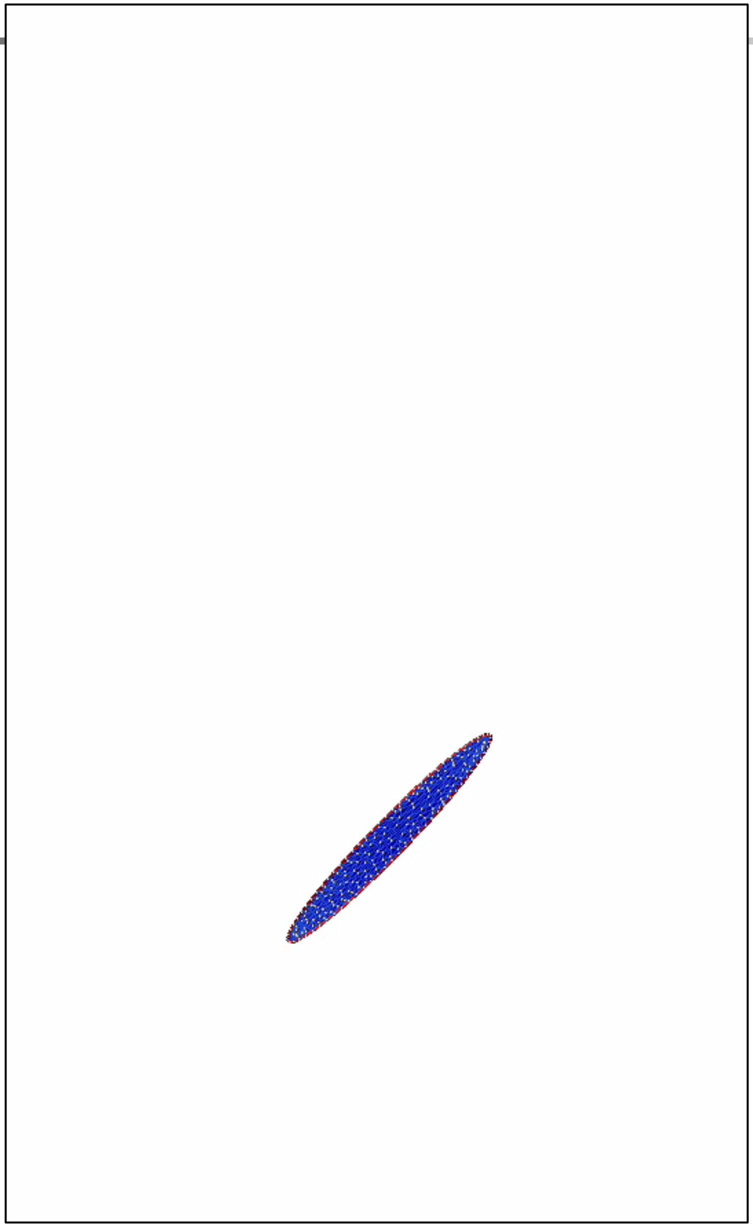
Fracture Re-Orientation



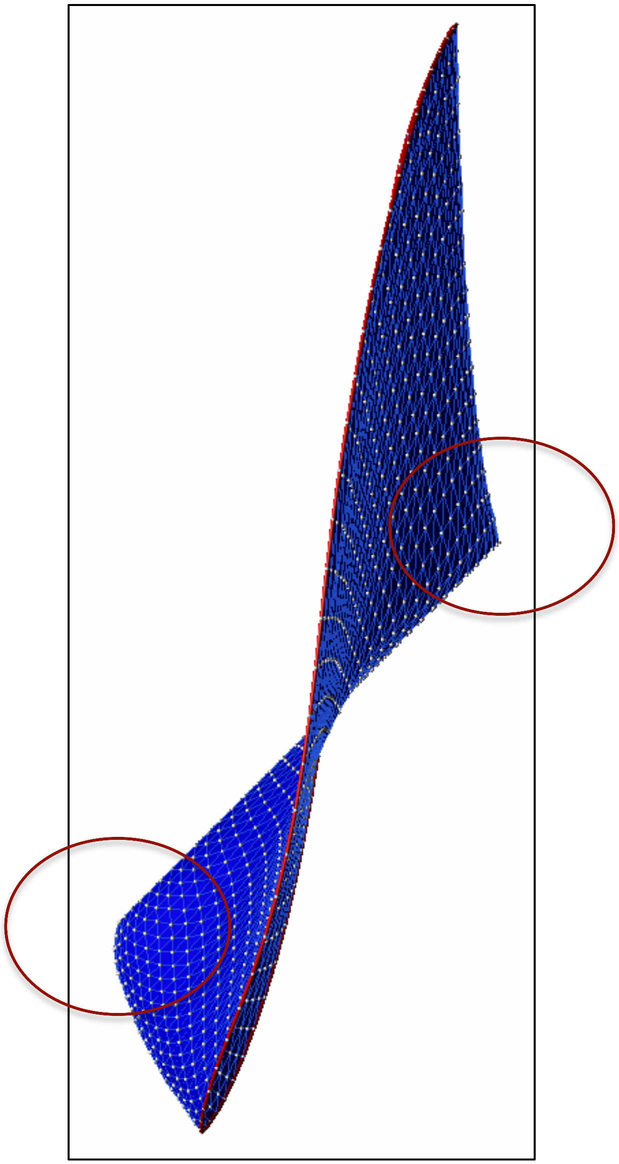


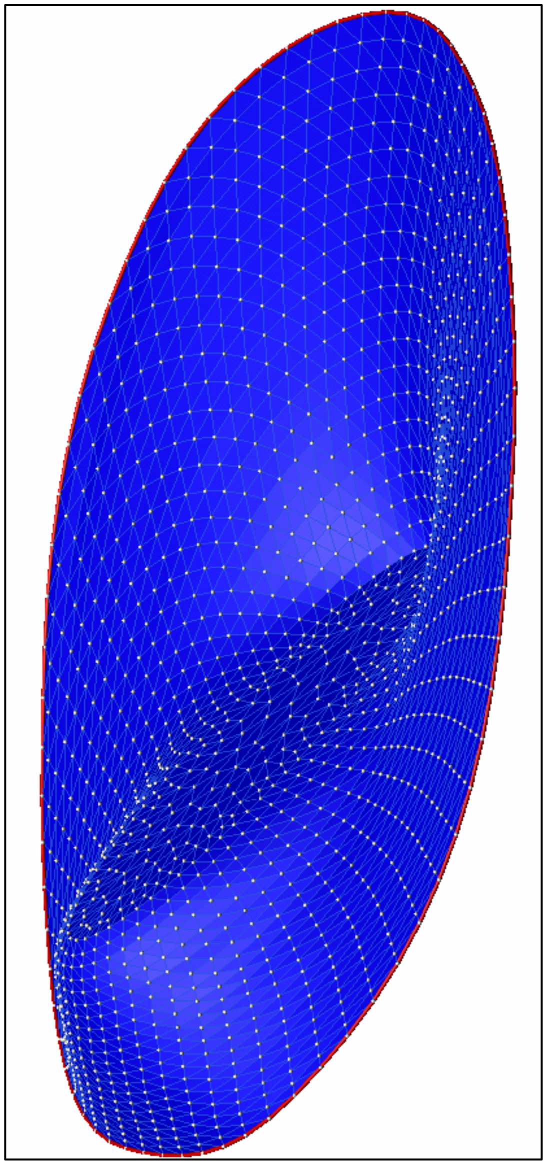


Fracture Re-Orientation

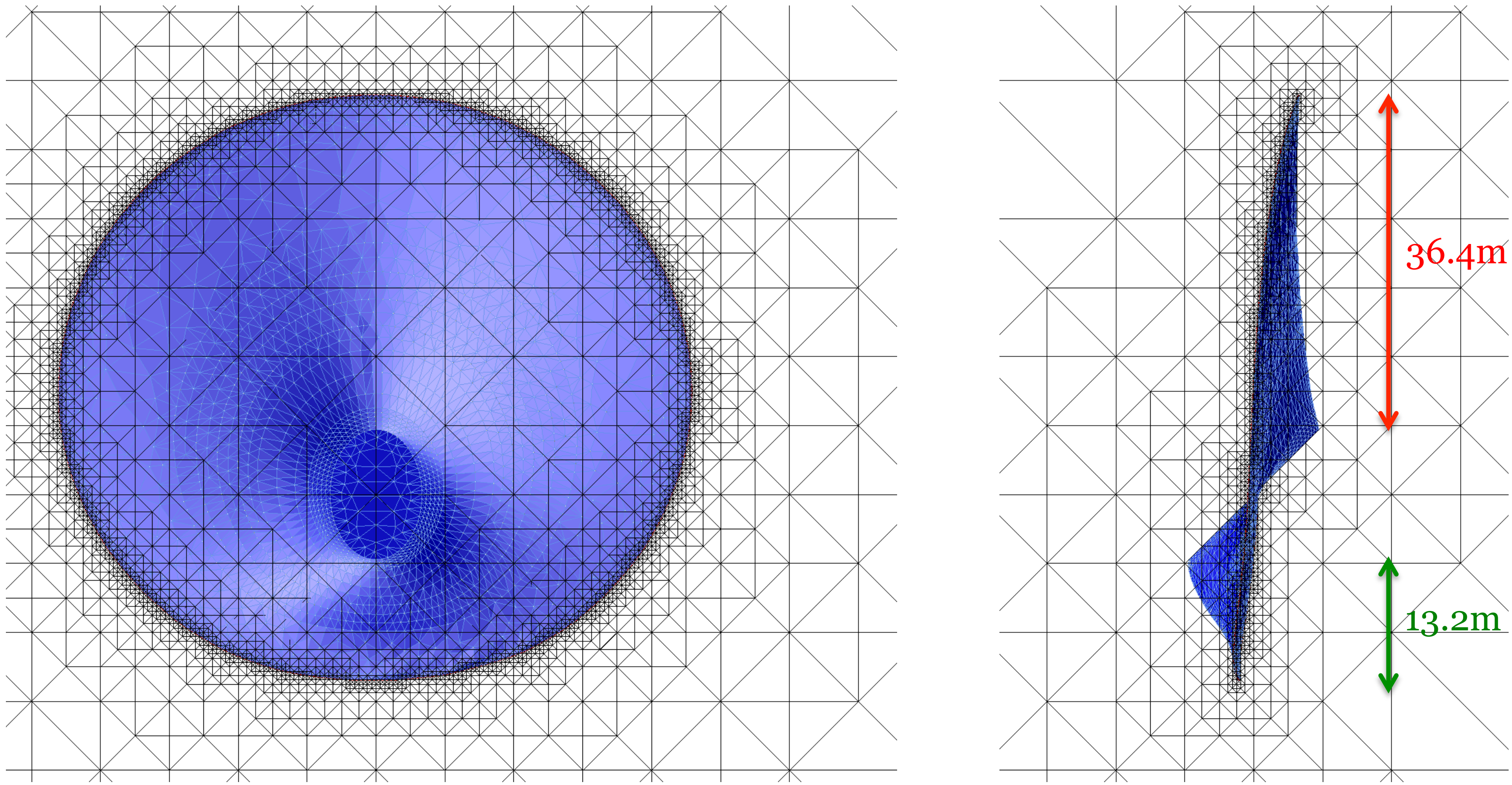


Fracture Re-Orientation: Step 20



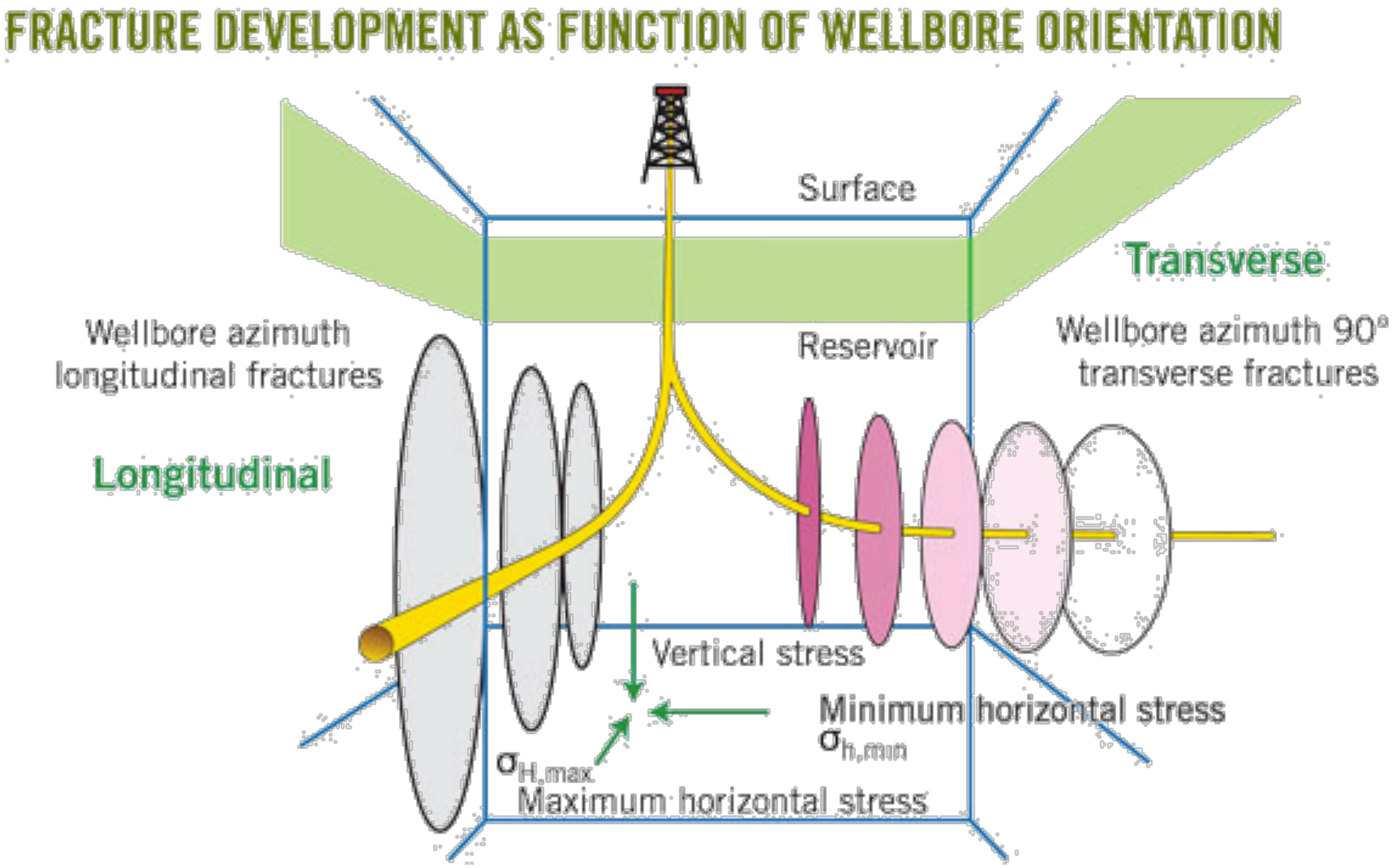


Fracture Re-Orientation: Adaptive Mesh



- Adaptive refinement along fracture front
- Sharp features are preserved
- High fidelity of fracture surface, regardless of computational mesh

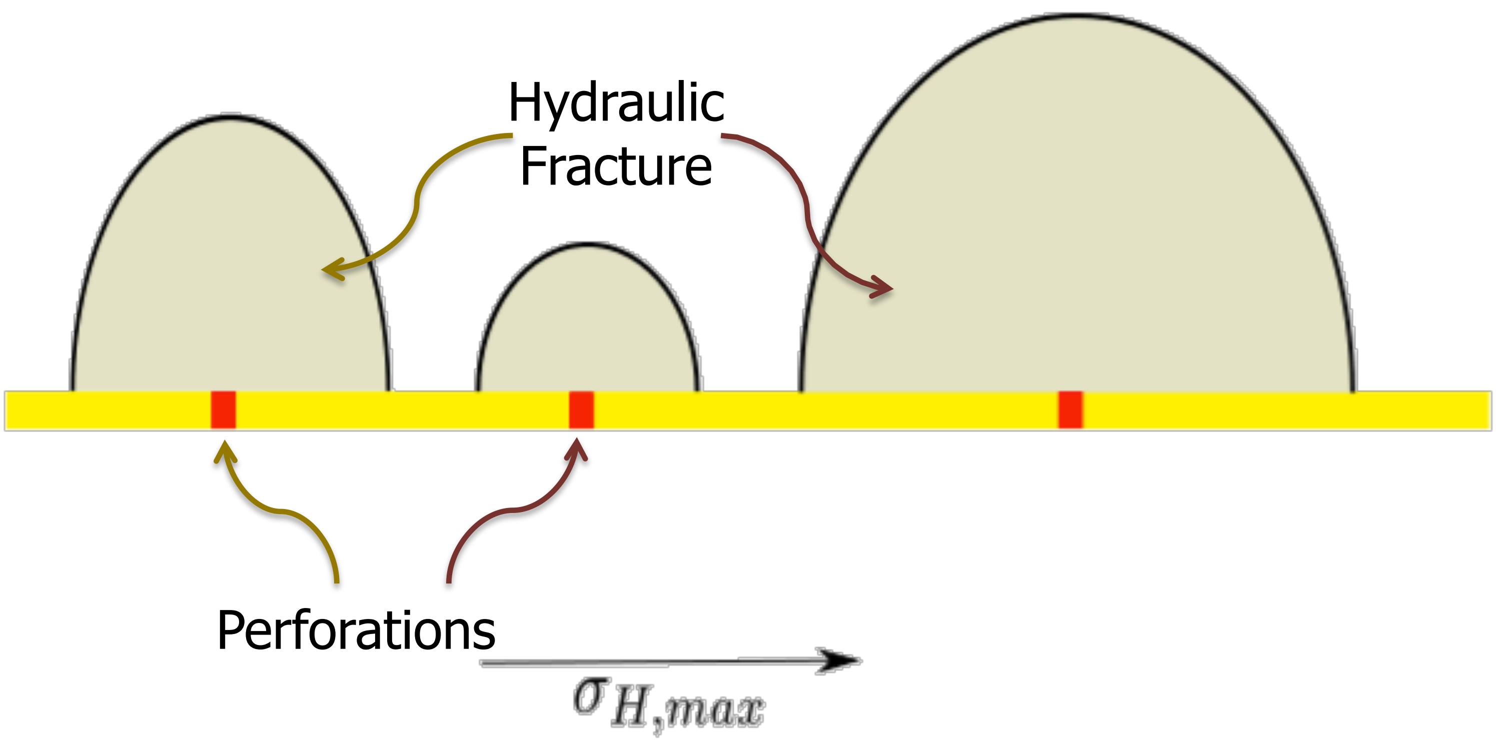
Typical Hydraulic Fracturing



[Z. Rahim et al., 2012]

Longitudinal Fractures

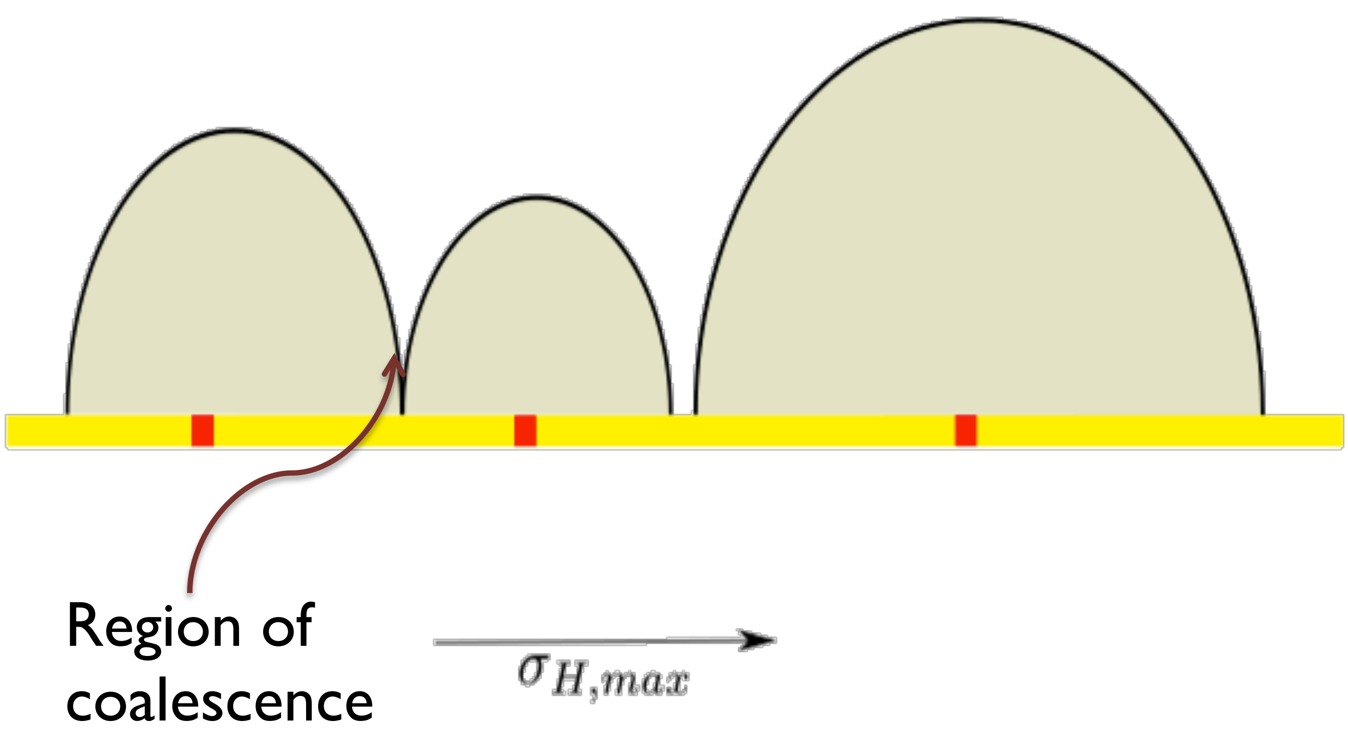
- Develop perpendicular to minimum in-situ stress
- Fractures along the length of the wellbore
- Planar fractures from the perforation



Coalescence of Longitudinal Fractures

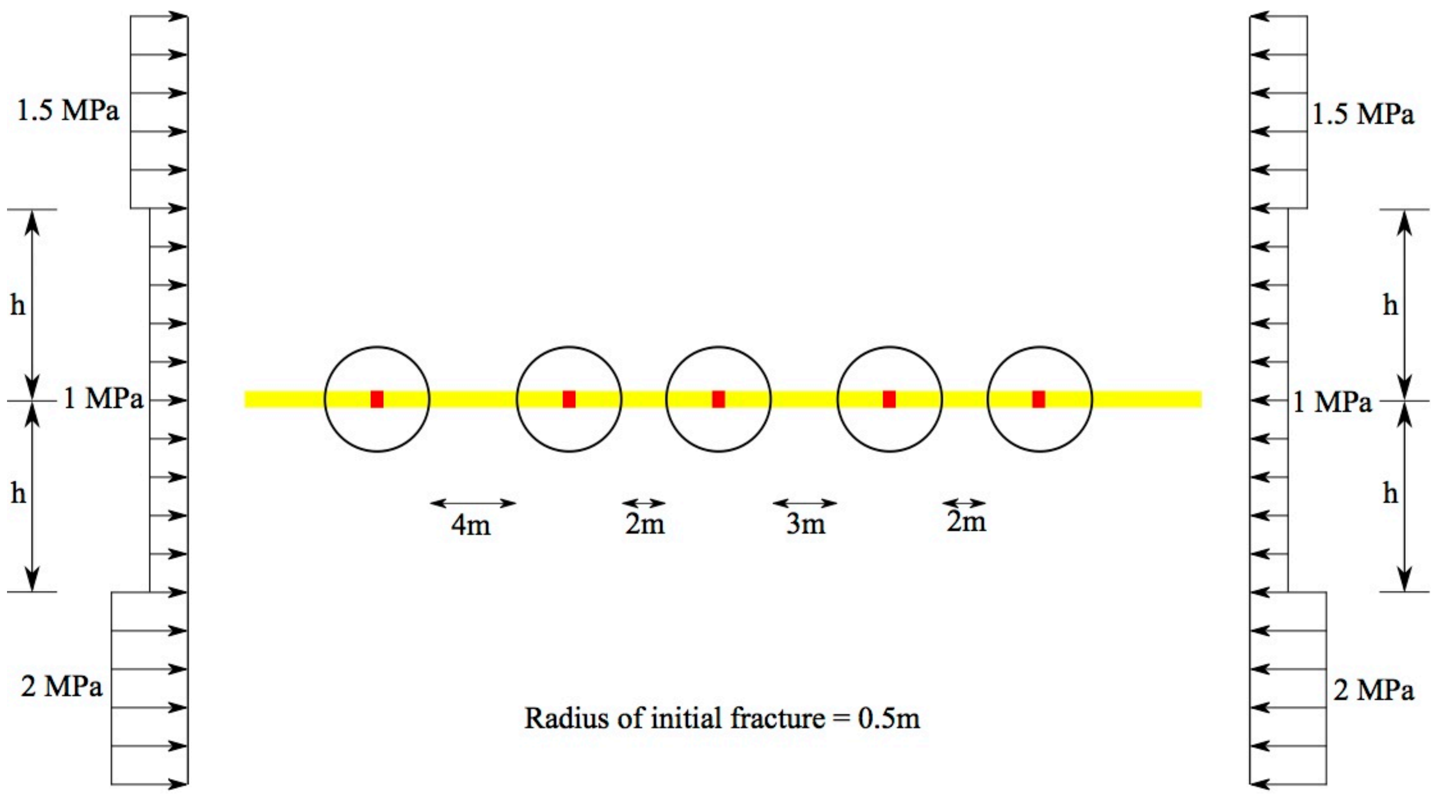
Challenges

- Propagation and coalescence of multiple fractures
- Highly non-convex fracture front after coalescence



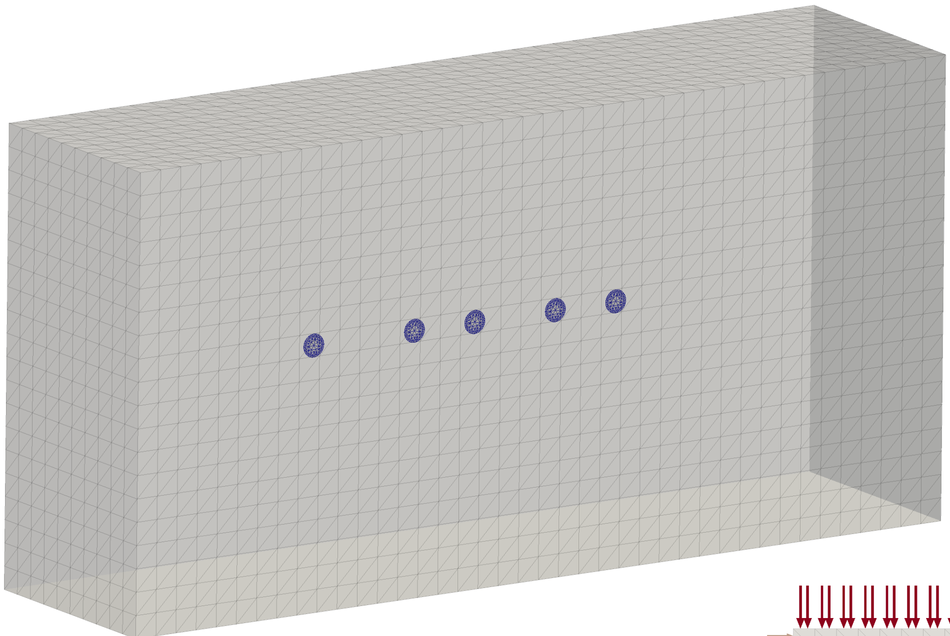
Coalescence of Longitudinal Fractures

- Propagation and coalescence from a horizontal well

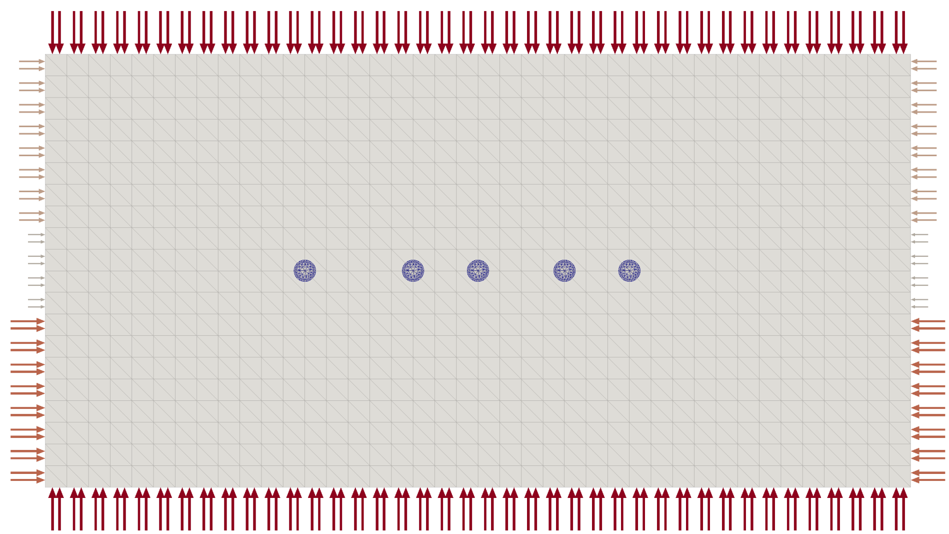


h = 2 m
p = 3.5 MPa

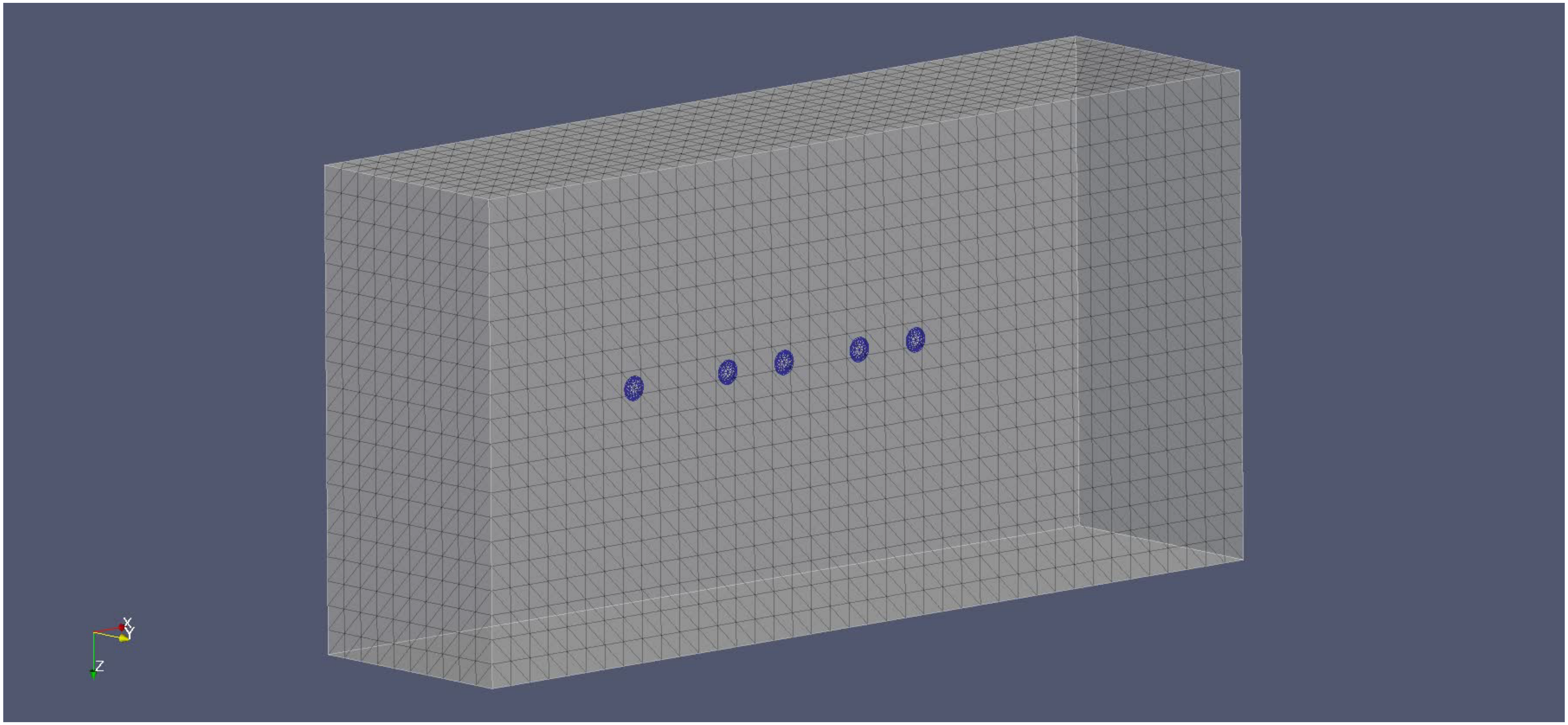
Coalescence of 3-D Fractures: GFEM Model



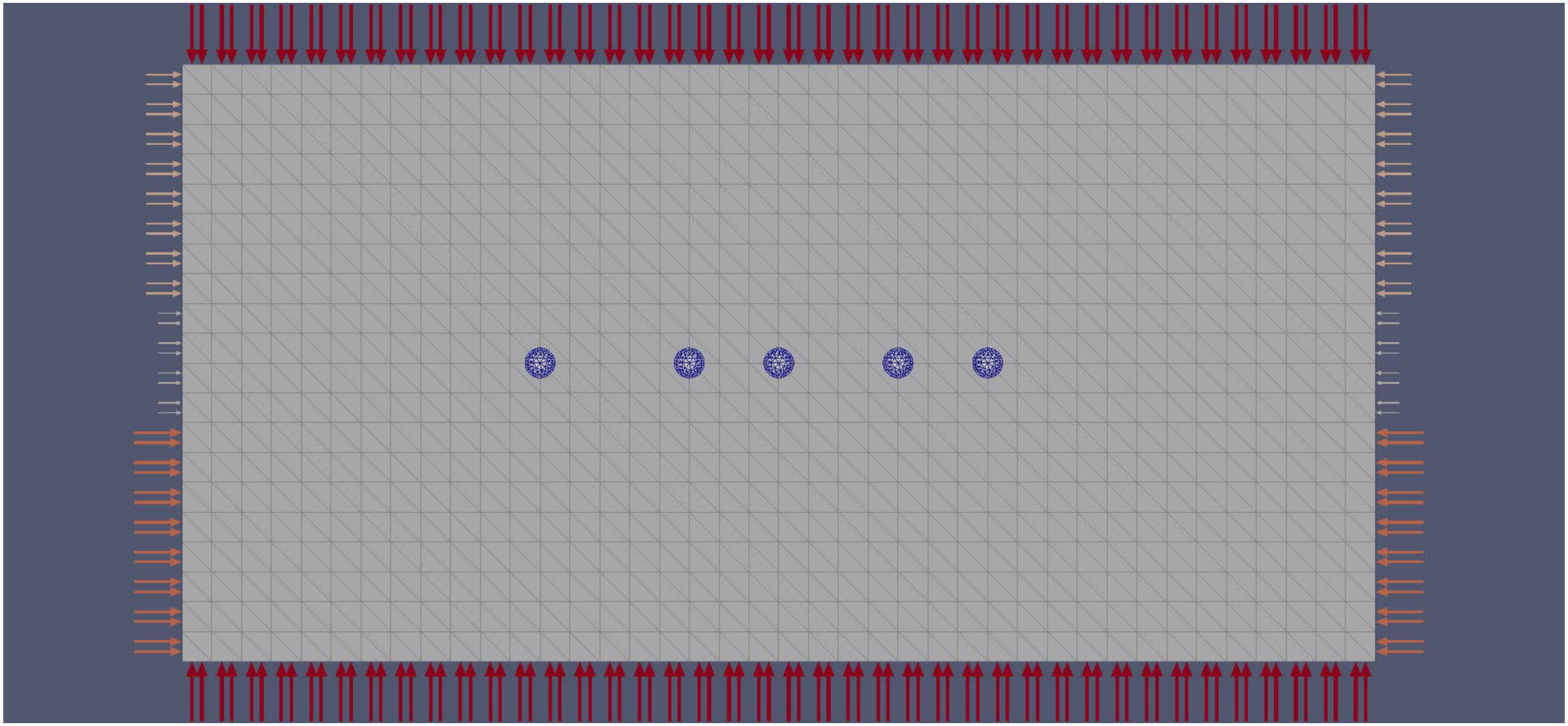
- Input mesh and fracture surfaces for GFEM simulation



Coalescence of 3-D Fractures

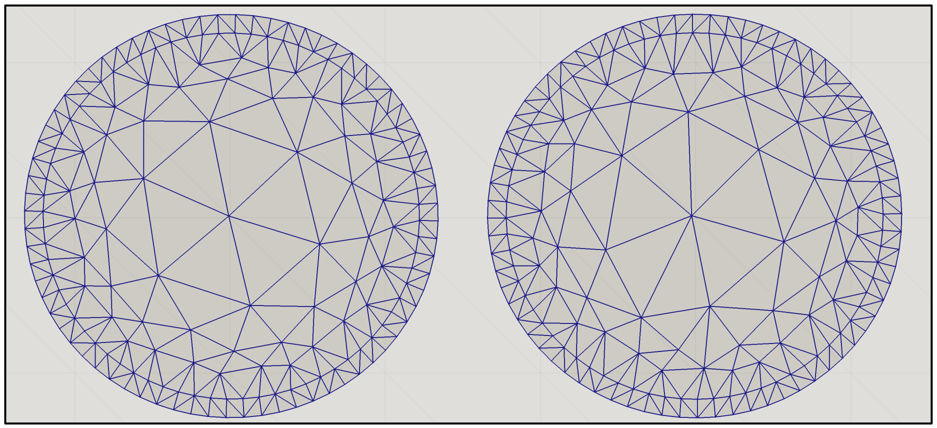


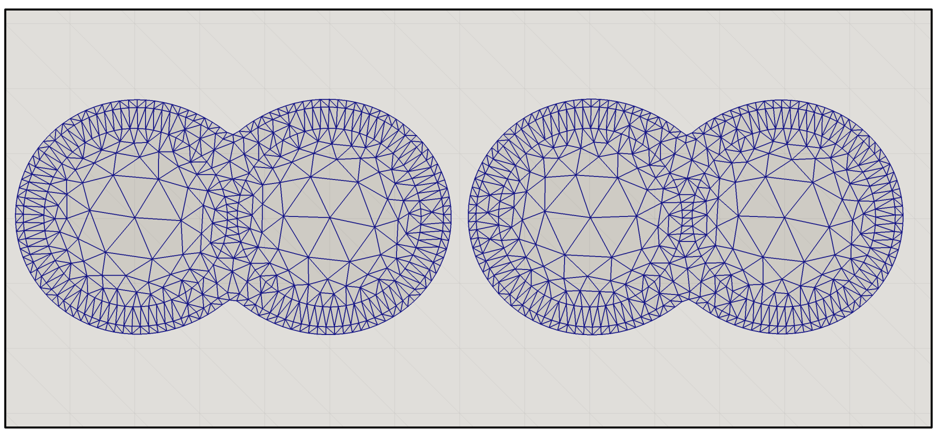
Coalescence of 3-D Fractures



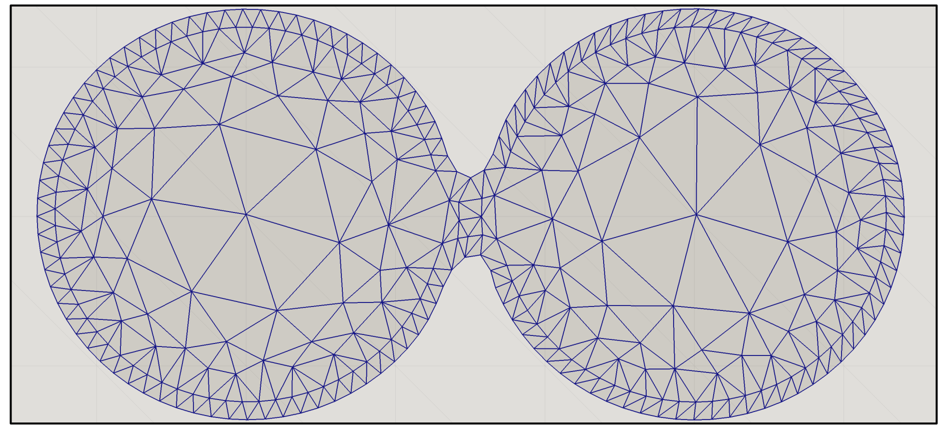
Coalescence of 3-D Fractures

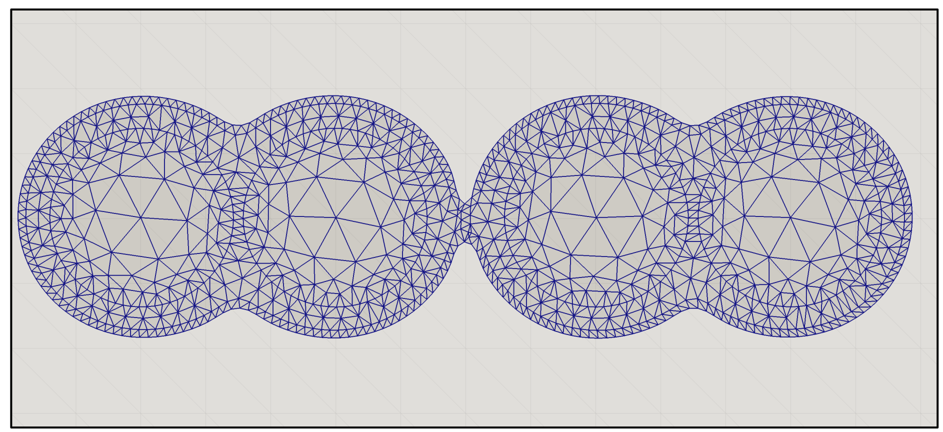
Fractures just prior to coalescence



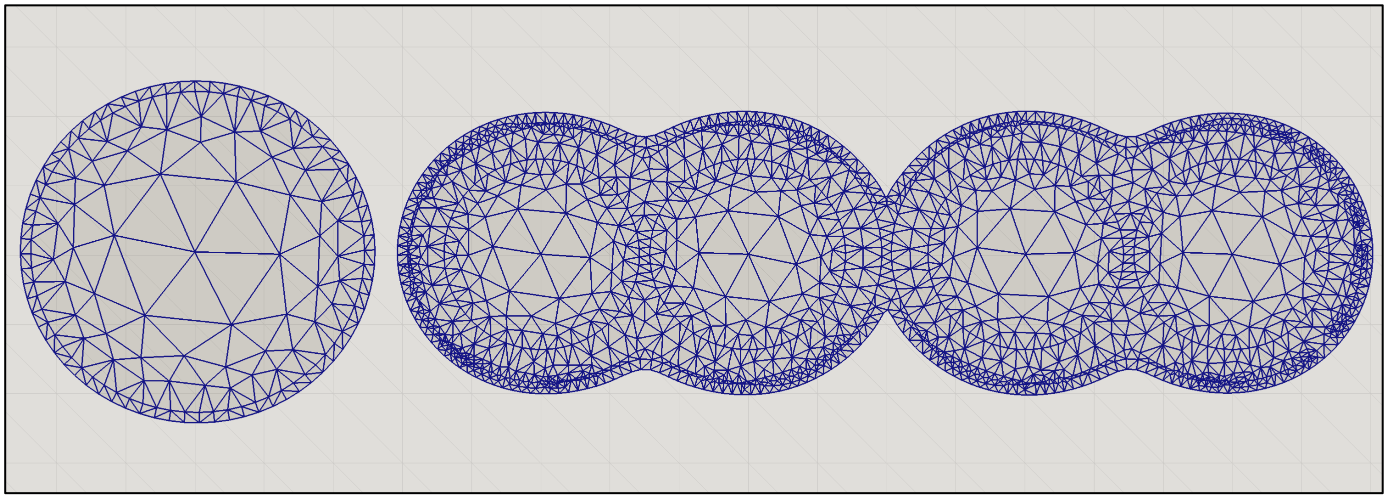


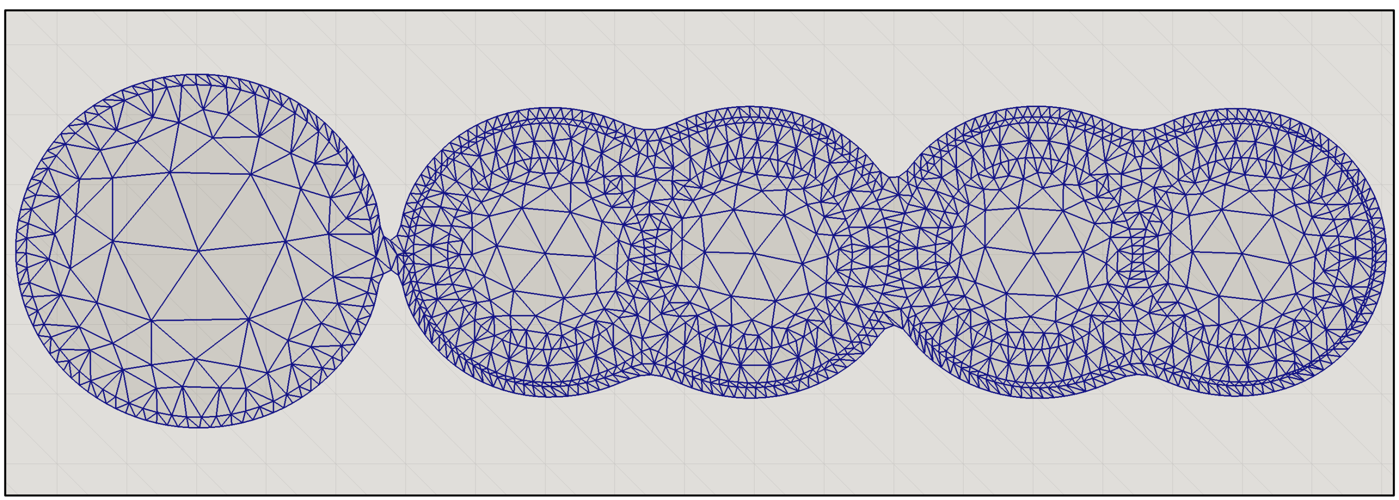
Fractures just after coalescence





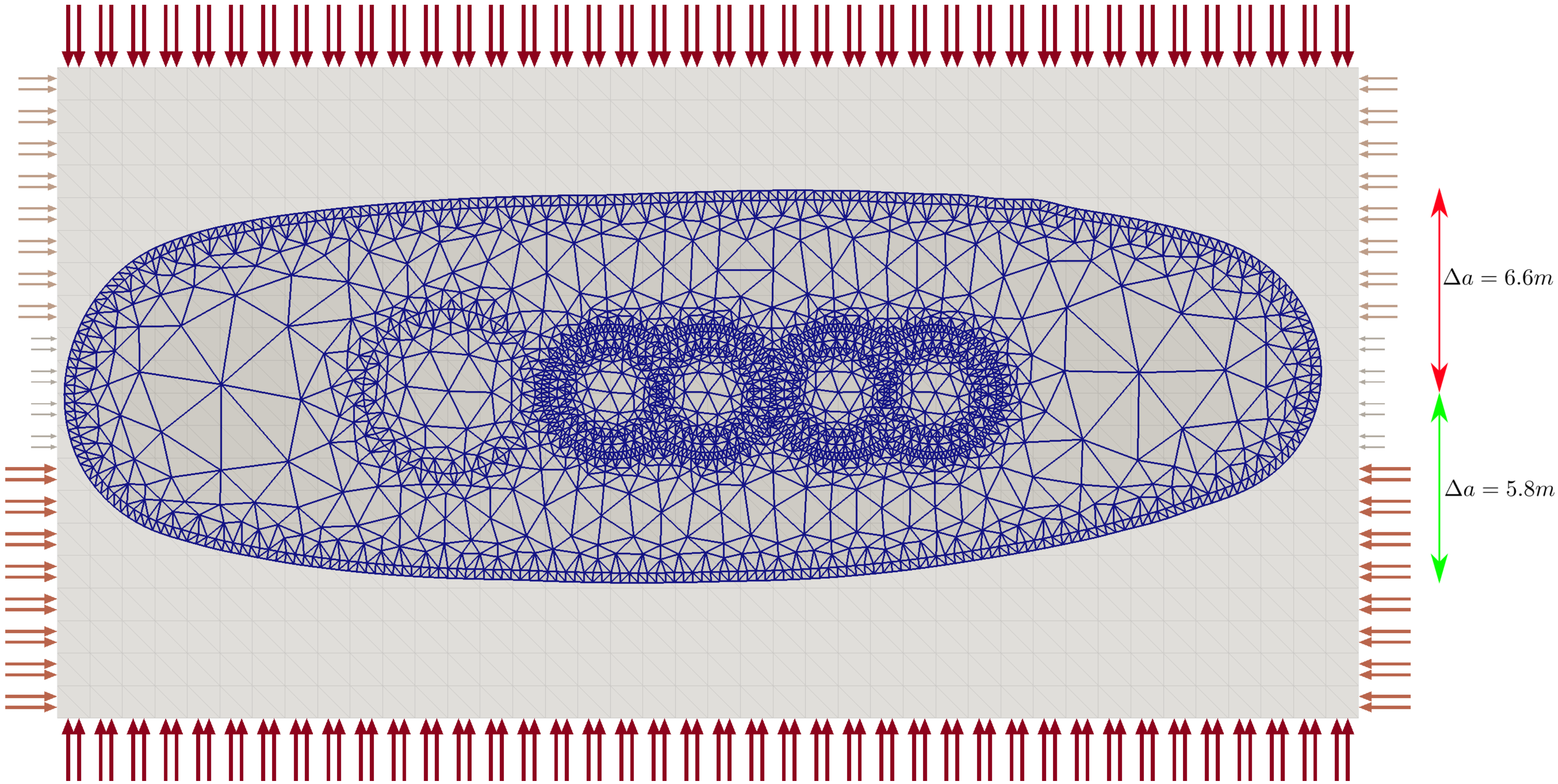
Coalescence of 3-D Fractures





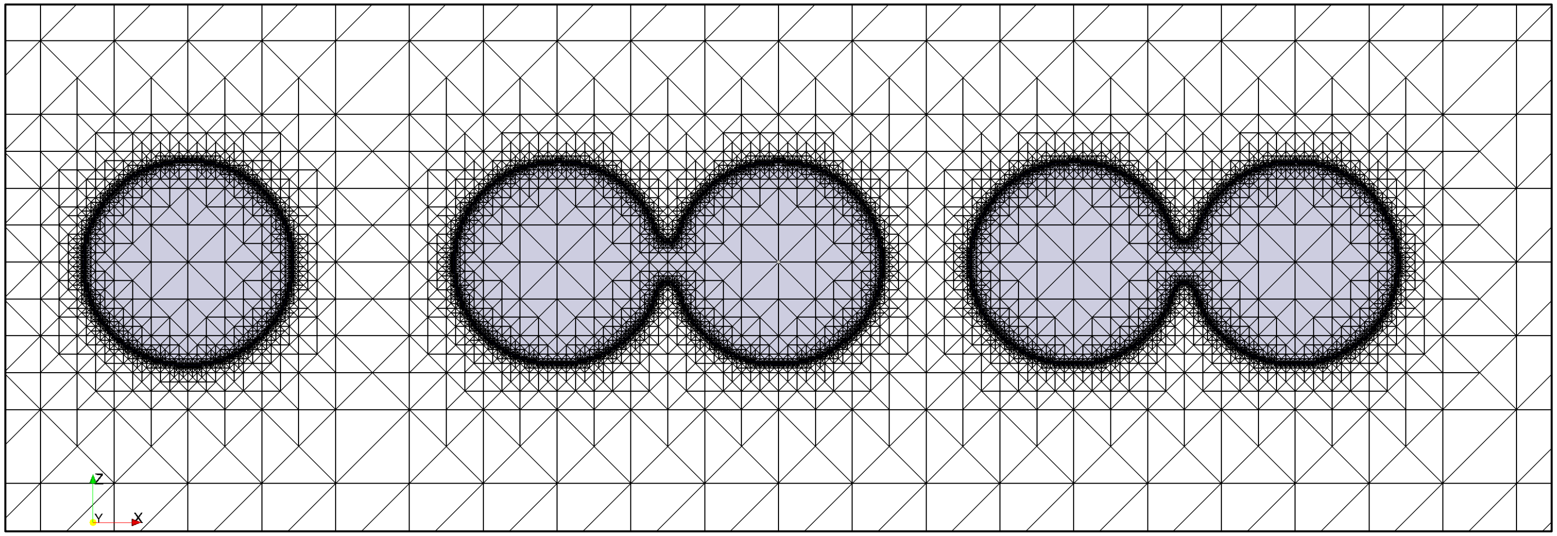
Coalescence of 3-D Fractures

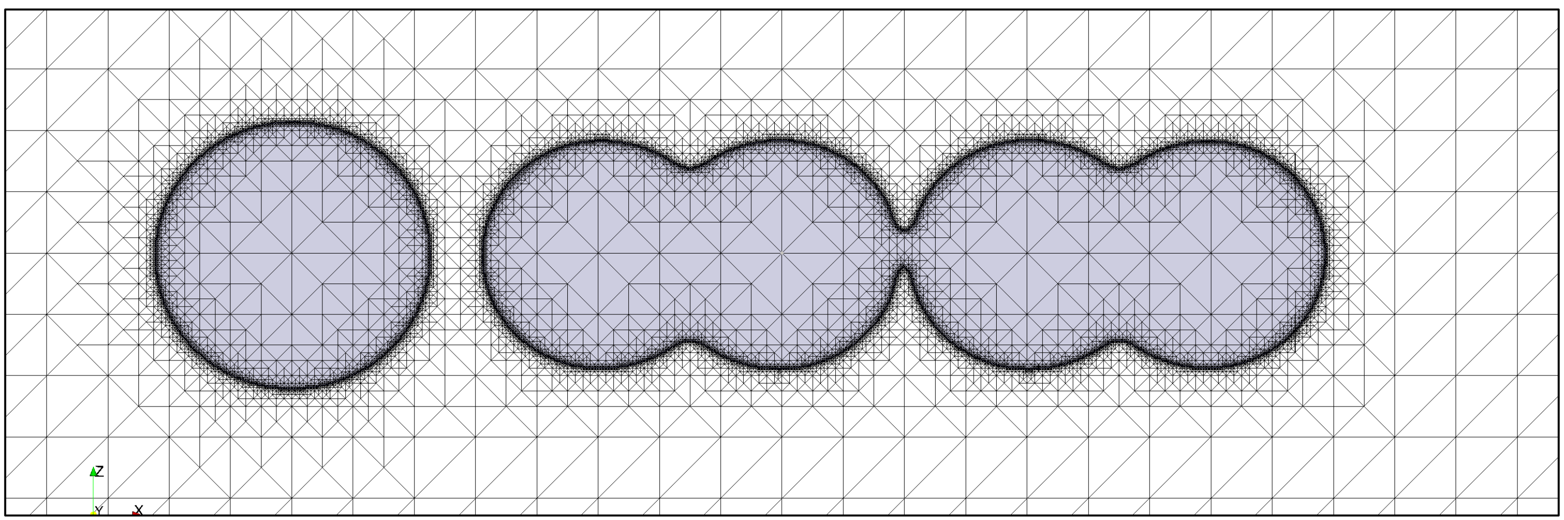
- Coalesced fracture at end of simulation



Coalescence of 3-D Fractures

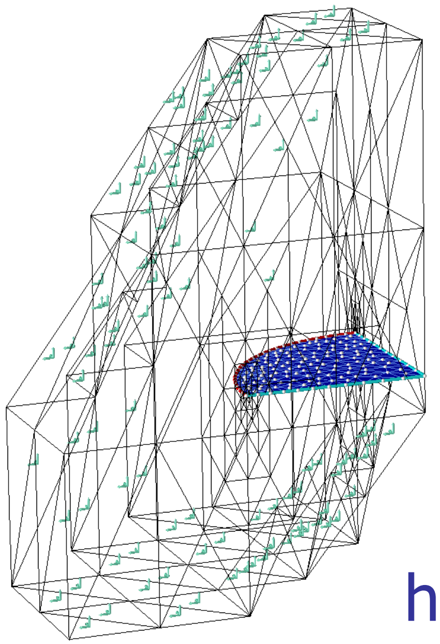
- Adaptive refinement along fracture fronts





Conclusions

- Generalized/Extended FEM removes several limitations of FEM
- It enables the solution of problems that are difficult or not practical with the FEM
- This is the case of three-dimensional fracture problems involving
  - ✓ Complex crack surfaces
  - ✓ Fluid-induced fracturing
  - ✓ Coalescence of 3-D fractures, etc.
- Open issues under investigation include
  - ✓ Coupling with fluid flow on fracture
  - ✓ Coalescence of non-planar fractures near a wellbore



Questions?

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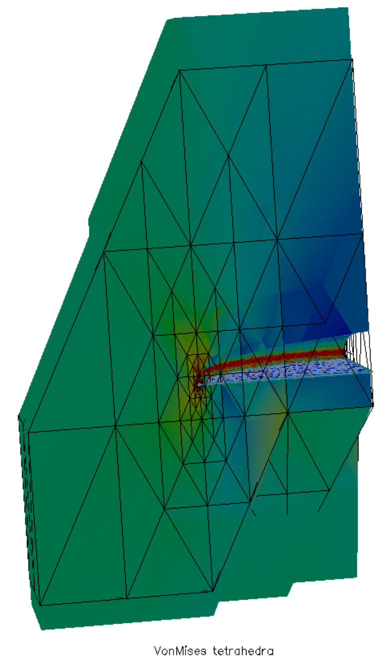
http://gfem.cee.illinois.edu/



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