UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

What Does It Take for a New Computational Method to Be Adopted by Engineers?

To the memory of Prof. Ted Belytschko

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Computational Science and Engineering

Presented at Meshfree Workshop for Large Scale CSE
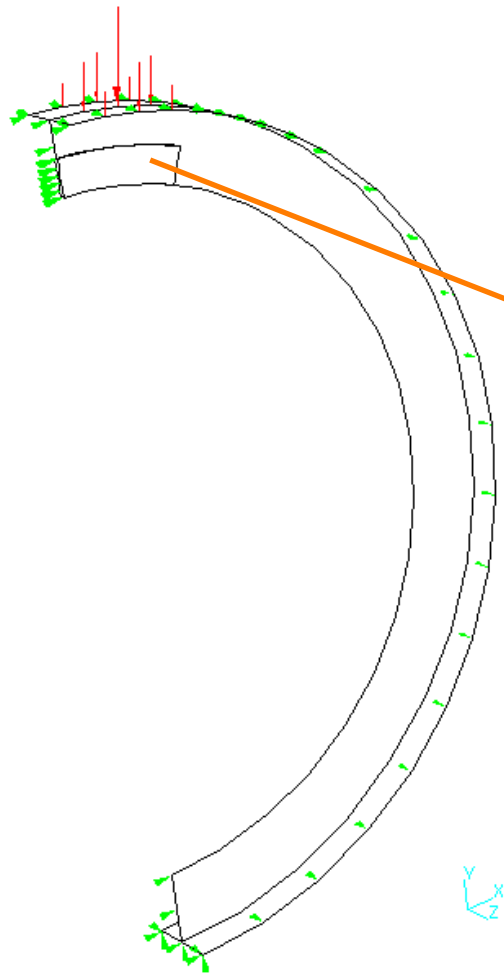Tampa, FL, October 27-28, 2014

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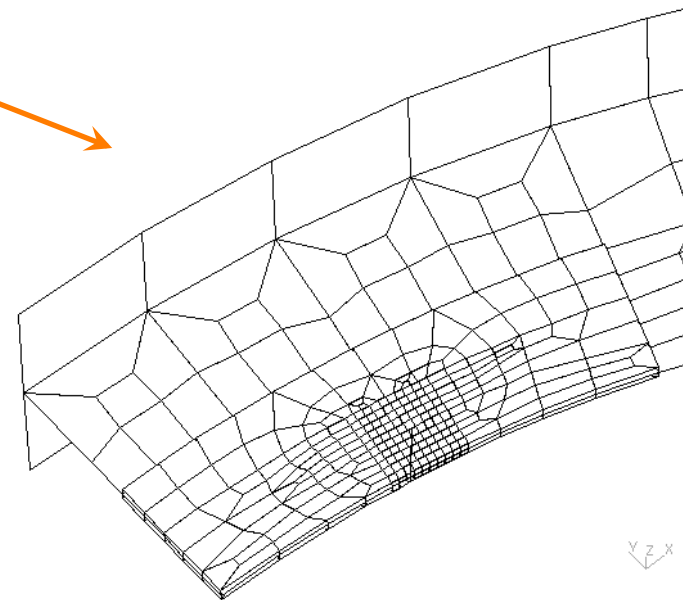# Lessons learned about method development while working at Altair Engineering

- Generalized FEM for 3-D, dynamic crack propagation (1999)

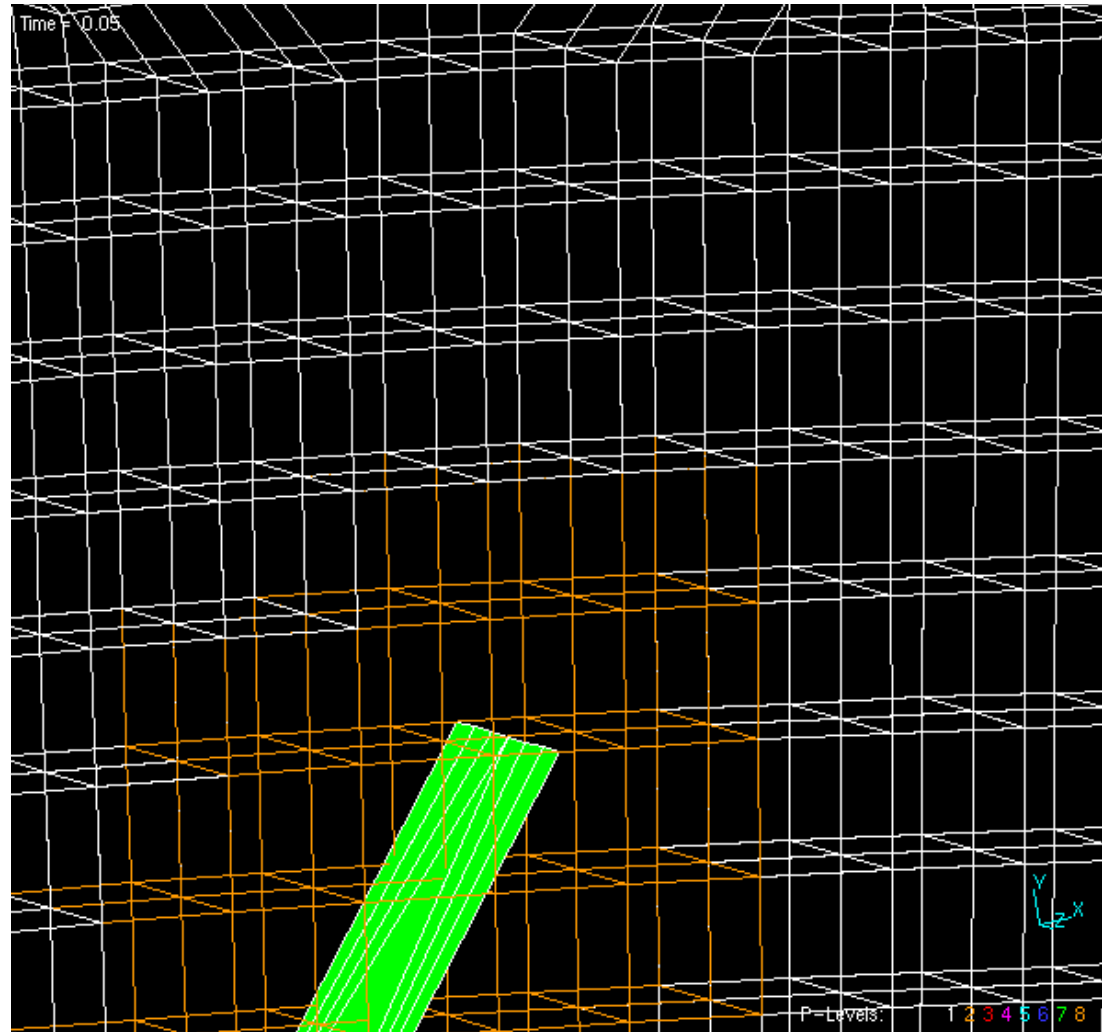http://www.tx.altair.com/

Cracked cylinder with ribs

3-D discretization with crack cutting finite elements

Lessons learned about method development while working at Altair Engineering

- Evolution of crack surface under dynamic loading

Lessons learned about method development while working at Altair Engineering

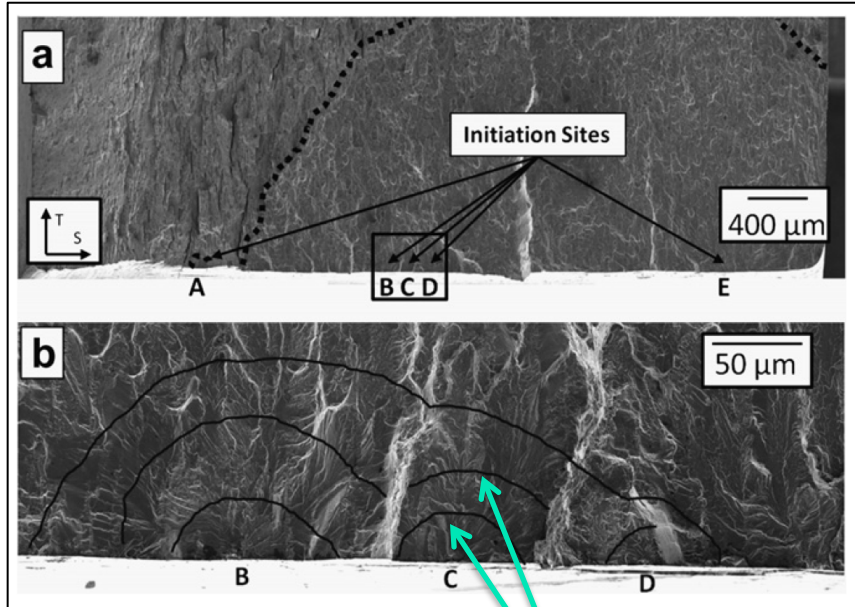What does it take for a new computational method to be adopted by engineers?

- Must demonstrate that new method can solve problems that are difficult or impossible to be solved by available methods

- Computational performance is important but....

- Just being faster than available methods is <u>not</u> enough!

- Robustness of method must be a top priority

- It must be possible to integrate the new method in an existing analysis flow within an engineering or research group.

- **Focus of this presentation:**
  - Generalized/Extended Finite Element Method (G/XFEM)
  - Simulation of interaction and coalescence of 3-D fractures

- **Goal:** Demonstrate that the GFEM meets the above requirements for this class of problems
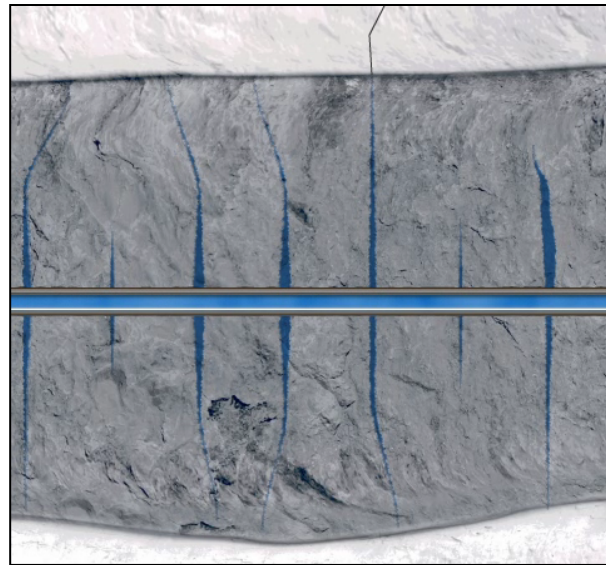
Crack Growth and Coalescence

✓ Understanding crack coalescence is of great importance in many applications

Coalescence of fatigue micro-cracks

Cluster of hydraulic fractures propagating from a horizontal well
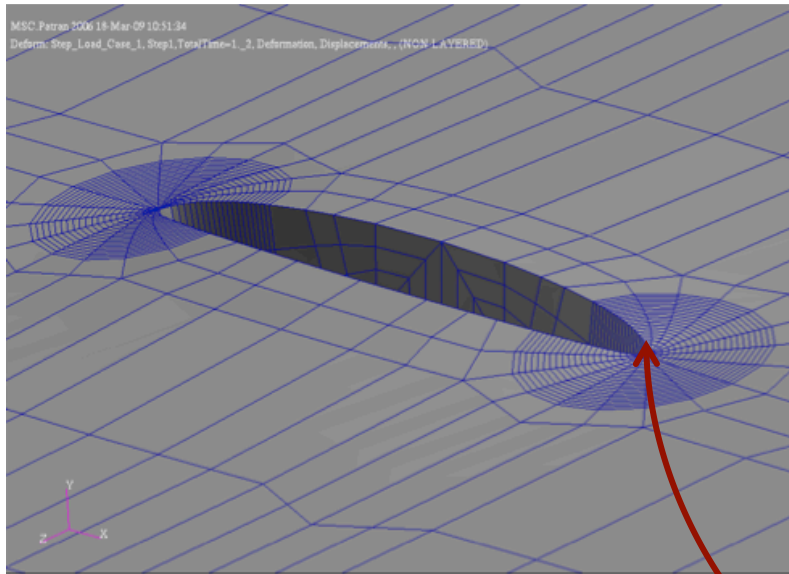

Crack fronts
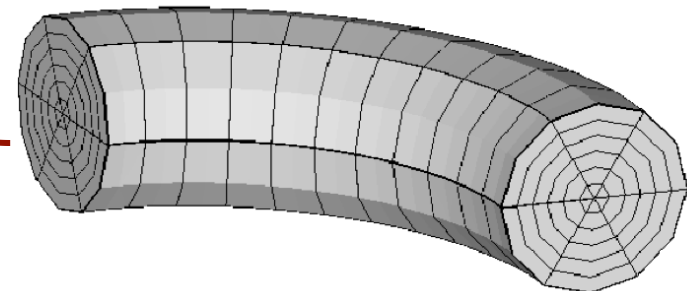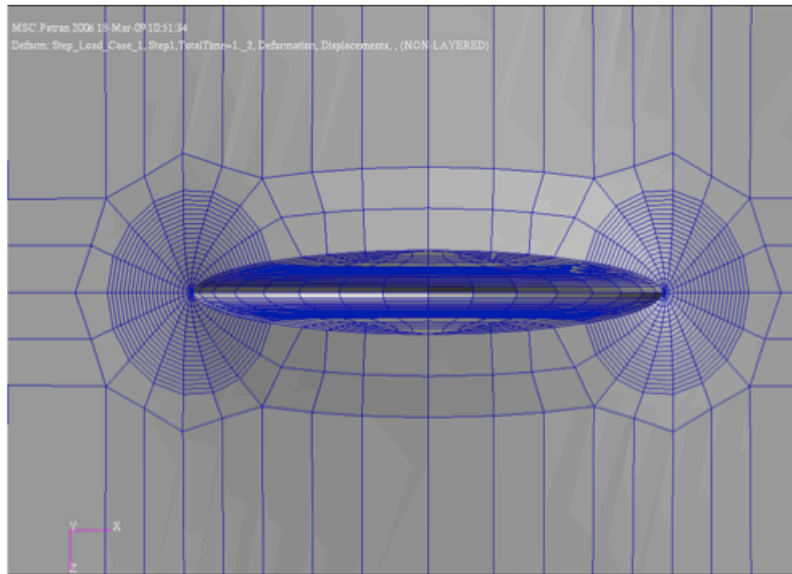
Reflective crack in asphalt overlay

Modeling 3-D Fractures: Limitations of Standard FEM

- It is not "just" fitting the 3-D <u>evolving</u> fracture
- FEM meshes must satisfy special requirements for acceptable accuracy
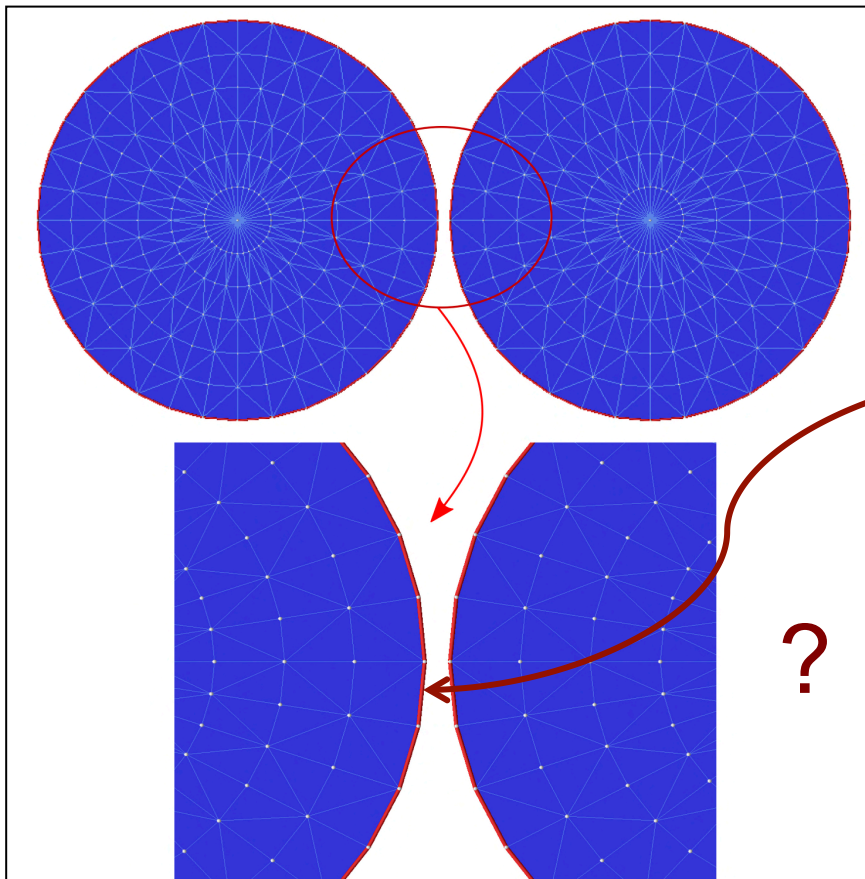

FEM mesh for a surface fracture
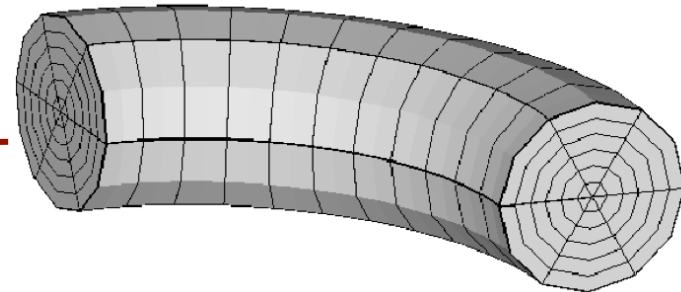
Mesh with quarter-point elements

Limitations of Standard FEM

- Difficulties arise if fracture front is close to complex geometrical features
- Fracture surfaces with sharp turns
- Coalescence of fractures

- Not possible in general to automatically create structured meshes along both fracture fronts when they are in close proximity

- Even with these crafted meshes and quarter-point elements, convergence rate of std FEM is slow (*controlled by singularity at fracture front*)

Outline

- Introduction

- Basic ideas of GFEM

- Application: Coalescence of 3-D hydraulic fractures

- Non-intrusive implementation of GFEM with global-local enrichment functions

- Conclusions and assessment

Generalized Finite Element Method

- GFEM is a Galerkin method with special test/trial space given by

$$\mathbb{S}_{GFEM} = \mathbb{S}_{FEM} + \mathbb{S}_{ENR}$$

Low order FEM space   Enrichment space with functions related to the given problem
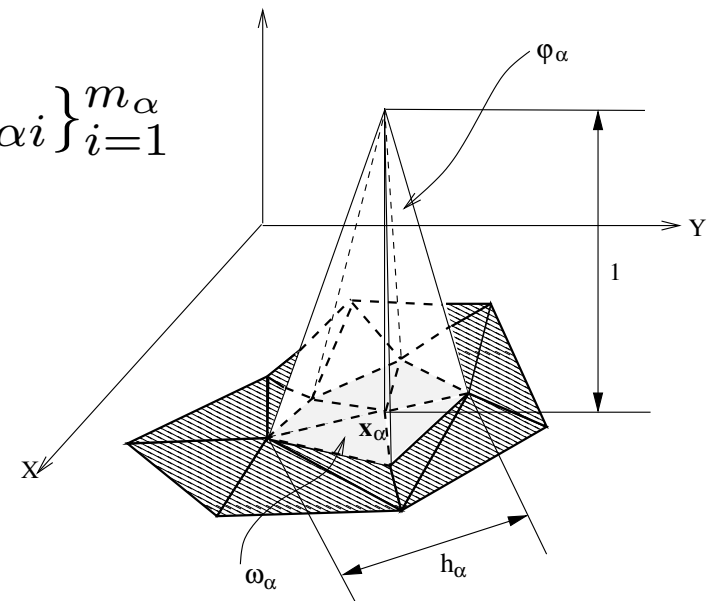
$$\mathbb{S}_{FEM} = \sum_{\alpha \in I_h} c_\alpha \varphi_\alpha, \quad c_\alpha \in \mathbb{R}$$

$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$L_{\alpha i} \in \chi_\alpha(\omega_\alpha)$$

Enrichment function   Patch space
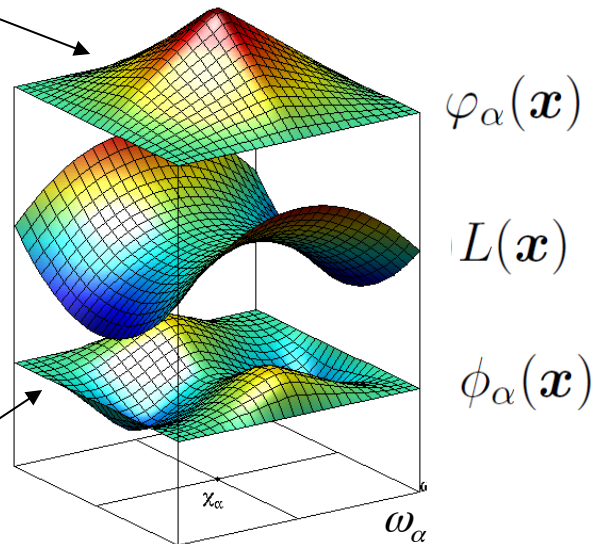
Generalized Finite Element Method

$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$\phi_{\alpha i}(x) = \varphi_\alpha(x) L_{\alpha i}(x) \qquad \sum_\alpha \varphi_\alpha(\boldsymbol{x}) = 1$$
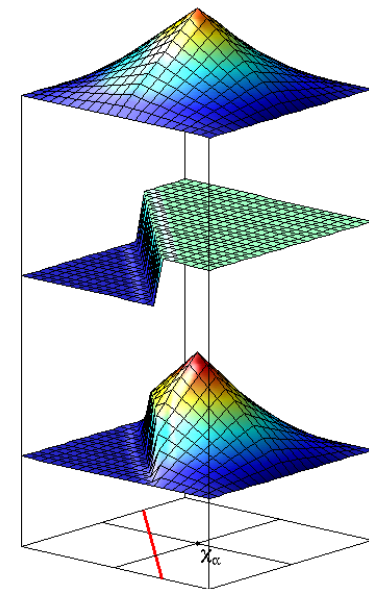
Linear FE shape function

Enrichment function

GFEM shape function

$\varphi_\alpha(\boldsymbol{x})$

$L(\boldsymbol{x})$

$\phi_\alpha(\boldsymbol{x})$

[Oden, Duarte & Zienkiewicz, 1996]

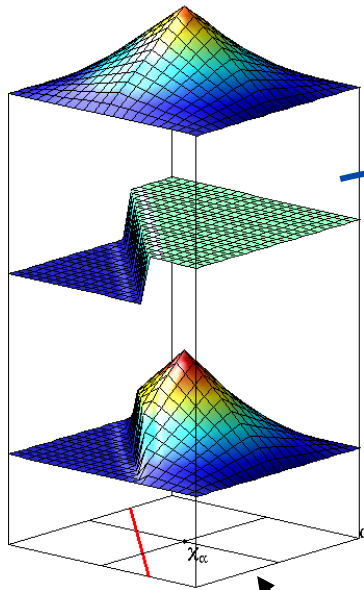• Allows construction of shape functions incorporating a-priori knowledge about solution

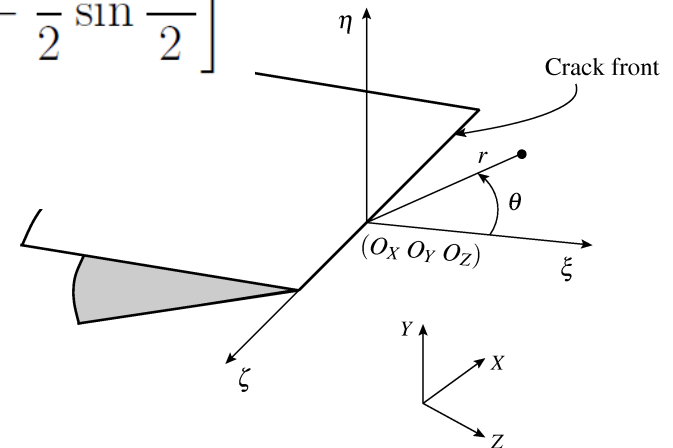Discontinuous enrichment [Moes et al., 1999]

GFEM Approximation for 3-D Fractures

$$\mathbb{S}_{GFEM}(\Omega) = \left\{ \boldsymbol{u}^{hp} = \sum_{\alpha \in I_h} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$
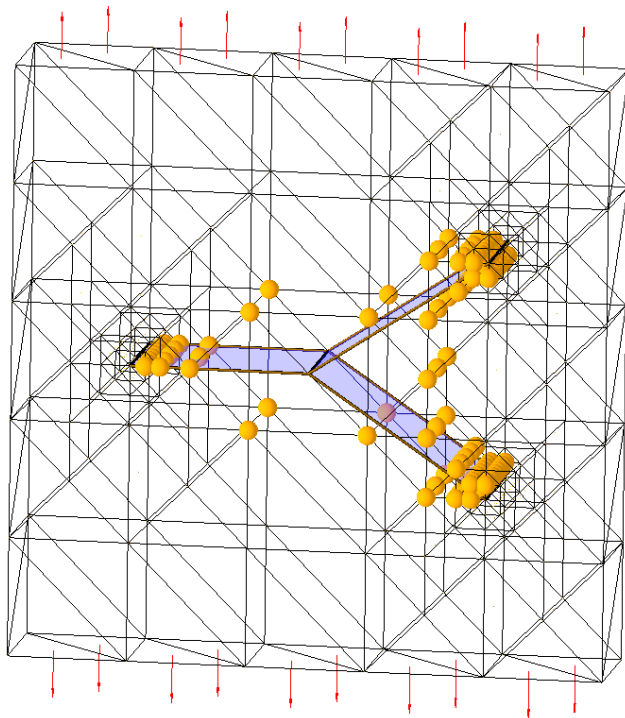
$$\breve{L}^\xi_{\alpha 1}(r, \theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

[Duarte & Oden 1996]

$$\breve{L}^\eta_{\alpha 1}(r, \theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

$$\breve{L}^\zeta_{\alpha 1}(r, \theta) = \sqrt{r}\sin\frac{\theta}{2}$$
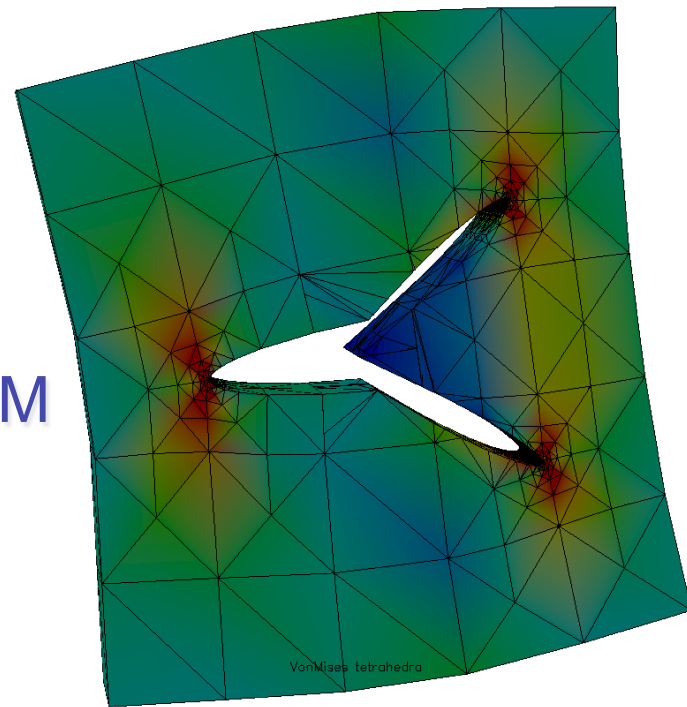
patch ω_α

Modeling Fractures with the GFEM

- Fractures are modeled via enrichment functions, *not* the FEM mesh
- Mesh refinement *still required* for acceptable accuracy

hp-GFEM

● = Nodes with discontinuous enrichments

Von Mises stress

[Duarte et al., International Journal Numerical Methods in Engineering, 2007]
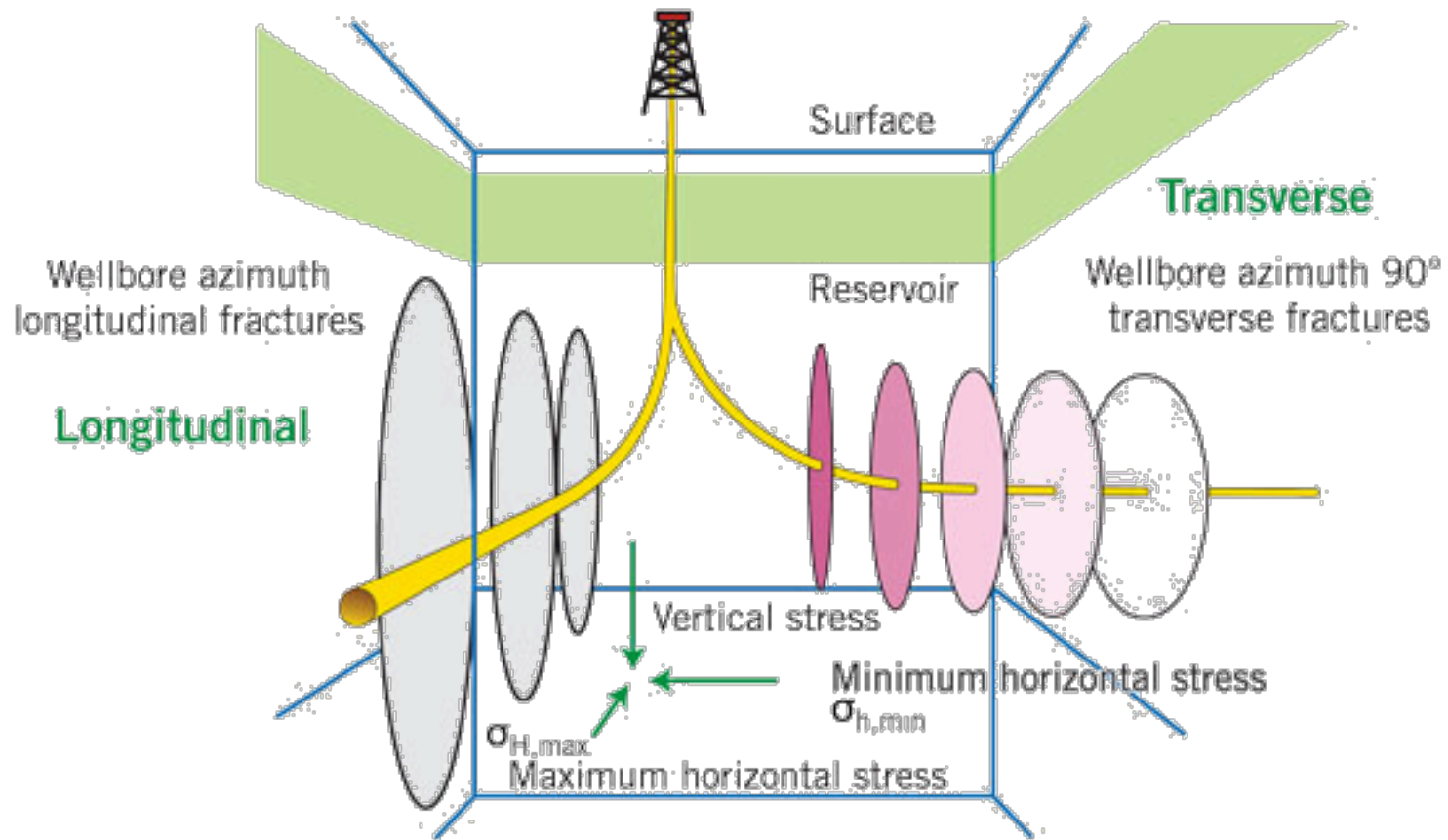
Outline

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- Basic ideas of GFEM

- Application: Coalescence of 3-D hydraulic fractures

- Non-intrusive implementation of GFEM with global-local enrichment functions

- Conclusions and assessment

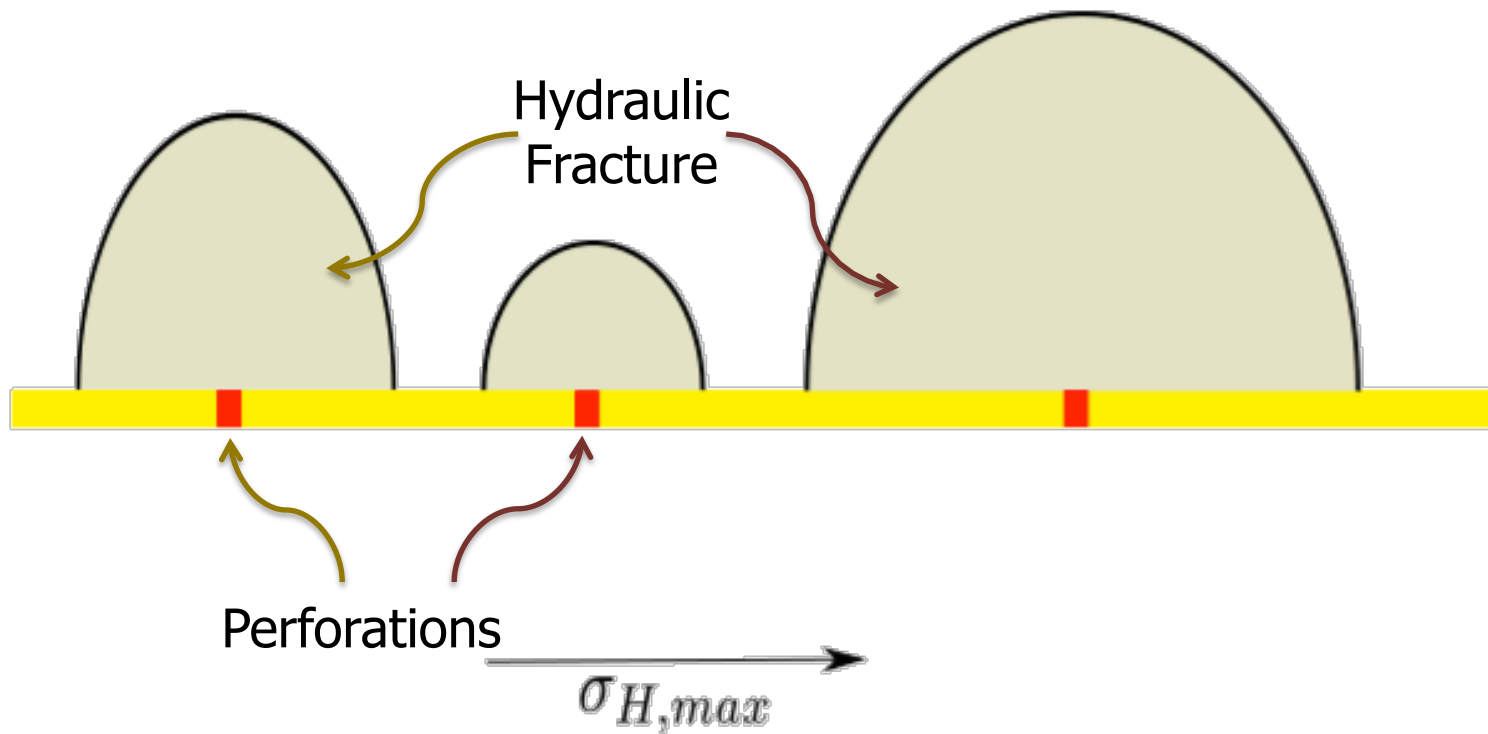# Typical Hydraulic Fracturing Clusters

[Z. Rahim et al., 2012]

Longitudinal Fractures

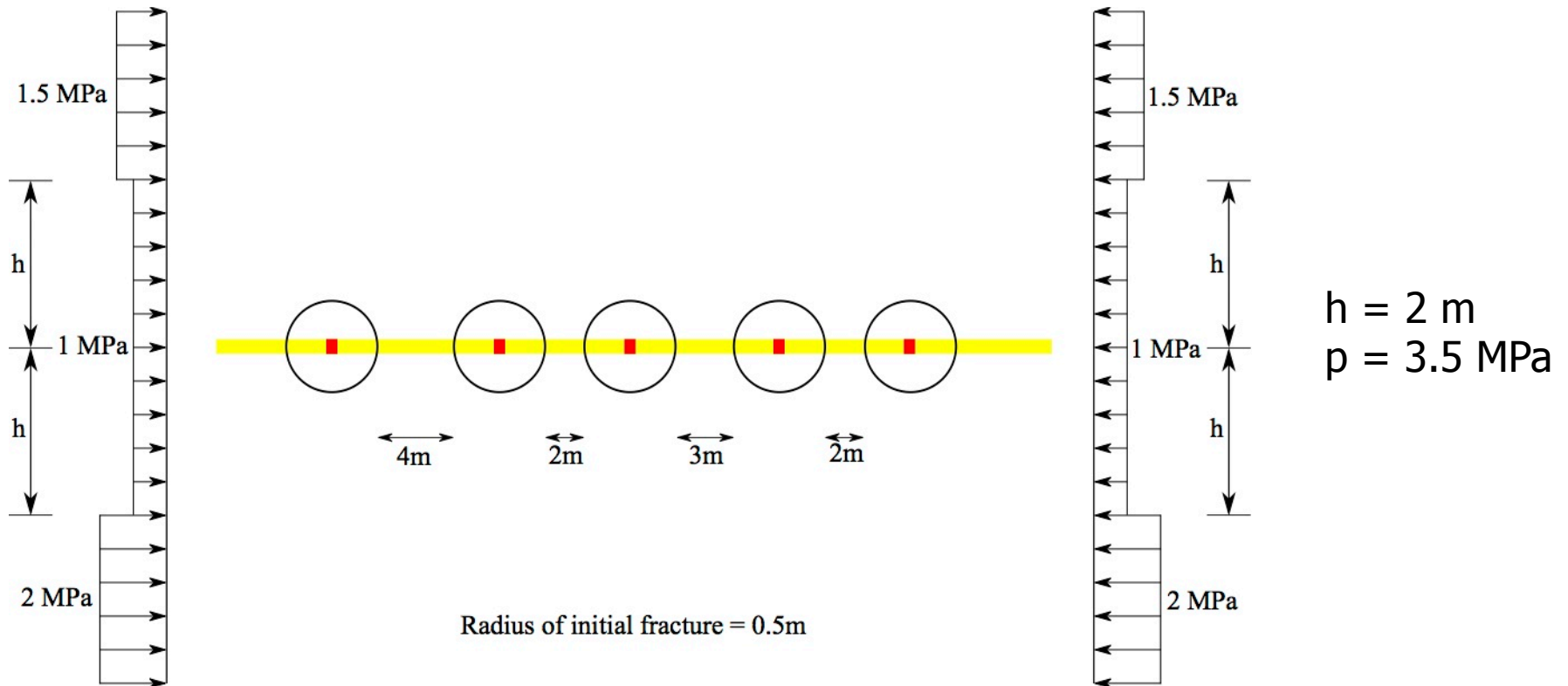- Develop perpendicular to minimum in-situ stress
- Fractures along the length of the wellbore
- Planar fractures from the perforation

Hydraulic
Fracture

Perforations

$$\sigma_{H,max}$$

Coalescence of Longitudinal Fractures

- Propagation and coalescence from a horizontal well

1.5 MPa

h

1 MPa

h

2 MPa

4m   2m   3m   2m

Radius of initial fracture = 0.5m

1.5 MPa
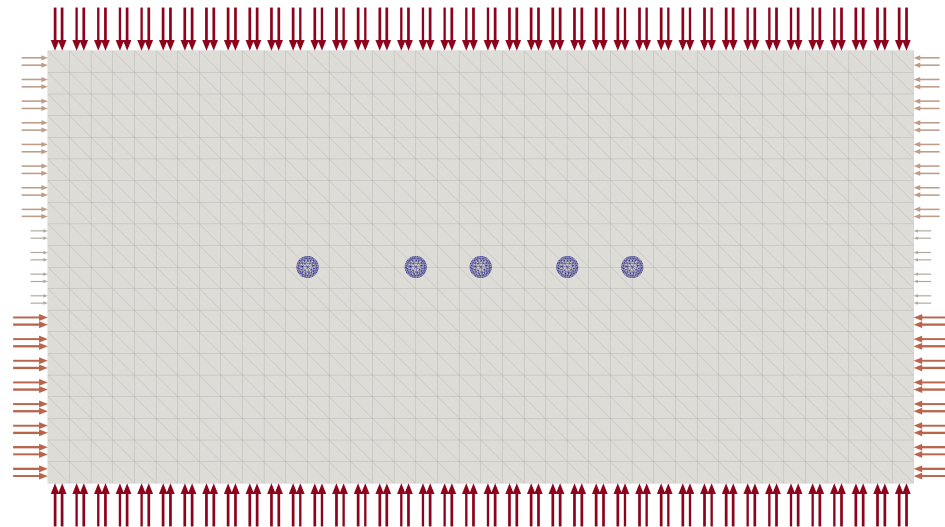
h

1 MPa

h

2 MPa

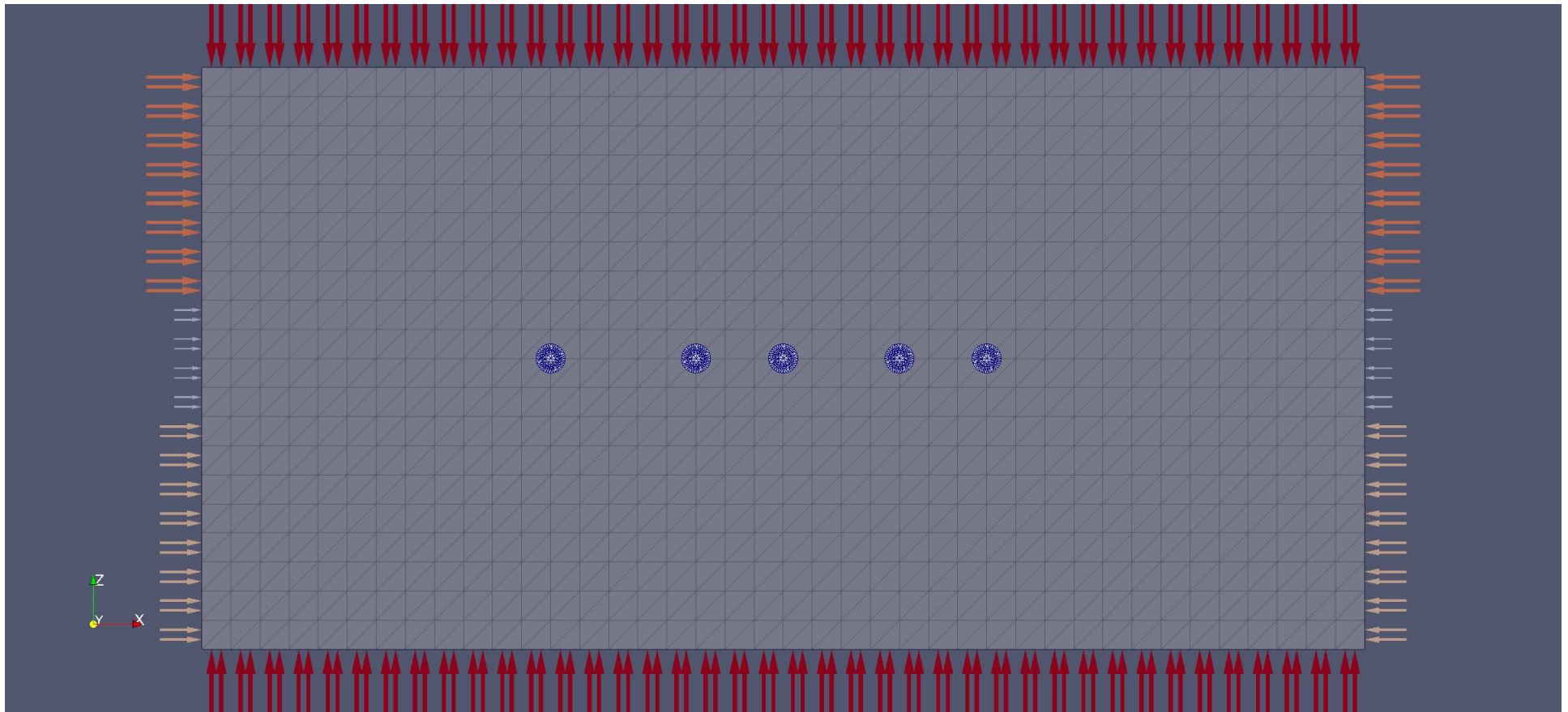h = 2 m
p = 3.5 MPa

Coalescence of 3-D Fractures: GFEM Model
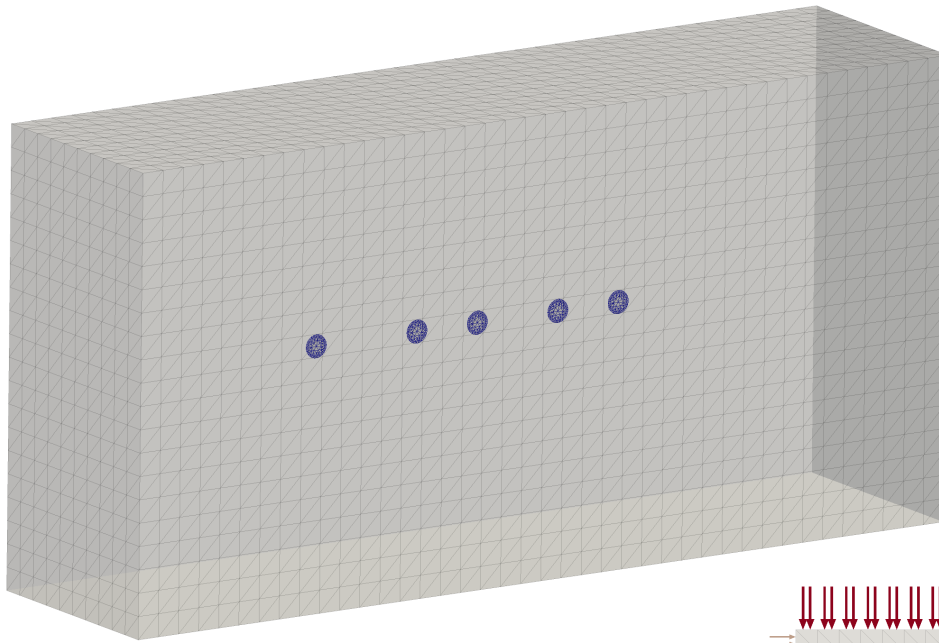
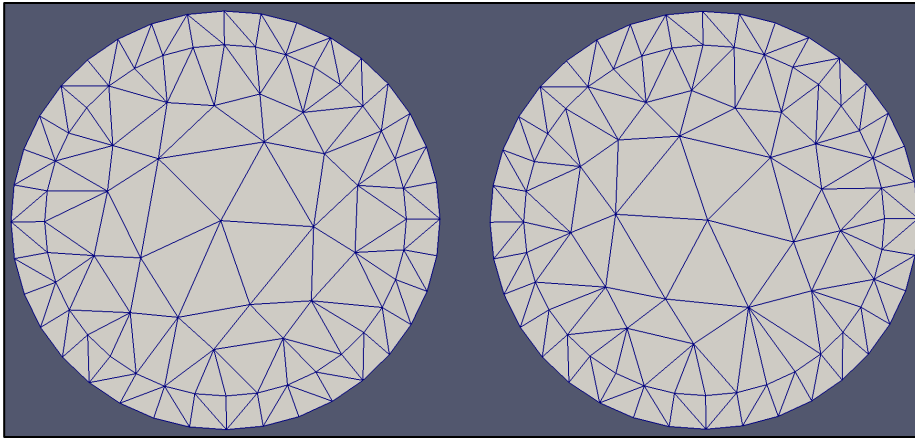- Input mesh and fracture surfaces for GFEM simulation

Coalescence of 3-D Fractures

Coalescence of 3-D Fractures

Fractures just prior to coalescence

Fractures just after coalescence
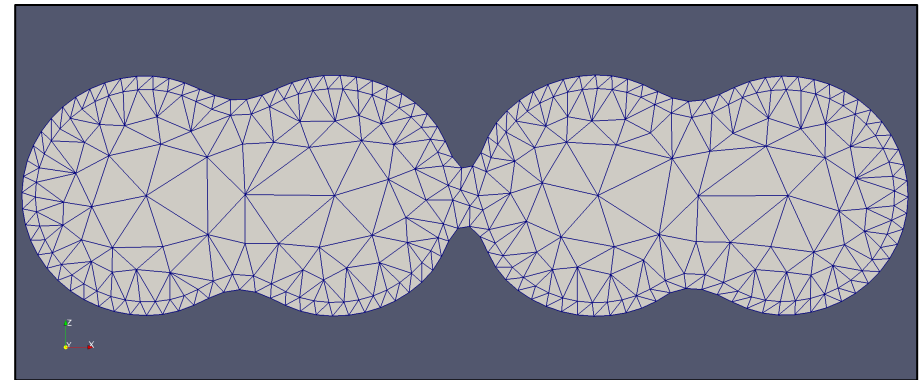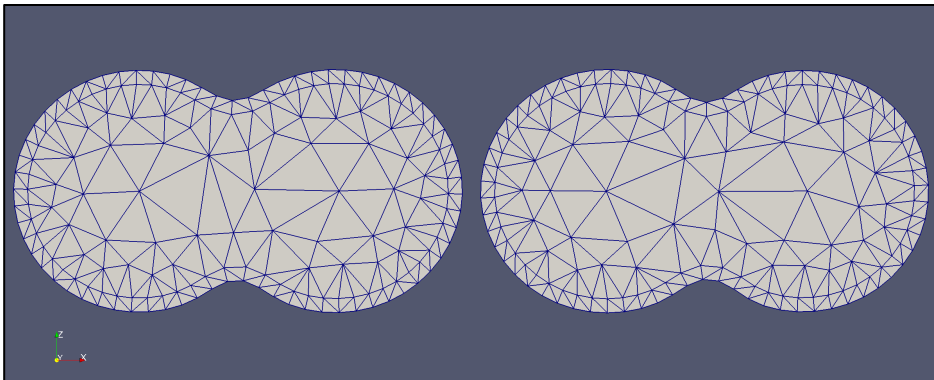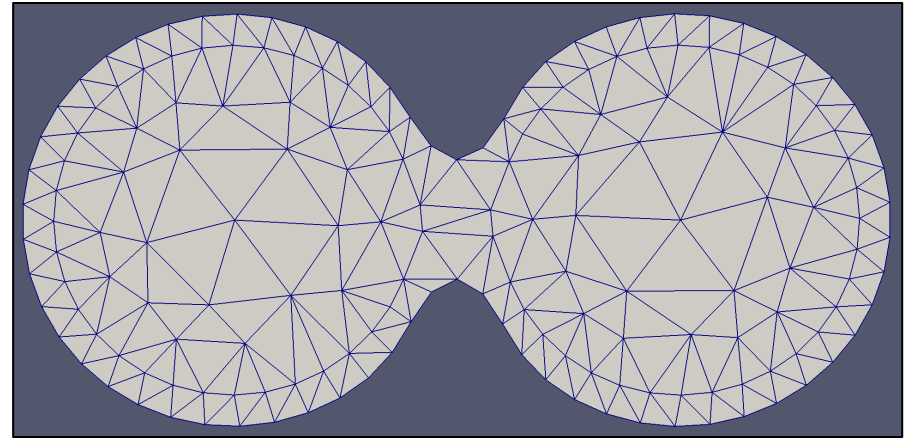
Coalescence of 3-D Fractures

- Adaptive refinement along fracture fronts
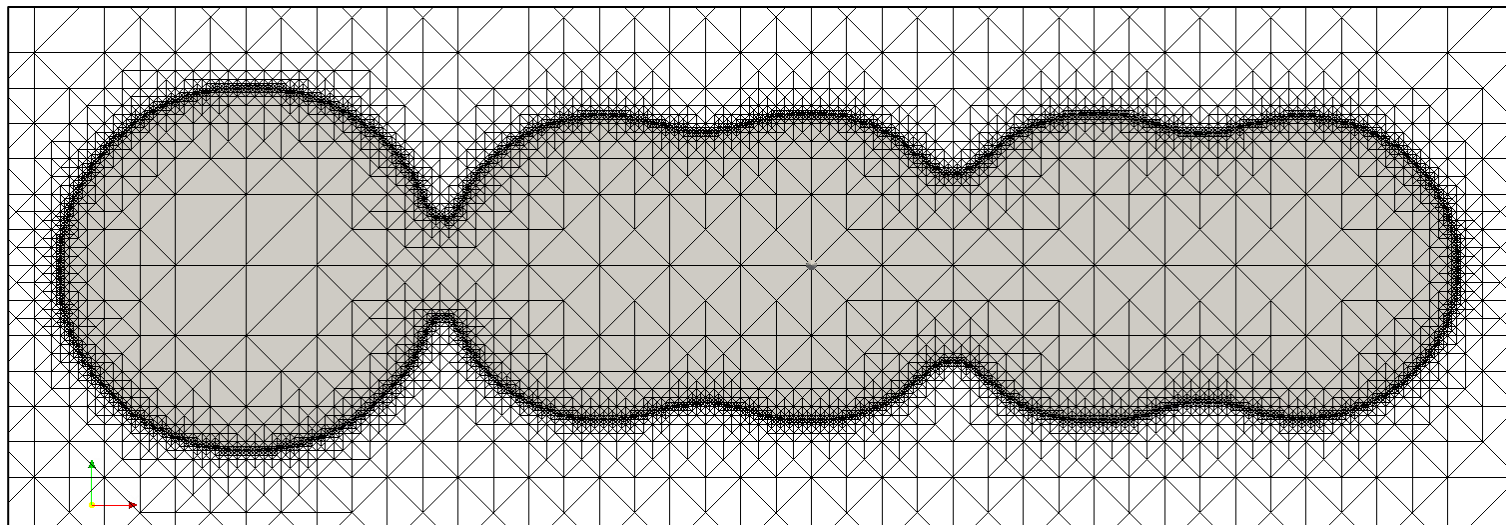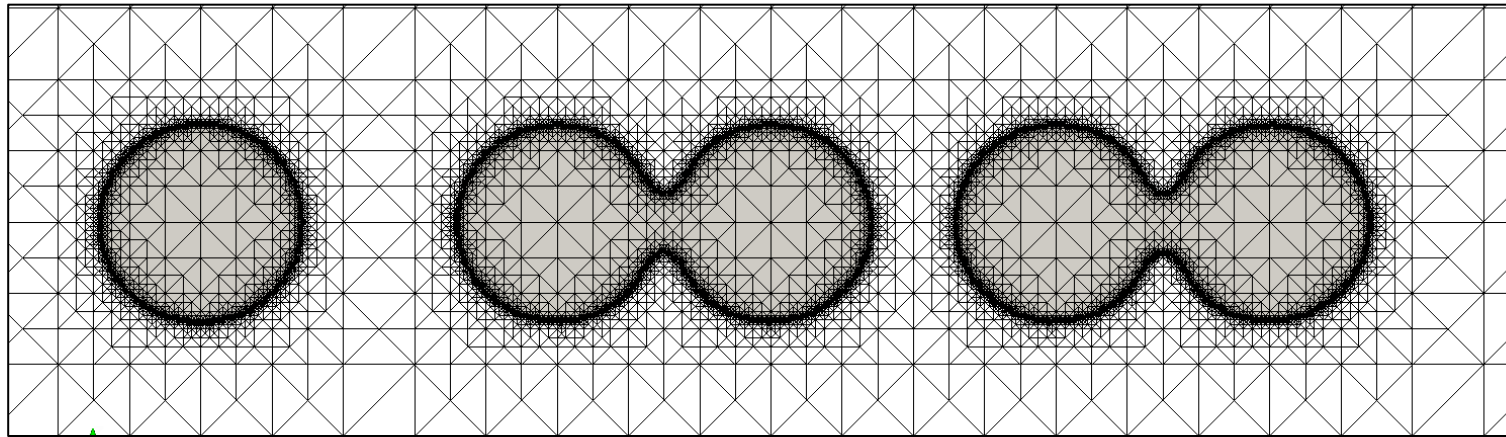
How to transition this method?

- Implementation of 3-D adaptive methods in legacy FEM codes is non trivial
- Numerical integration of singular and discontinuous functions is much more difficult than polynomial shape functions used in the FEM

Strategy:

- Non-intrusive implementation of GFEM with global-local enrichment functions

Bridging Scales with Global-Local Enrichment Functions*

■ Enrichment functions computed from solution of local boundary value problems: <u>Global-Local enrichment functions</u>

Linear FE shape function

GFEM shape function

χ_α

Enrichment = Numerical solutions of BVP
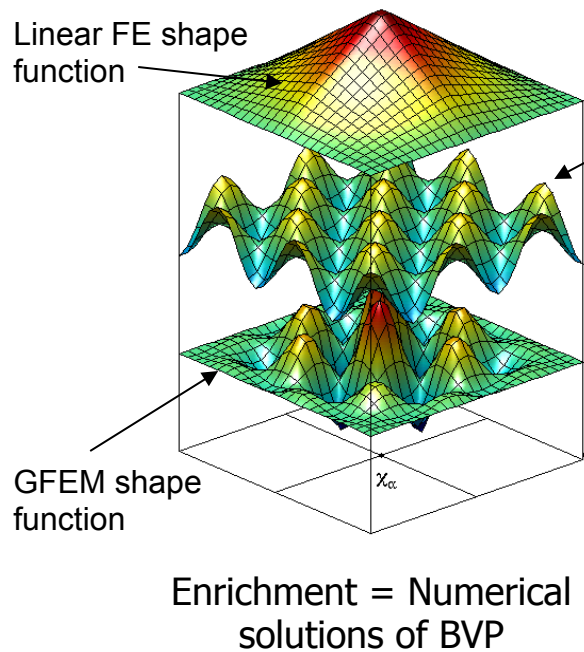
■ **Idea:** *Use available numerical solution at a simulation step to build shape functions for next step* (quasi-static, transient, non-linear, etc.)

■ Enrichment functions are produced numerically on-the-fly through a global-local analysis

■ Use a *coarse* mesh enriched with Global-Local (GL) functions

■ GFEM^{gl} = GFEM with global-local enrichments

*[Duarte et al. 2005, 2007, 2008, 2010, 2011, 2014]

Global-Local Enrichments for 3-D Fractures

Key Idea: Use solution of global problem at simulation *k* to build enrichment functions for step *k+1*

BCs from step *k*

Solve local problem *using best available method*

Enrichment for step *k+1*

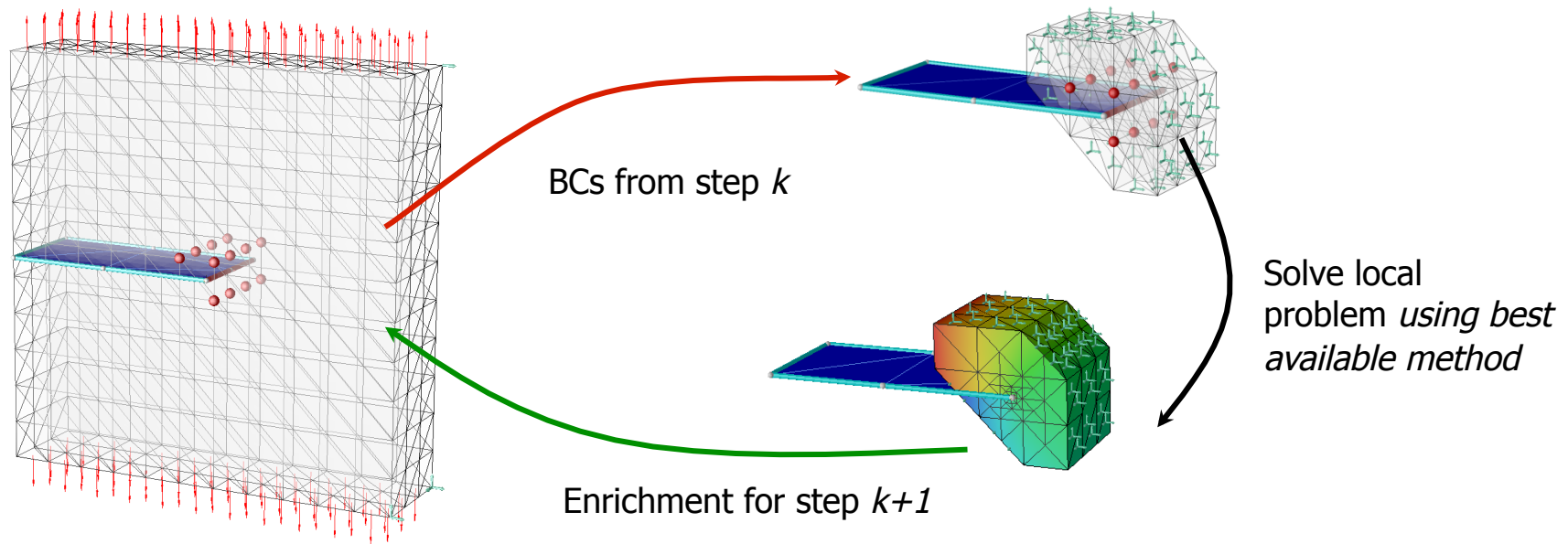- Discretization spaces updated on-the-fly with global-local enrichment functions

$$
\boldsymbol{X}_G^{k+1}(\Omega_G) = \left\{ \boldsymbol{u} = \underbrace{\sum_{\alpha=1}^{N} \varphi_\alpha(\boldsymbol{x})\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{coarse-scale approx.}} + \underbrace{\sum_{\beta \in \mathcal{I}_{gl}^k} \varphi_\beta(\boldsymbol{x})\boldsymbol{u}_\beta^{gl(k)}(\boldsymbol{x})}_{\text{fine-scale approx.}} \right\} \quad \boldsymbol{u}_\beta^{gl(k)} = \text{G-L enrichment}
$$

Computation of Solution at a Crack Step

$$u_G = \underbrace{\tilde{u}^0}_{\text{coarse scale (polynomial)}} + \underbrace{u^{gl}}_{\text{fine scale (G-L)}} = [N^0 N^{gl}] \begin{bmatrix} \tilde{u}^0 \\ \underline{u}^{gl} \end{bmatrix}$$

$\underline{\tilde{u}}^0$ = DOFs associate with coarse scale discretization

$\underline{u}^{gl}$ = DOFs associate with G-L (hierarchical) enrichments

$$\dim(\underline{u}^{gl}) << \dim(\underline{\tilde{u}}^0)$$
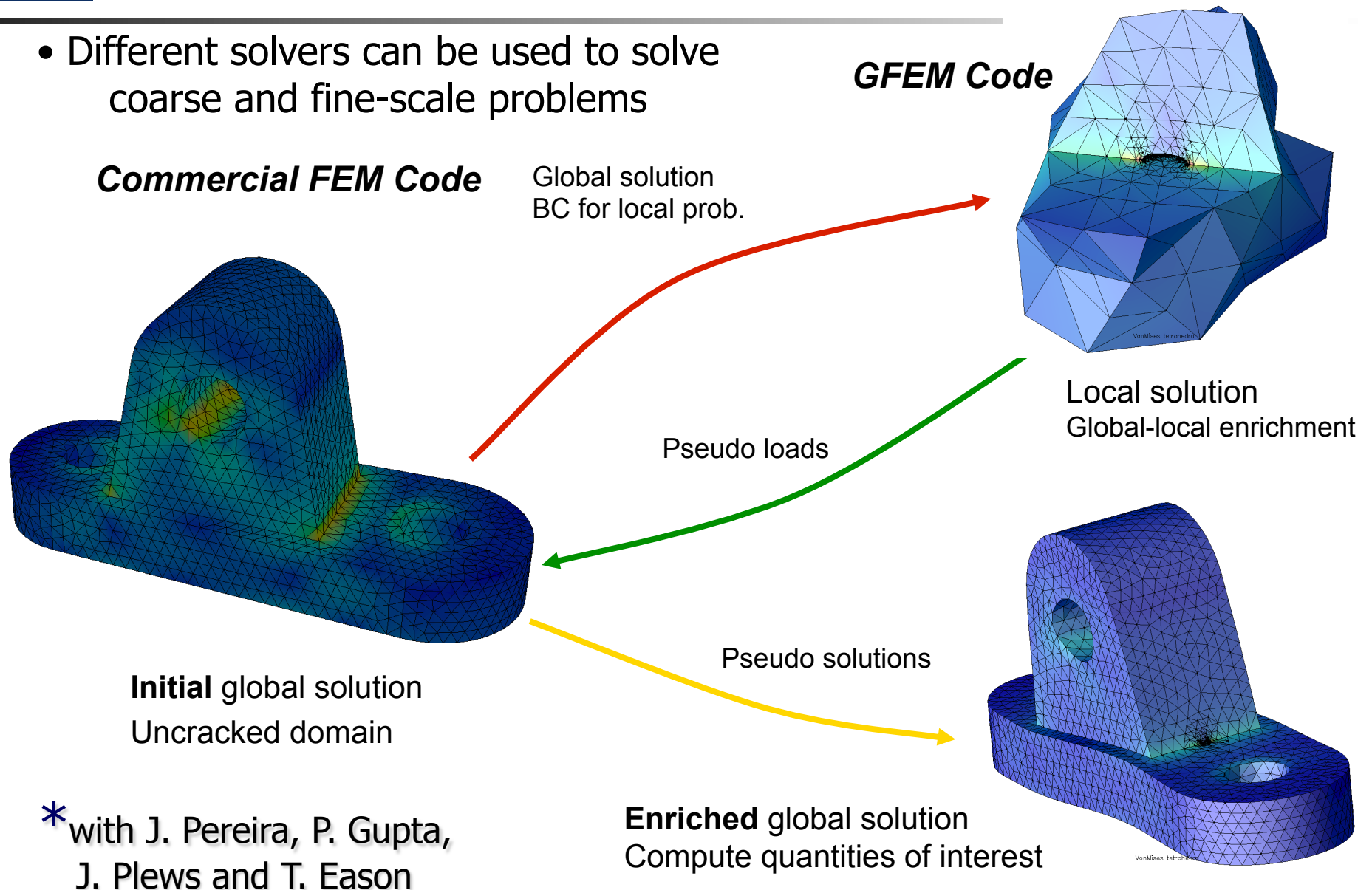
This leads to

$$\begin{bmatrix} K^0 & K^{0,gl} \\ K^{gl,0} & K^{gl} \end{bmatrix} \begin{bmatrix} \underline{\tilde{u}}^0 \\ \underline{u}^{gl} \end{bmatrix} = \begin{bmatrix} F^0 \\ F^{gl} \end{bmatrix}$$

Computed by FEM code

Solve using, e.g., static condensation of $\underline{u}^{gl}$

Non-Intrusive Implementation in Existing FEM Codes*

- Different solvers can be used to solve coarse and fine-scale problems

GFEM Code

Commercial FEM Code

Global solution
BC for local prob.

Local solution
Global-local enrichment

Pseudo loads

Pseudo solutions

Initial global solution
Uncracked domain

Enriched global solution
Compute quantities of interest

*with J. Pereira, P. Gupta, J. Plews and T. Eason

Related Non-Intrusive Methods

- Krause R, Rank, E. Multiscale computations with a combination of the h- and p-versions of the finite-element method. CMAME, 2003

- Gendre L, Allix O, Gosselet P, Comte F. Non-intrusive and exact global/local techniques for structural problems with local plasticity. CM, 2009

- Gendre L, Allix O, Gosselet P. A two-scale approximation of the Schur complement and its use for non-intrusive coupling. IJNME, 2011
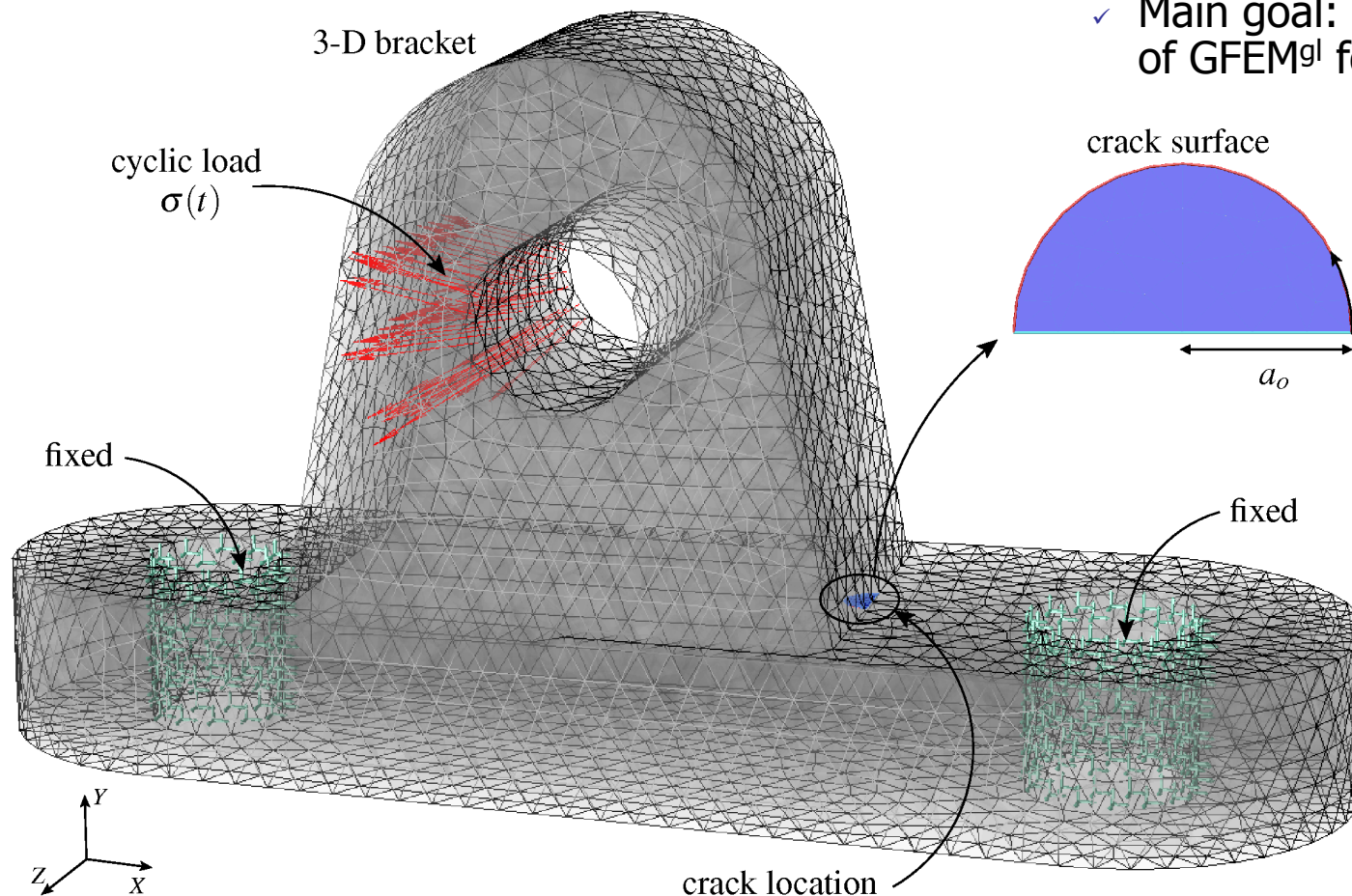
Computational Efficiency

- **Bracket with half-penny shaped crack**
  - ✓ *hp*-GFEM as reference solution
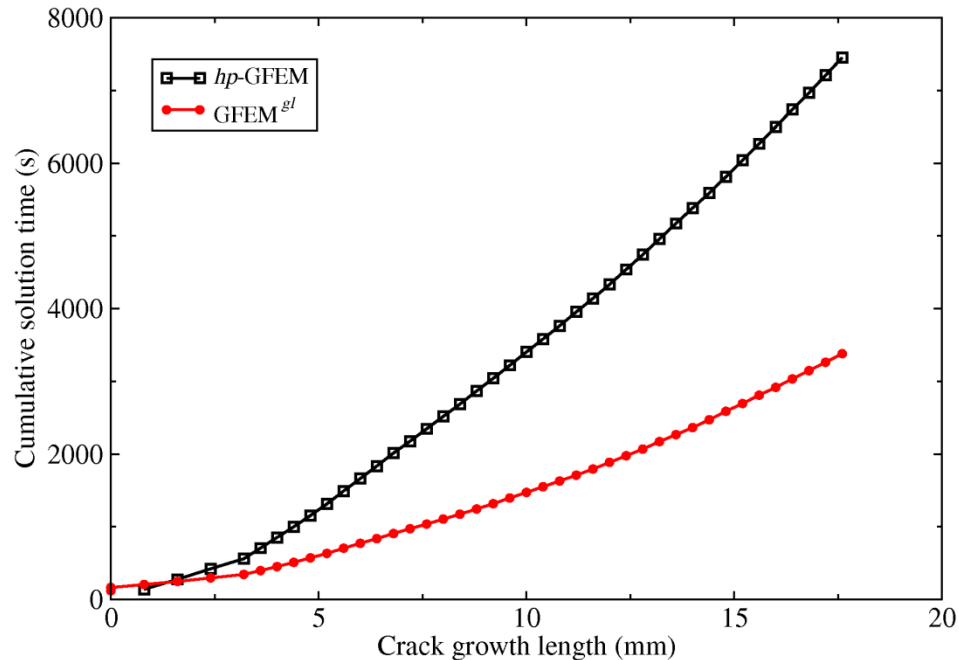 - ✓ Main goal: computational efficiency of GFEM^{gl} for crack growth

3-D bracket

cyclic load $\sigma(t)$

fixed

fixed

crack location

crack surface

ζ

a_o

Movie

Y

Z X

3-D mesh courtesy of Altair Engineering

Computational Efficiency

Computational cost analysis

- GFEM^{gl}:
  115,470 + 27 *dofs* (min)
  115,470 + 84 *dofs* (max)

- *hp*-GFEM:
  186,666 global *dofs* (min)
  255,618 global *dofs* (max)

Strain Energy

- ~ 60% computational cost reduction

- *hp*-GFEM and GFEM^{gl} solutions show good agreement
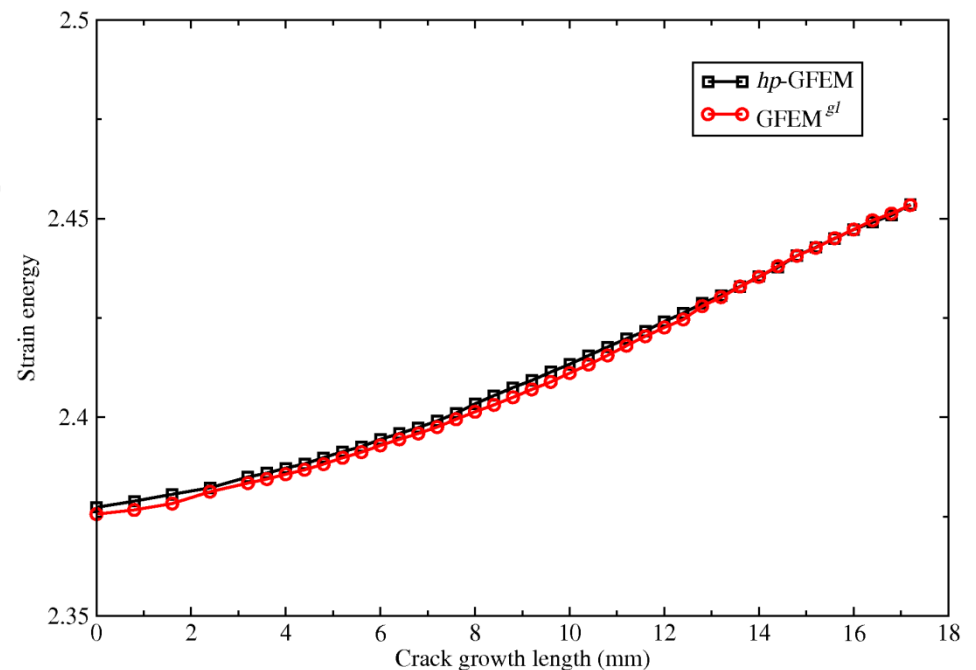
Conclusions and Assessment

- Generalized FEMs offer significant **flexibility** and attractive features

- It enables the solution of problems that are difficult or not practical with the FEM:

  - Coalescence of 3-D fractures: Hydraulic fracturing of oil and gas reservoirs

  - Multiscale problems:

    - Fine-scale computations are naturally parallelizable

    - Can adopt different discretization methods at each scale without introduction of additional fields (LM, mortar, etc.)

- Robust: Stable GFEM (Uday Banerjee talk on Tuesday)

- Transition to Labs and Industries: Non-intrusive integration with existing FEA software

Acknowledgements

AFRL-University Collaborative Center in
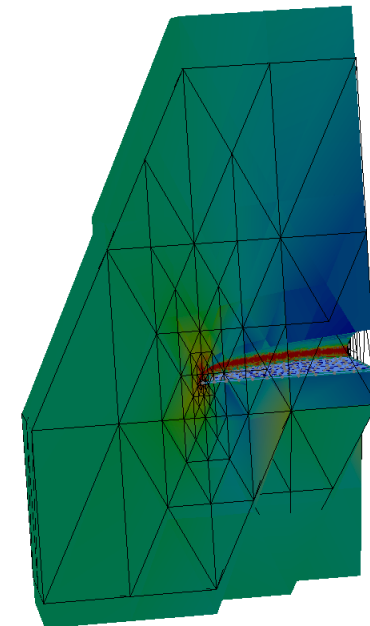Structural Sciences (C²S²)
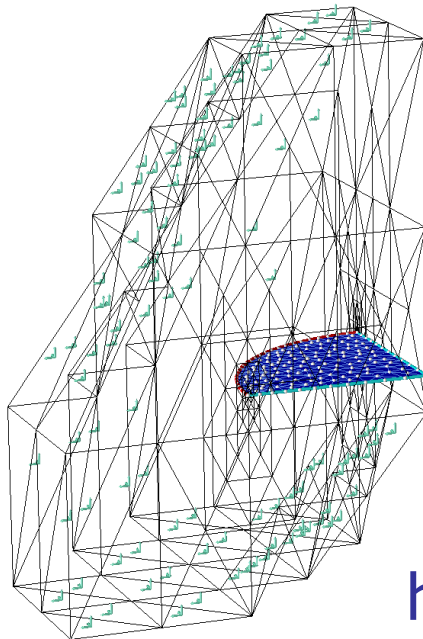
Questions?

caduarte@illinois.edu

http://gfem.cee.illinois.edu/

VonMises tetrahedra