UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Bridging Singularities and Nonlinearities Across Scales

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Presented at Dept. of Aeronautics
Imperial College London, Oct. 15, 2014

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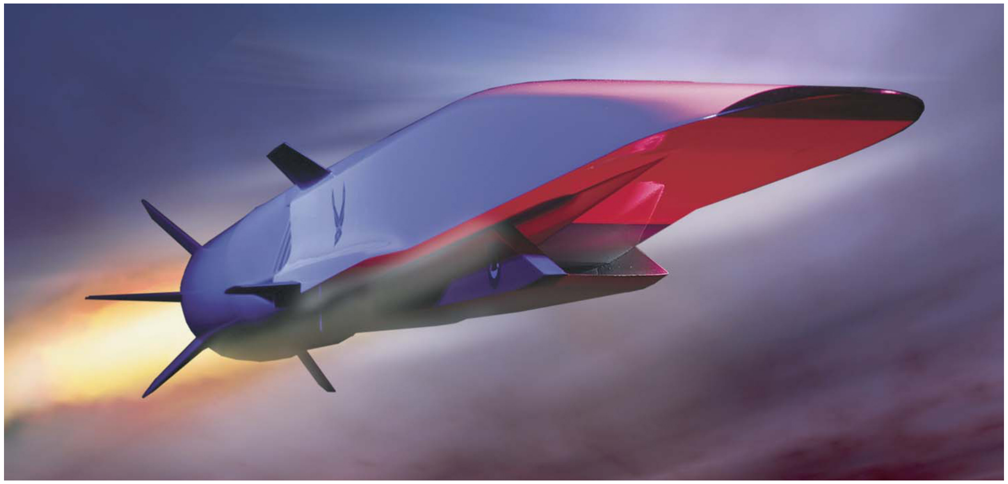
Motivation: Multiscale Structural Analysis

The US Air Force has expended six decades and untold resources in attempts to field a reusable hypersonic vehicle*

Scientific challenge:

“An inability to computationally capture the material evolution and degradation within a structural component”

X-15 (1959)

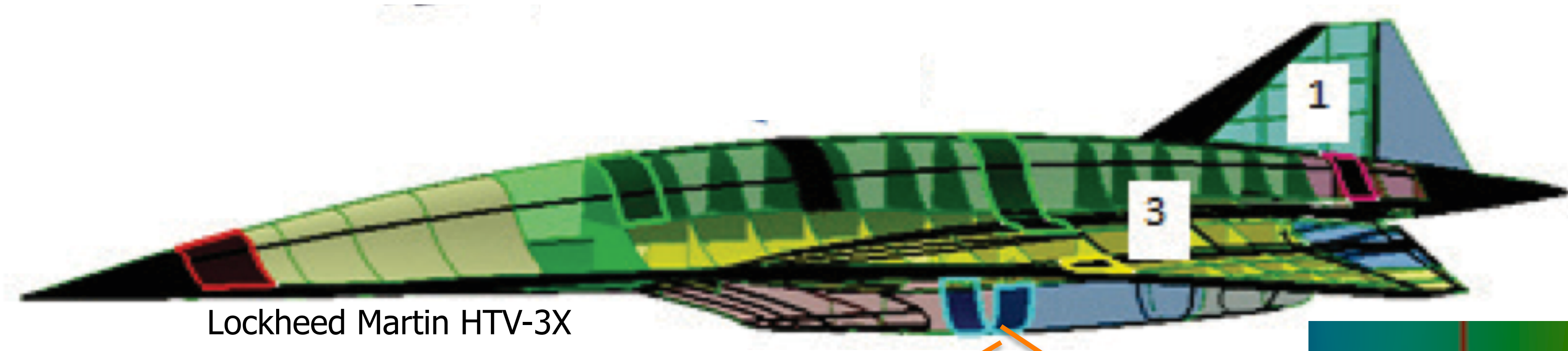


X-51a (2010)

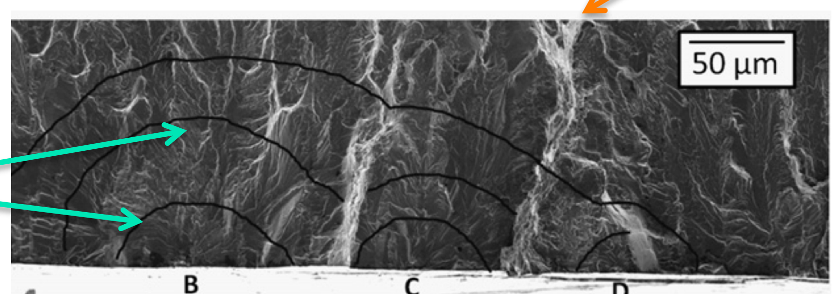
*[T. G. Eason et al., 2013]

Motivation: Multiscale Structural Analysis

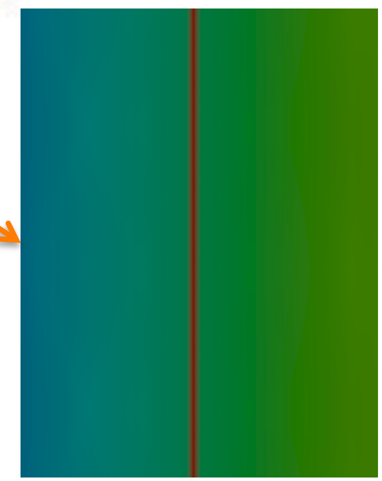
- The analysis of hypersonic aircrafts involves multiple spatial and temporal scales: From airframe scale to material microscale, loading scale, etc



Lockheed Martin HTV-3X
[AFRL-RB-WP-TR-2010-3069]



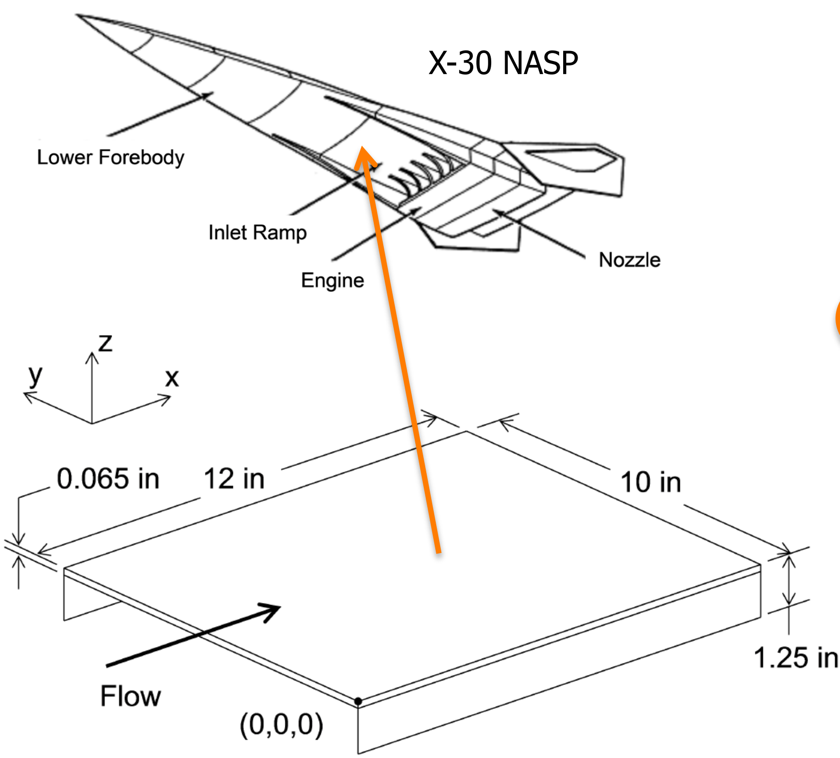
Coalescence of micro-cracks [Burns et al., IJF 2012]



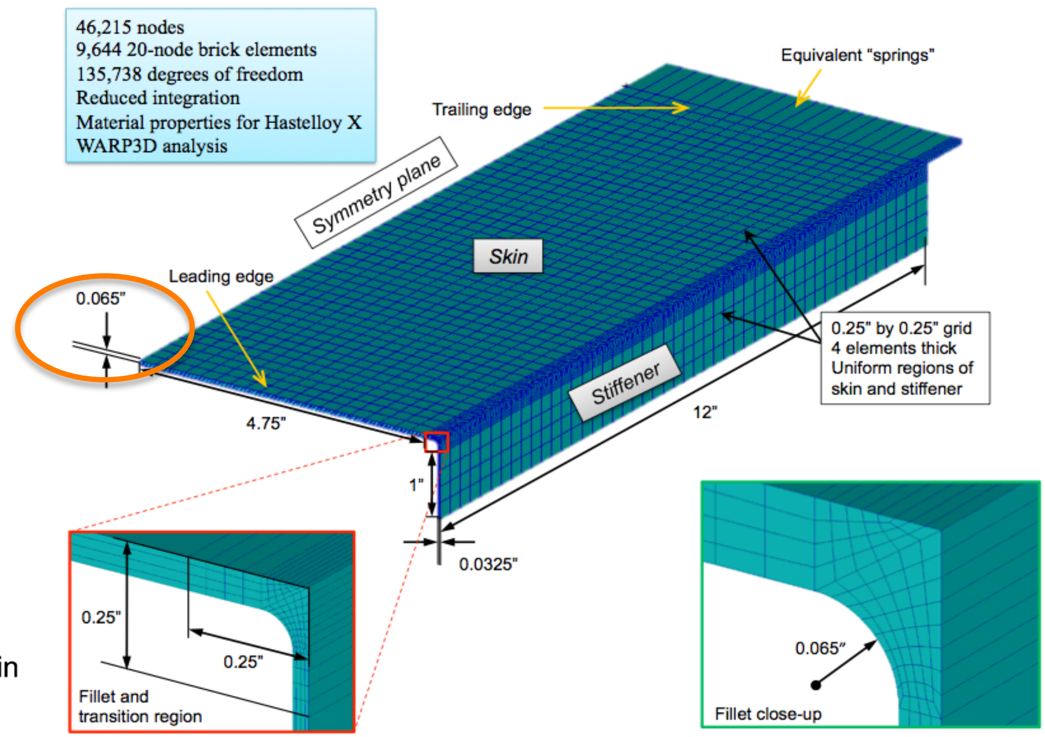
Shock impingement
Color = Temperature

Motivation: Multiscale Structural Analysis

- Thermo, mechanical and acoustic loads lead to highly localized non-linear 3-D stress fields: Finite element models with fine meshes are required



Representative hypersonic skin panel [Sobotka et al., 2013]



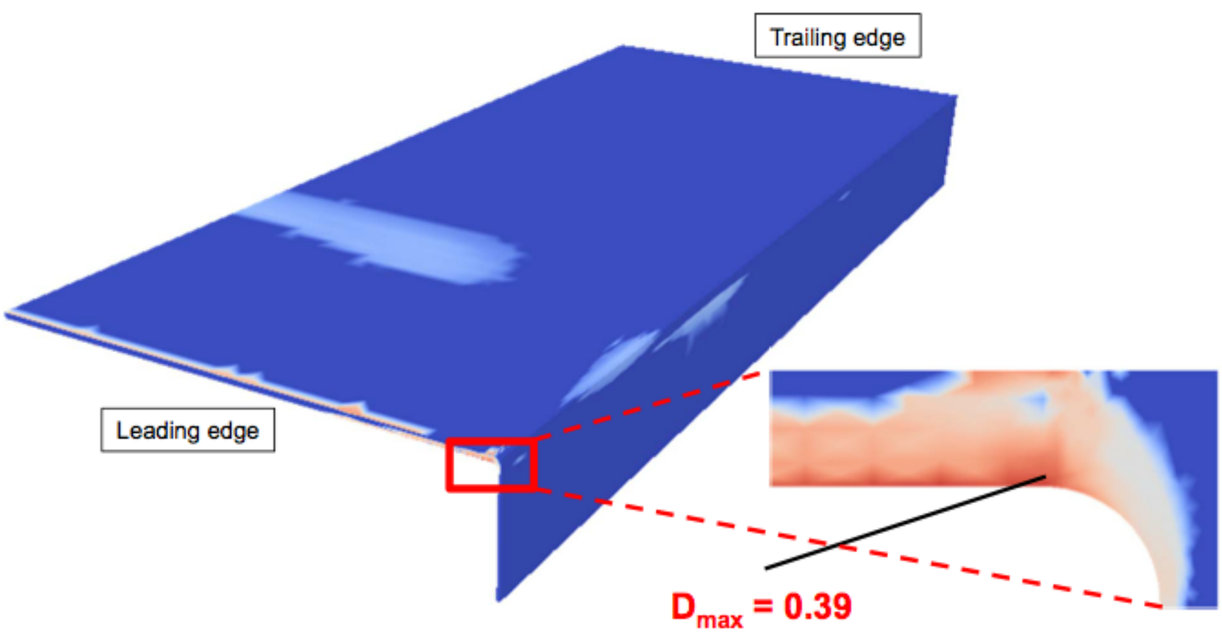
3-D FEM: Large aspect ratio of elements may lead to numerical instabilities during analysis [Sobotka et al., 2013]

Motivation: Multiscale Structural Analysis

- Highly localized non-linear 3-D effects
- Most of the structure remains linear elastic

- Q: How to efficiently capture these localized non-linear 3-D effects?
- Q: How to avoid numerical stability issues caused by aspect ratio of elements?

- Strategy: A Generalized FEM for multiscale structural analysis



[Sobotka et al., 2013]

Outline

- Motivation
- Generalized finite element methods: Basic ideas
- Bridging scales with GFEM:
  - Global-local enrichments for localized non-linearities
  - Global-local enrichment for heterogeneous materials and parallelization of fine-scale computations
- Conclusions and outlook

Early works on Generalized FEMs

- Babuska, Caloz and Osborn, 1994 (Special FEM).
- Duarte and Oden, 1995 (Hp Clouds).
- Babuska and Melenk, 1995 (PUFEM).
- Oden, Duarte and Zienkiewicz, 1996 (Hp Clouds/GFEM).
- Duarte, Babuska and Oden, 1998 (GFEM).
- Belytschko et al., 1999 (Extended FEM).
- Strouboulis, Babuska and Copps, 2000 (GFEM).

- Basic idea:
  - Use a partition of unity to build Finite Element shape functions

- Review paper

  Belytschko T., Gracie R. and Ventura G. A review of extended/generalized finite element methods for material modeling, *Mod. Simul. Matl. Sci. Eng.,* 2009

  "The XFEM and GFEM are basically <u>identical</u> methods: the name generalized finite element method was adopted by the Texas school in 1995–1996 and the name extended finite element method was coined by the Northwestern school in 1999."

Generalized Finite Element Method

- GFEM is a Galerkin method with special test/trial space given by

$$\mathbb{S}_{GFEM} = \mathbb{S}_{FEM} + \mathbb{S}_{ENR}$$

Low order FEM space

Enrichment space with functions related to the given problem

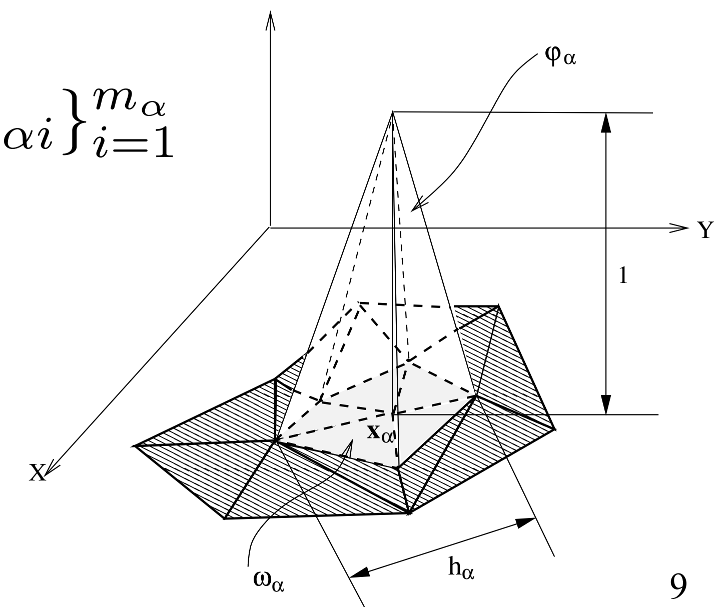
$$\mathbb{S}_{FEM} = \sum_{\alpha \in I_h} c_\alpha \varphi_\alpha, \quad c_\alpha \in \mathbb{R}$$

$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \quad \chi_\alpha = \mathrm{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$L_{\alpha i} \in \chi_\alpha(\omega_\alpha)$$

Enrichment function

Patch space



Generalized Finite Element Method

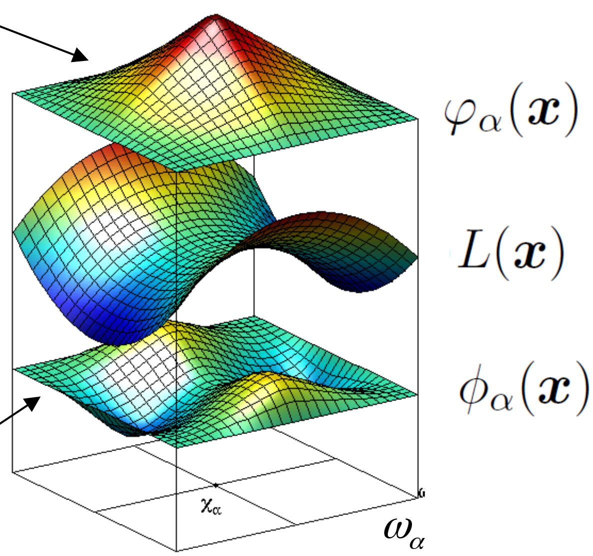
$$\mathbb{S}_{ENR} = \sum_{\alpha \in I_h^e \subset I_h} \varphi_\alpha \chi_\alpha; \qquad \chi_\alpha = \operatorname{span}\{L_{\alpha i}\}_{i=1}^{m_\alpha}$$

$$\phi_{\alpha i}(x) = \varphi_\alpha(x) L_{\alpha i}(x) \qquad\qquad \sum_\alpha \varphi_\alpha(\boldsymbol{x}) = 1$$

Linear FE shape function

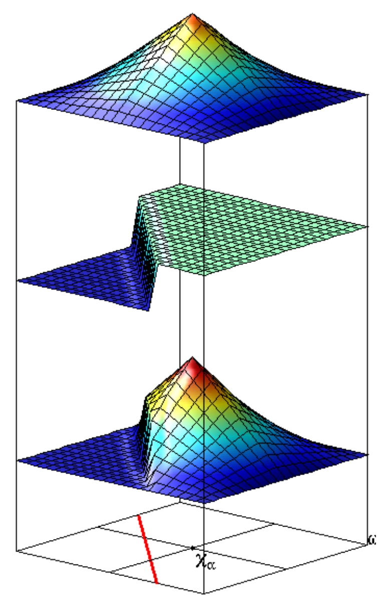
Enrichment function

GFEM shape function



[Oden, Duarte & Zienkiewicz, 1996]

- Allows construction of shape functions incorporating a-priori knowledge about solution



Discontinuous enrichment [Moes et al., 1999]

Multiscale Structural Analysis

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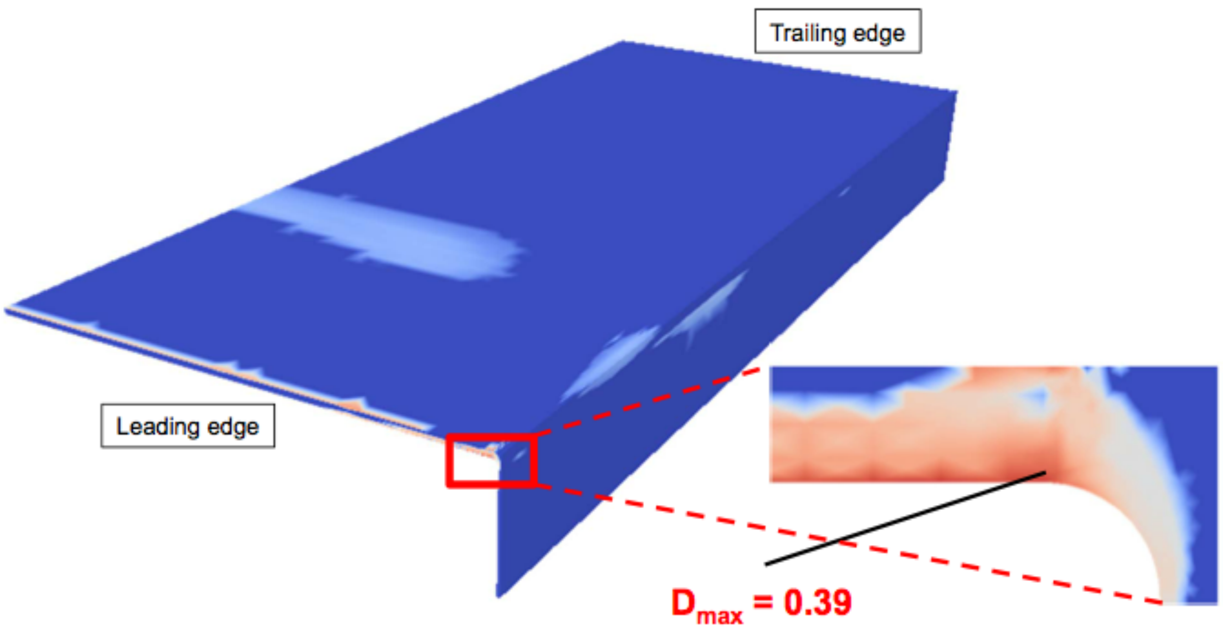
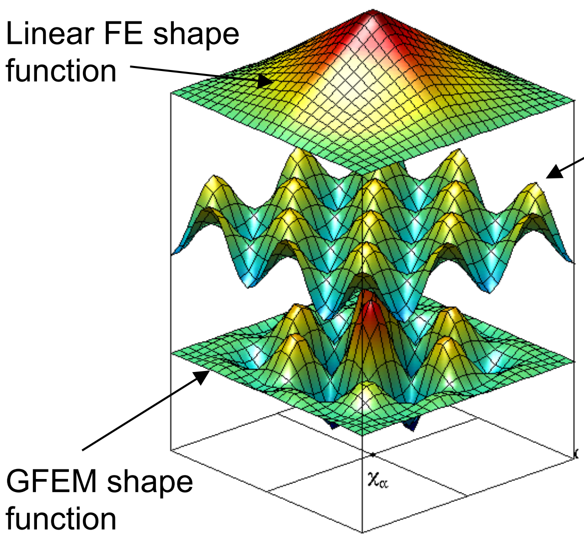


[Sobotka et al., 2013]

Bridging Scales with Global-Local Enrichment Functions*

- Enrichment functions computed from solution of local boundary value problems: Global-Local enrichment functions



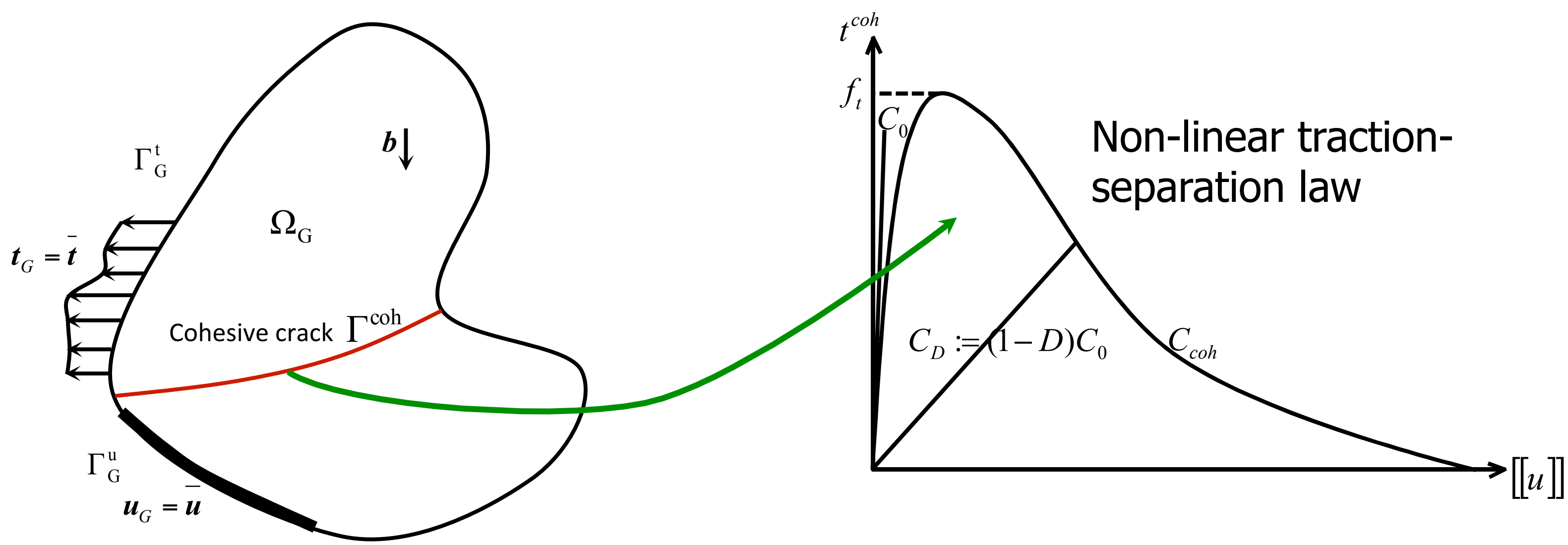
Enrichment = Numerical solutions of BVP

- **Idea:** *Use available numerical solution at a simulation step to build shape functions for next step* (quasi-static, transient, non-linear, etc.)
- Enrichment functions are produced numerically on-the-fly through a global-local analysis
- Use a *coarse* mesh enriched with Global-Local (GL) functions
- $GFEM^{gl}$ = GFEM with global-local enrichments

*[Duarte et al. 2005, 2007, 2008, 2010, 2011, 2014]

Global-Local Enrichments for Problems with Localized Non-Linearities*

- **Model Problem**: Simulation of propagating cracks using cohesive fracture models



Find $\boldsymbol{u} \in H^1(\Omega_G)$, such that $\forall\ \delta\boldsymbol{u} \in H^1(\Omega_G)$

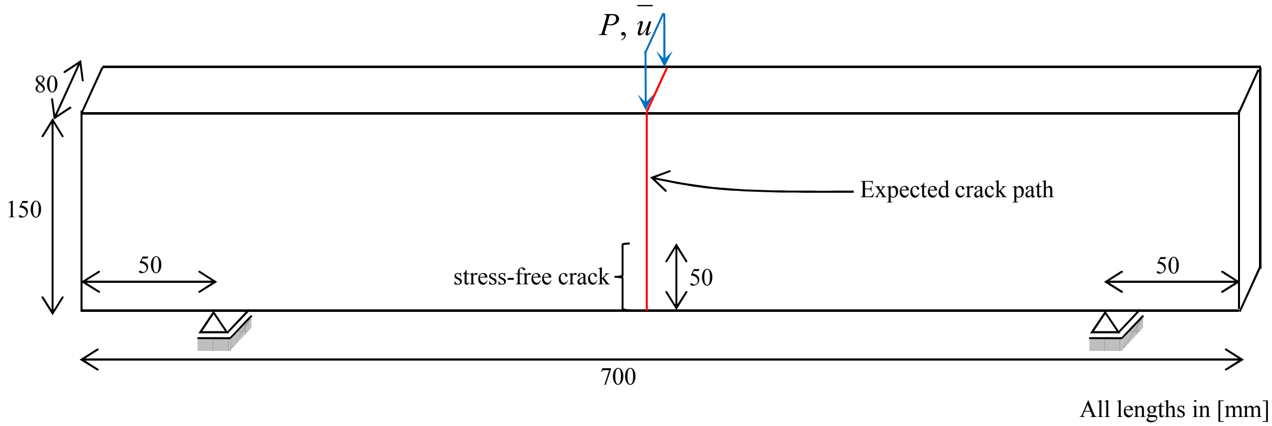
$$\int_{\Omega_G} \nabla^s(\delta\boldsymbol{u}) : \boldsymbol{\sigma}(\boldsymbol{u})\ \mathrm{dV} + \int_{\Gamma^{coh}} \delta[\![\boldsymbol{u}]\!] \cdot \boldsymbol{t}^{coh}([\![\boldsymbol{u}]\!])\ \mathrm{dS} + \eta \int_{\Gamma_G^u} \delta\boldsymbol{u} \cdot \boldsymbol{u}\ \mathrm{dS}$$

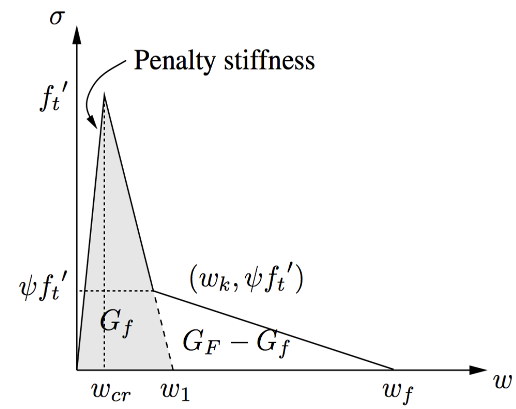
$$= \int_{\Omega_G} \delta\boldsymbol{u} \cdot \boldsymbol{b}\ \mathrm{dV} + \int_{\Gamma_G^t} \delta\boldsymbol{u} \cdot \bar{\boldsymbol{t}}\ \mathrm{dS} + \eta \int_{\Gamma_G^u} \delta\boldsymbol{u} \cdot \bar{\boldsymbol{u}}\ \mathrm{dS}$$

*[with Jongheon Kim]

Global-Local Enrichments for Problems with Localized Non-Linearities

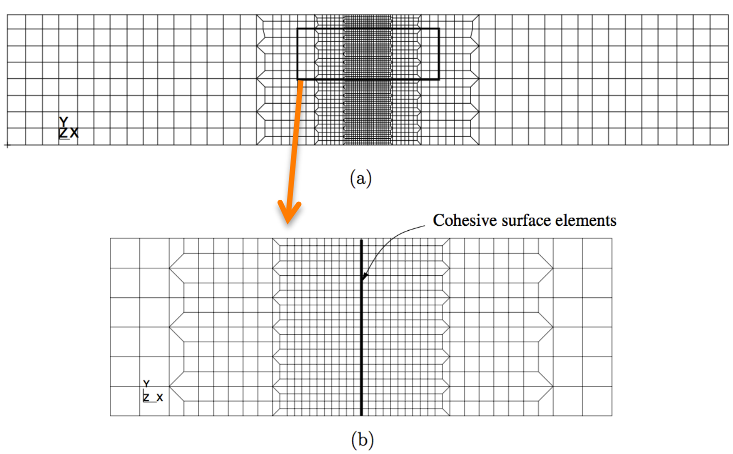
- Three-Point Bending Beam





Bi-linear intrinsic CZM [Park et al. 2008]

- Typical FEM discretization [Park et al. 2008]

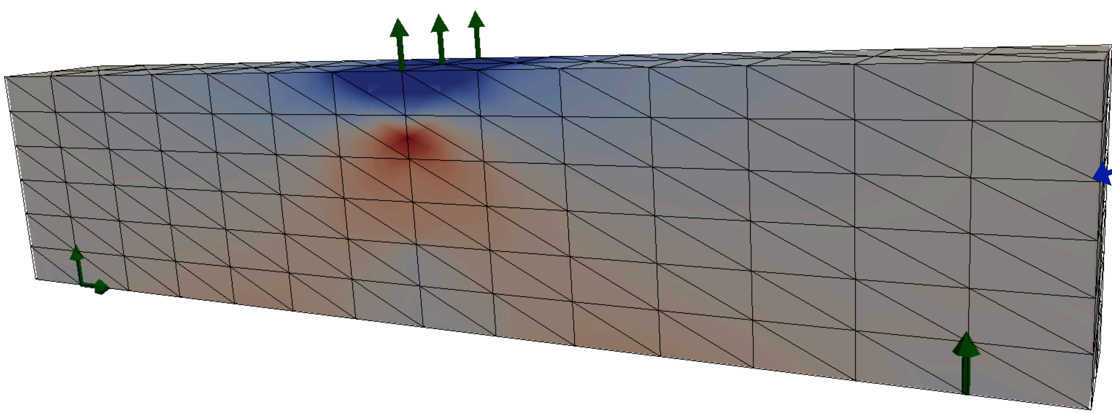


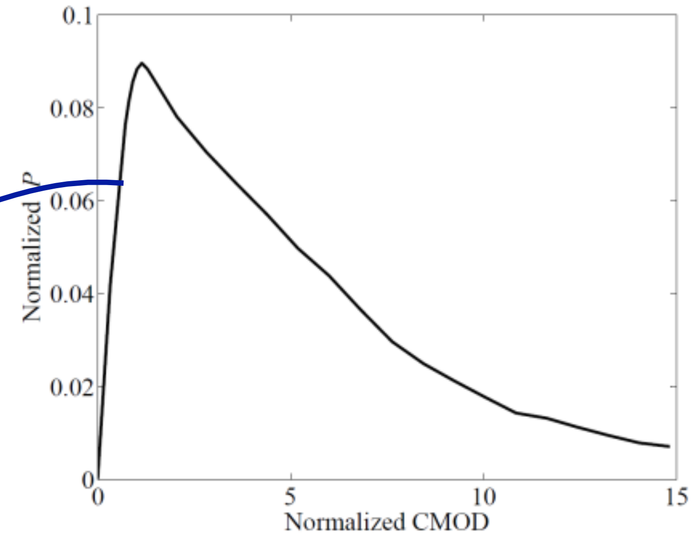
Goals:

- Solve problem on a coarse global mesh.
- Non-linear iterations at fine scales <u>only</u>.

Global-Local Enrichments for Problems with Localized Non-Linearities

Let $\boldsymbol{u}_G^n \in \mathbb{S}_G^n(\Omega)$ and $\boldsymbol{u}_L^n \in \mathbb{S}_L^n(\Omega)$, GFEM solutions of global and local problems at load step n

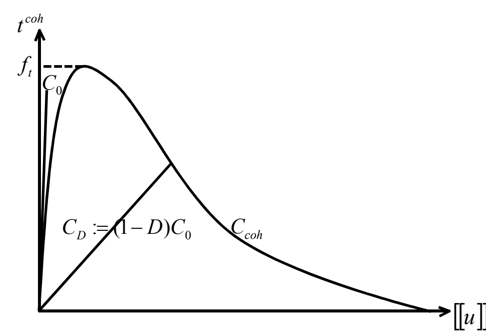




Find $\boldsymbol{u}_G^n \in \mathbb{S}_G^n(\Omega_G)$ such that, $\forall\ \delta\boldsymbol{u}_G^n \in \mathbb{S}_G^n(\Omega_G)$

$$\int_{\Omega_{\mathrm{G}}} \nabla^{\mathrm{s}}\left(\delta\boldsymbol{u}_G^n\right) : \boldsymbol{\sigma}\left(\boldsymbol{u}_G^n\right)\ \mathrm{dV} + \int_{\Gamma^{\mathrm{coh}}} \delta[\![\boldsymbol{u}_G^n]\!] \cdot \boldsymbol{C}_D(\boldsymbol{u}_L^n)[\![\boldsymbol{u}_G^n]\!]\ \mathrm{dS} + \eta \int_{\Gamma_{\mathrm{G}}^{\mathrm{u}}} \delta\boldsymbol{u}_G^n \cdot \boldsymbol{u}_G^n\ \mathrm{dS}$$

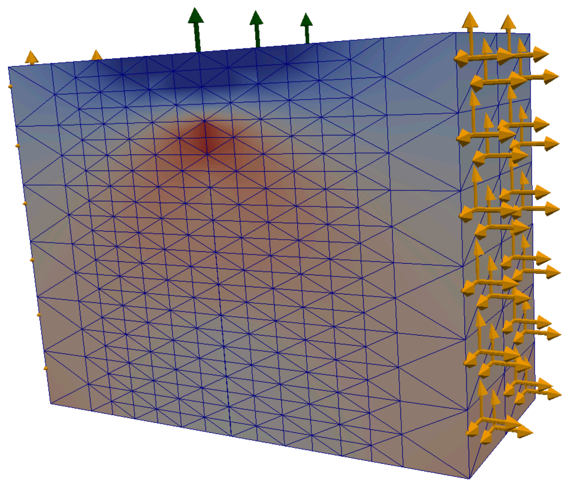
$$= \int_{\Omega_{\mathrm{G}}} \delta\boldsymbol{u}_G^n \cdot \boldsymbol{b}\ \mathrm{dV} + \int_{\Gamma_{\mathrm{G}}^{\mathrm{t}}} \delta\boldsymbol{u}_G^n \cdot \bar{\boldsymbol{t}}\ \mathrm{dS} + \eta \int_{\Gamma_{\mathrm{G}}^{\mathrm{u}}} \delta\boldsymbol{u}_G^n \cdot \bar{\boldsymbol{u}}\ \mathrm{dS}$$

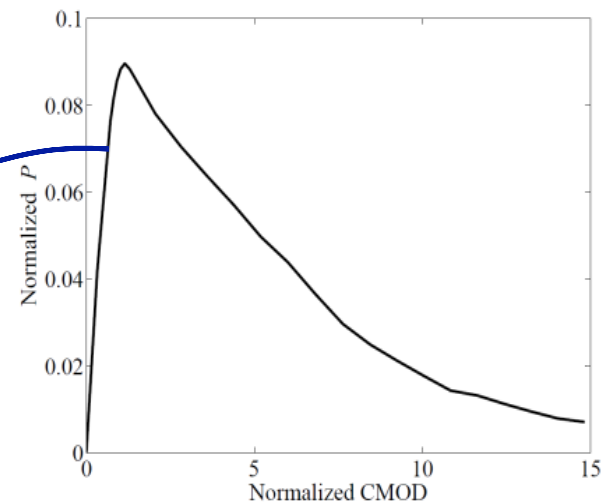


Define $\boldsymbol{u}_{G,0}^{n+1} = \frac{n+1}{n}\boldsymbol{u}_G^n$

Global-Local Enrichments for Problems with Localized Non-Linearities

- Solve following non-linear *local* problem at load step n+1 using, e.g., *hp*-GFEM



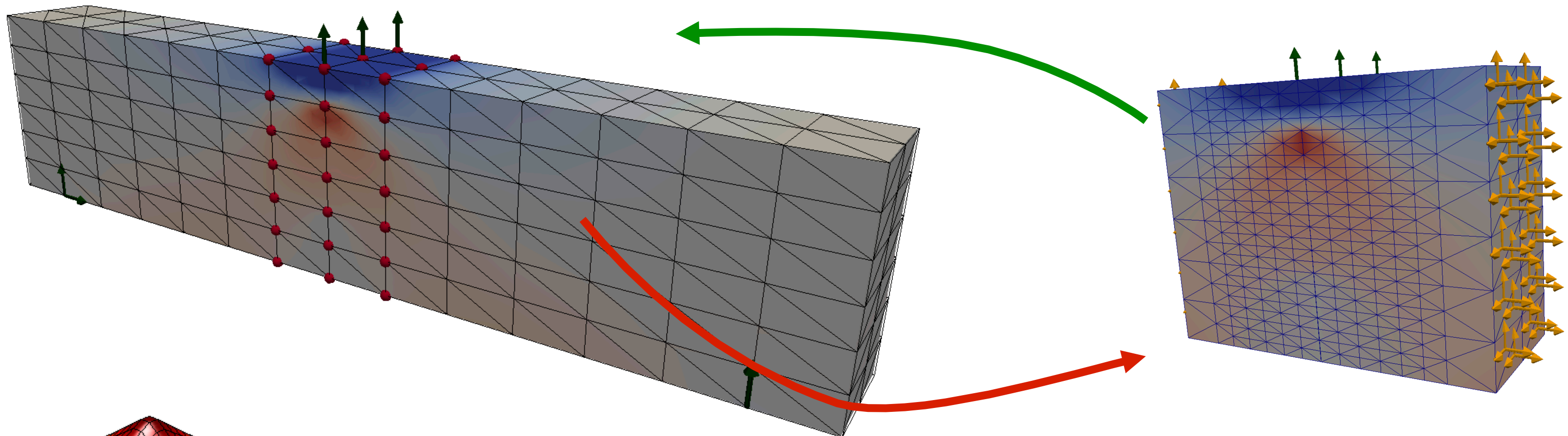


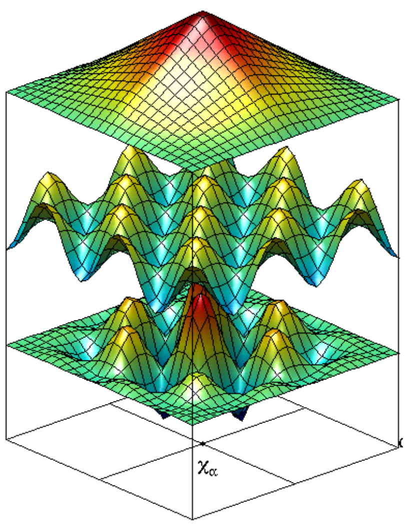
Find $\boldsymbol{u}_L^{n+1} \in \mathbb{S}_L^{n+1}(\Omega_L)$ such that, $\forall\ \delta\boldsymbol{u}_L^{n+1} \in \mathbb{S}_L^{n+1}(\Omega_L)$

$$
\begin{aligned}
&\int_{\Omega_\mathrm{L}} \nabla^\mathrm{s}\left(\delta\boldsymbol{u}_L^{n+1}\right) : \boldsymbol{\sigma}\left(\boldsymbol{u}_L^{n+1}\right)\ \mathrm{dV} + \int_{\Gamma^\mathrm{coh}} \delta[\![\boldsymbol{u}_L^{n+1}]\!] \cdot \boldsymbol{t}_L^\mathrm{coh}([\![\boldsymbol{u}_L^{n+1}]\!])\ \mathrm{dS} + \eta \int_{\Gamma_L \cap \Gamma_\mathrm{G}^\mathrm{u}} \delta\boldsymbol{u}_L^{n+1} \cdot \boldsymbol{u}_L^{n+1}\ \mathrm{dS} \\
&+ \kappa \int_{\Gamma_\mathrm{L} \backslash \left(\Gamma_\mathrm{L} \cap (\Gamma_\mathrm{G}^\mathrm{u} \cup \Gamma_\mathrm{G}^\mathrm{t})\right)} \delta\boldsymbol{u}_L^{n+1} \cdot \boldsymbol{u}_L^{n+1}\ \mathrm{dS} = \int_{\Omega_\mathrm{L}} \delta\boldsymbol{u}_L^{n+1} \cdot \boldsymbol{b}\ \mathrm{dV} + \int_{\Gamma_L \cap \Gamma_\mathrm{G}^\mathrm{t}} \delta\boldsymbol{u}_L^{n+1} \cdot \bar{\boldsymbol{t}}\ \mathrm{dS} \\
&+ \eta \int_{\Gamma_L \cap \Gamma_\mathrm{G}^\mathrm{u}} \delta\boldsymbol{u}_L^{n+1} \cdot \bar{\boldsymbol{u}}\ \mathrm{dS} + \int_{\Gamma_\mathrm{L} \backslash \left(\Gamma_\mathrm{L} \cap (\Gamma_\mathrm{G}^\mathrm{u} \cup \Gamma_\mathrm{G}^\mathrm{t})\right)} \delta\boldsymbol{u}_L^{n+1} \cdot \left[\boldsymbol{t}_G^{n+1}\left(\boldsymbol{u}_{G,0}^{n+1}\right) + \kappa \boldsymbol{u}_{G,0}^{n+1}\right] \mathrm{dS}
\end{aligned}
$$

Global-Local Enrichments for Problems with Localized Non-Linearities

- **Defining Step:** Global space is enriched with non-linear local solution





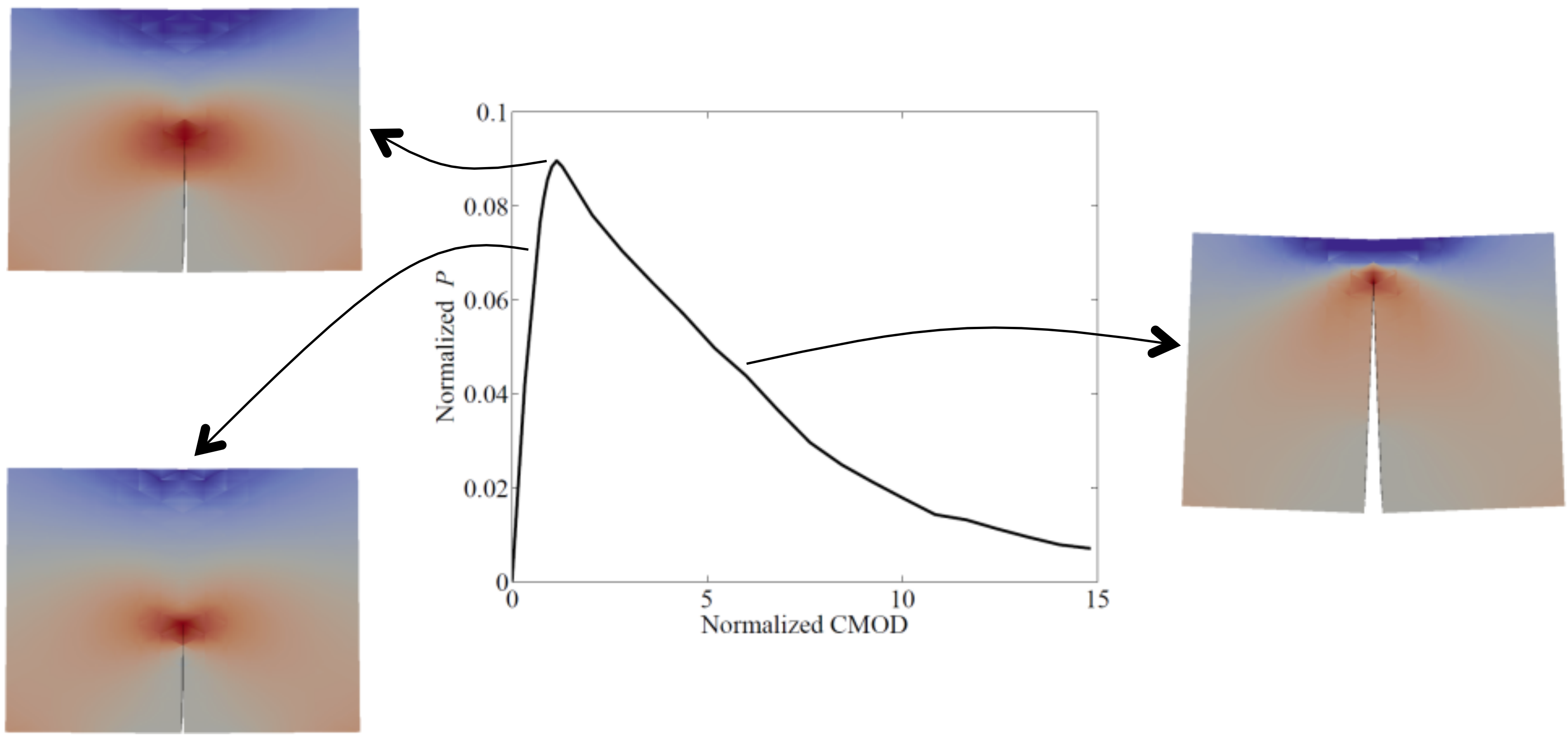
$$\phi_\alpha^{n+1}(\boldsymbol{x}) = \varphi_\alpha(\boldsymbol{x})\boldsymbol{u}_L^{n+1}(\boldsymbol{x})$$

$$u_G^{n+1}(\boldsymbol{x}) \in \mathbb{S}_G^{n+1}(\Omega_G) = \mathbb{S}_G^{\mathrm{FEM}} + \left\{\varphi_\alpha \boldsymbol{u}_\alpha^{\mathrm{gl},n+1},\ \alpha \in \mathcal{I}^{\mathrm{gl}}\right\}$$

$$\text{where } \boldsymbol{u}_\alpha^{\mathrm{gl},n+1}(\boldsymbol{x}) = \left\{\begin{array}{c} \underline{u}_\alpha \boldsymbol{u}_L^{n+1,<0>}(\boldsymbol{x}) \\ \underline{v}_\alpha \boldsymbol{u}_L^{n+1,<1>}(\boldsymbol{x}) \\ \underline{w}_\alpha \boldsymbol{u}_L^{n+1,<2>}(\boldsymbol{x}) \end{array}\right\},\ \underline{u}_\alpha, \underline{v}_\alpha, \underline{w}_\alpha \in \mathbb{R}$$

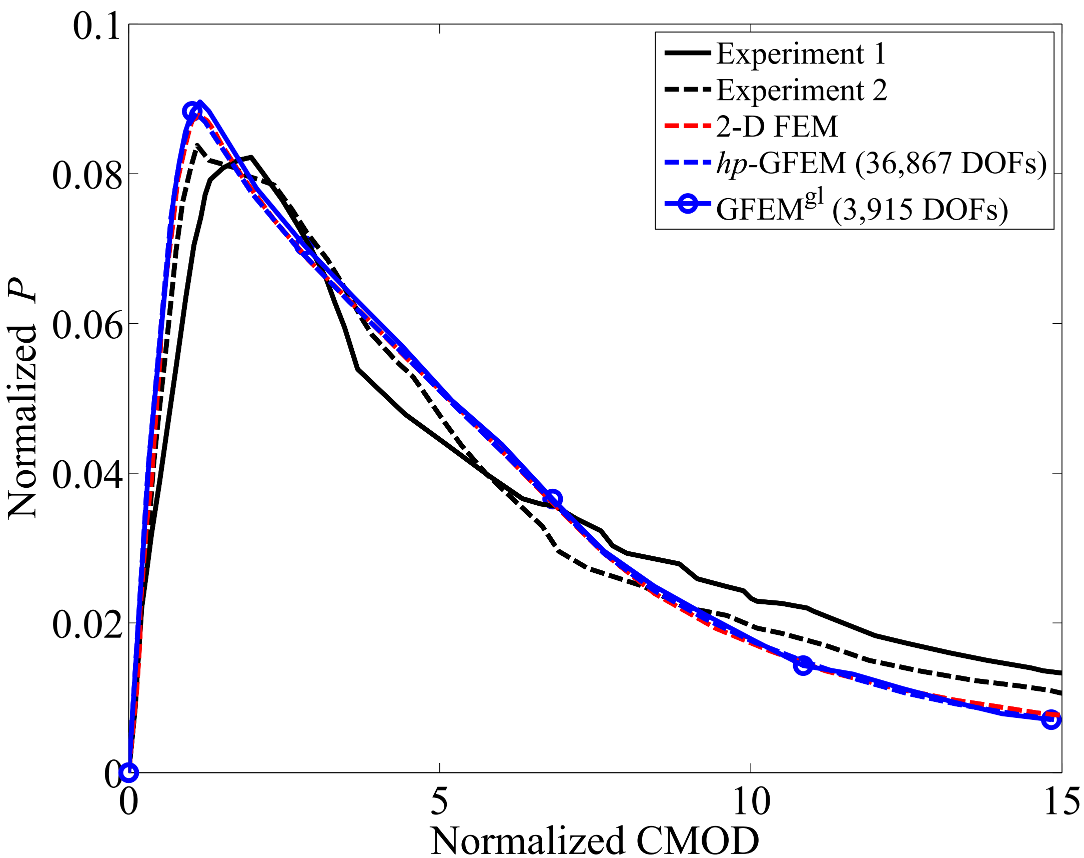
- Discretization spaces updated on-the-fly with global-local enrichment functions

Global-Local Enrichments for Problems with Localized Non-Linearities

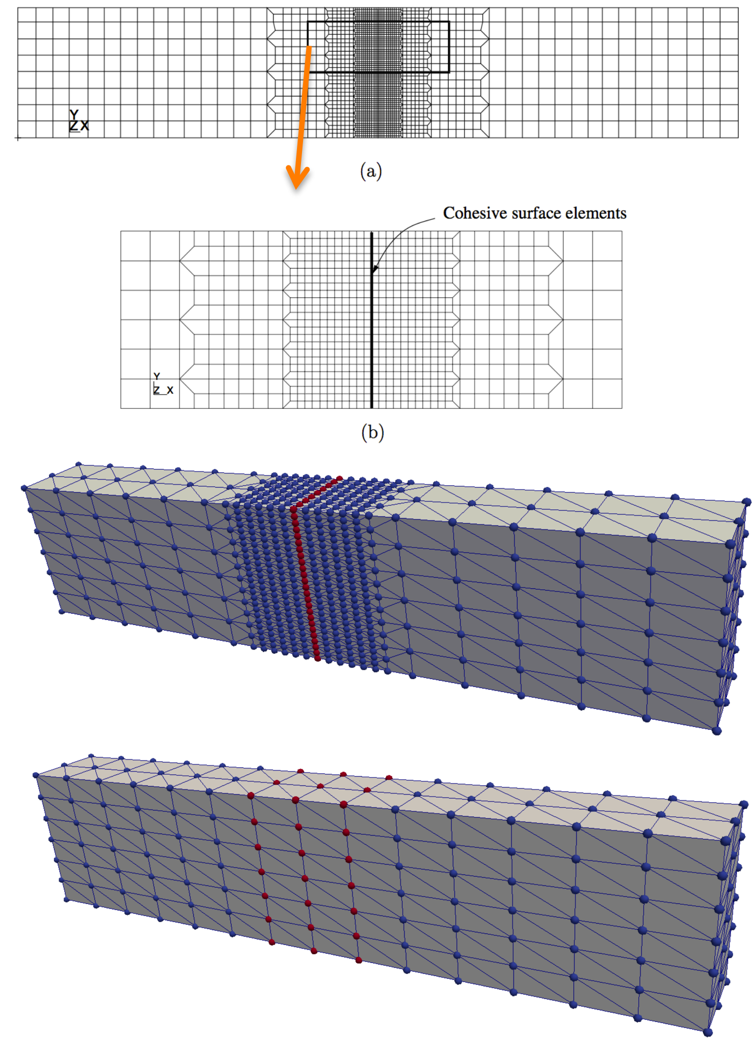


- On-the-fly updating of global-local enrichment functions during the non-linear iterative solution process

Global-Local Enrichments for Problems with Localized Non-Linearities



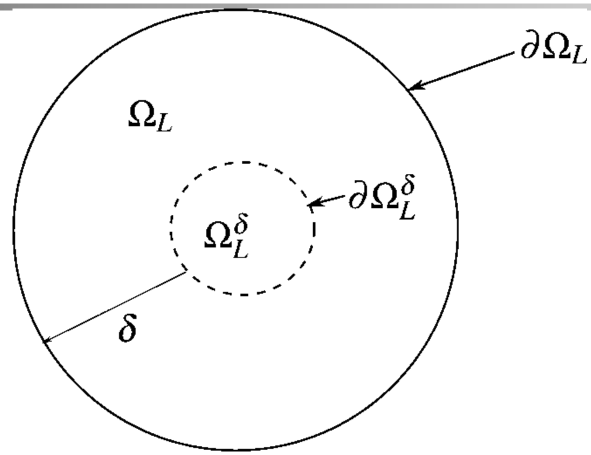
Experiments by [Roesler et al., 2007],
2-D FEM results by [Park et al. 2008]



2-D FEM, *hp*-GFEM, and GFEM^{gl} meshes

A-Priori Error Estimate*

- Local error estimate



$$\|\boldsymbol{u}^{exBC} - \boldsymbol{u}_h^{inexBC}\|_{\varepsilon(\Omega_L^\delta)} \le C \underbrace{\inf_{\boldsymbol{x}\in \boldsymbol{X}_L^{hp}(\Omega_L)} \|\boldsymbol{u}^{inexBC} - \boldsymbol{x}\|_{\varepsilon(\Omega_L)}}_{\text{Discretization error}} + \underbrace{\frac{C_1}{\delta}\|\boldsymbol{u}^{exBC} - \boldsymbol{u}^{inexBC}\|_{L^2(\Omega_L)}}_{\text{Effect of inexact BC controlled by } \delta}$$

- Global Error

$$\|\boldsymbol{u} - \boldsymbol{u}_G\|^2_{\varepsilon(\Omega)} \le C \sum_{\alpha=1}^{N} \inf_{\boldsymbol{u}_\alpha \in \chi_\alpha} \|\boldsymbol{u} - \boldsymbol{u}_\alpha\|^2_{\varepsilon(\omega_\alpha)} \le C \sum_{\alpha=1}^{N} \|\boldsymbol{u} - \boldsymbol{u}_h^{\text{inexBC}}\|^2_{\varepsilon(\omega_\alpha)}$$

where $\boldsymbol{u} \equiv \boldsymbol{u}^{\text{exBC}}$

*[Gupta and Duarte, CMAME, 2012]

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  - Global-local enrichment for heterogeneous materials and parallelization of fine-scale computations
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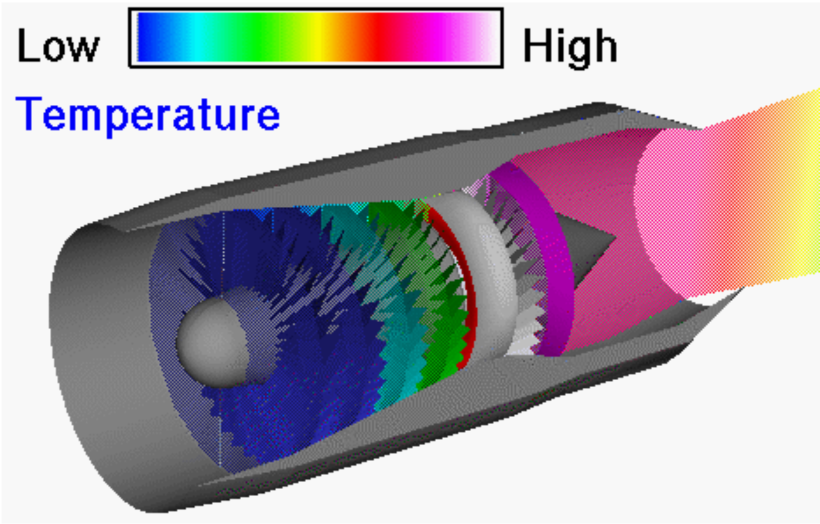
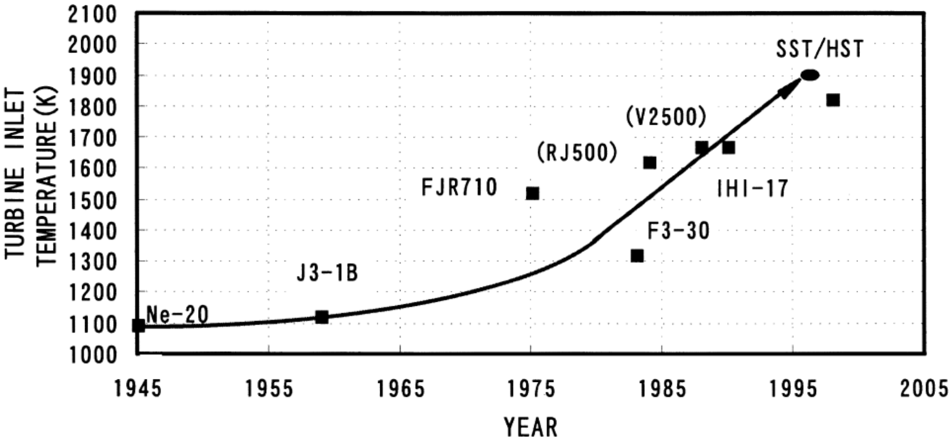
From Micro to Macro Scales

Courtesy of General Electric Co.

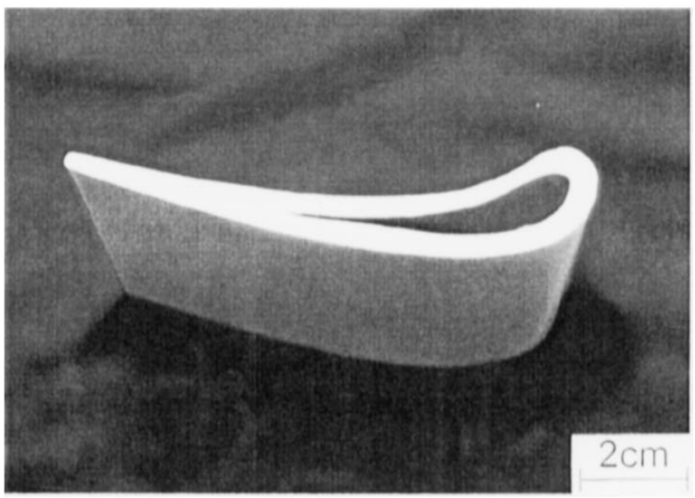




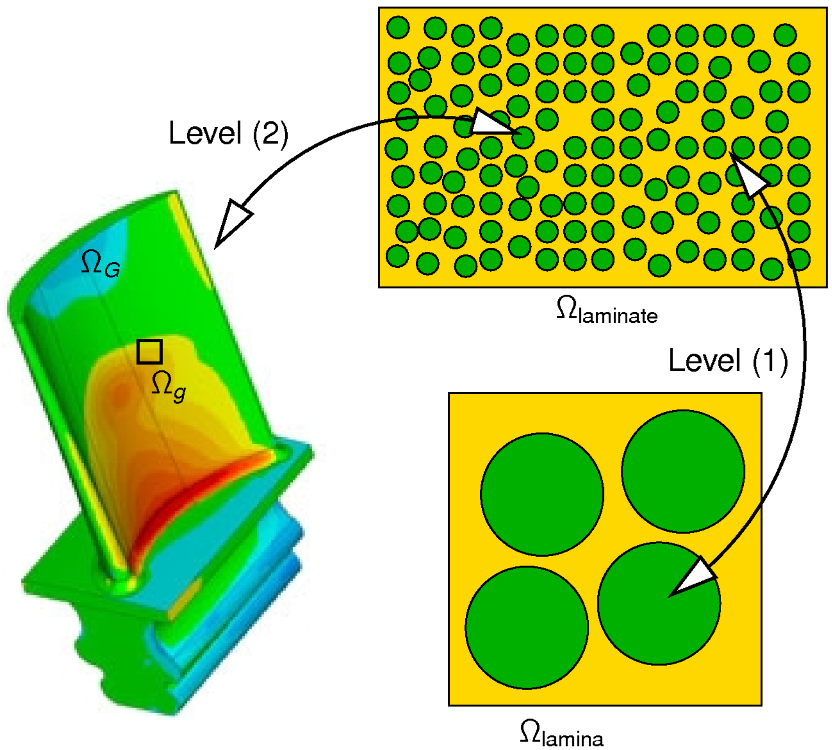
- The performance of a turbine increases with its operational temperature

From Micro to Macro Scales

- High operational temperatures require new materials like Ceramic Matrix Composites (CMC)



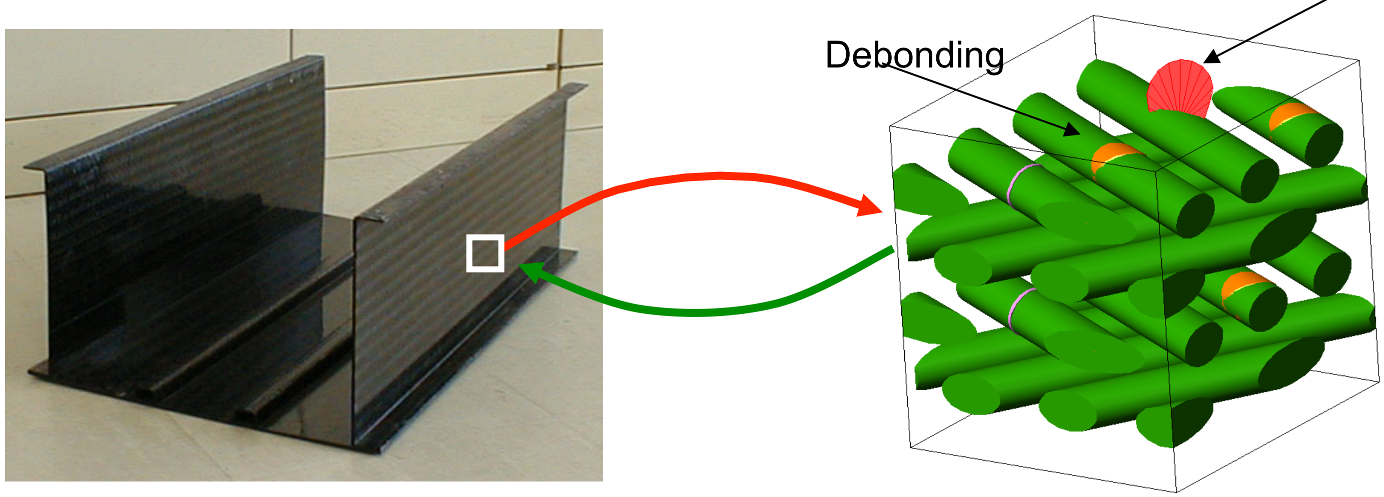
Turbine component made of CMC



Structural performance depends strongly on micro-scale details

From Micro to Macro Scales

- Failure of Heterogeneous Materials
  - Damage characterization in composite materials involves complex multi-scale phenomena

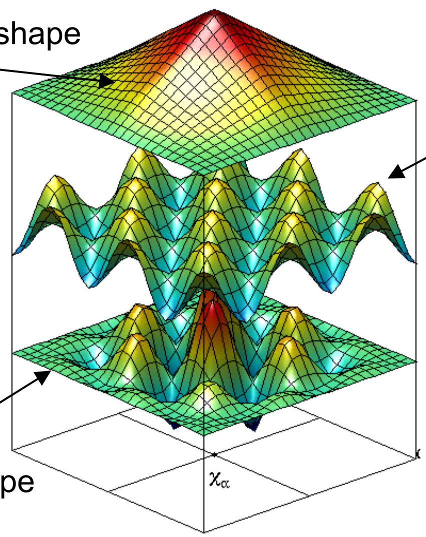


Damage in composite materials

- Homogenization approaches can not be used near singularities:
  - Can not predict local stress state since it converges in L_2 norm
  - Failure depends on local quantities as opposed to averaged [A. Needleman]

Bridging Material and Structural Scales with Global-Local Enrichment Functions

- Enrichment functions computed from solution of local boundary value problems: <u>Global-Local enrichment functions</u>



Enrichment = Numerical solutions of BVP

- **Idea:** *Use available numerical solution at a simulation step to build shape functions for next step* (quasi-static, transient, non-linear, etc.)
- Enrichment functions are produced numerically on-the-fly through a global-local analysis
- Use a *coarse* mesh enriched with Global-Local (GL) functions
- GFEM^{gl} = GFEM with global-local enrichments

Global-Local Enrichments for Heat Equation*

$$\rho c \frac{\partial u}{\partial t} = \nabla \left(\kappa(\boldsymbol{x}) \nabla u \right) + Q(\boldsymbol{x}, t) \quad \text{in} \quad \Omega$$

where $u(\boldsymbol{x}, t)$ is the temperature field, ρc is the volumetric heat capacity and $Q(\boldsymbol{x}, t)$ is the internal heat source. $\kappa(\boldsymbol{x})$ may be oscillatory.

$$-\kappa \frac{\partial u}{\partial n} = \eta \left(\bar{u} - u \right) \quad \text{on} \quad \Gamma_c$$

$$-\kappa \frac{\partial u}{\partial n} = \bar{f} \quad \text{on} \quad \Gamma_f$$

$$u(\boldsymbol{x}, 0) = u^0(\boldsymbol{x}) \quad \text{at} \quad t^0$$

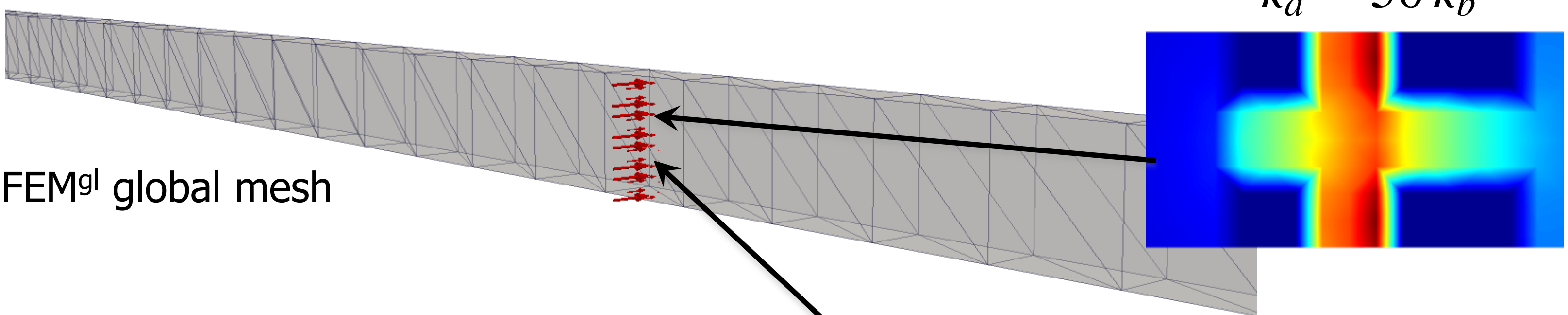
where $u^0(\boldsymbol{x})$ is the prescribed temperature field at time $t = t^0$

*[O'Hara et al., CMAME, 2011; Plews and Duarte, 2014]

Bridging Material and Structural Scales with Global-Local Enrichment Functions

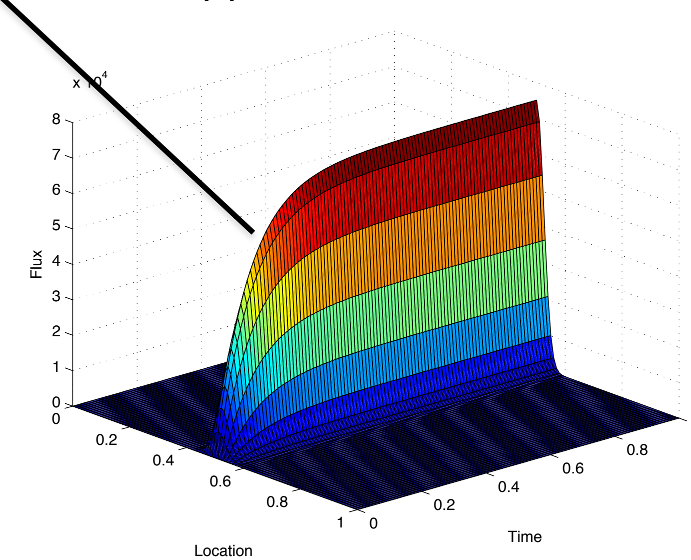
- **Goal**: Solve with GFEM^{gl} on the mesh shown below

Local material heterogeneity:
$\kappa_a = 50\,\kappa_b$



GFEM^{gl} global mesh

Sharp (Gaussian), localized heat flux applied as shown



Laser flux:

$$\bar{f}(\boldsymbol{x},t) = I_0 * f(t) * \frac{1}{2\pi a^2} * G(\boldsymbol{x},b,a)$$

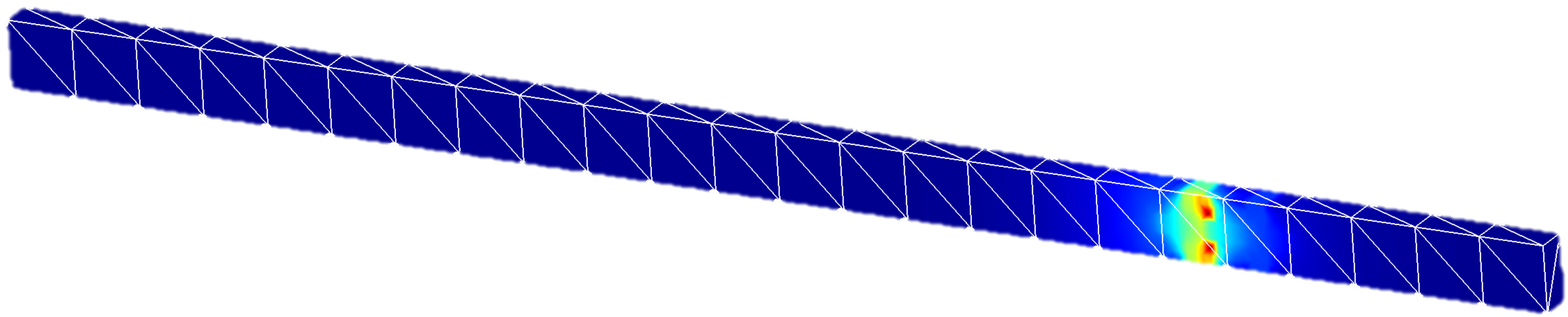
$$f(t) = 1 - \exp(-\gamma * t)$$

$$G(\boldsymbol{x},b,a) = \exp\left(\frac{-(x-b)^2}{2a^2}\right)$$

Convection BCs applied everywhere else

Global-Local Enrichments for Heat Equation

Let $u_G^n(\boldsymbol{x}) \in \mathbb{S}_G^{GFEM,n}(\Omega)$ be the GFEM solution at time $t = t^n = n\Delta t$



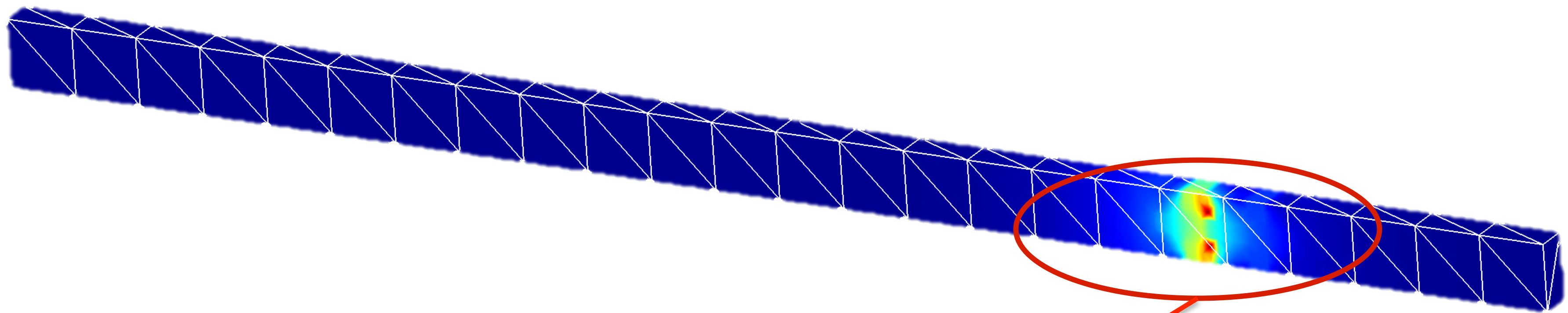
Find $u_G^n \in \mathbb{S}_G^{\mathrm{GFEM},n}(\Omega_G)$ such that, $\forall\ w_G^n \in \mathbb{S}_G^{\mathrm{GFEM},n}(\Omega_G)$

$$\frac{\rho c}{\Delta t}\int_\Omega w_G^n u_G^n d\Omega + \int_\Omega \left(\nabla w_G^n\right)^T \kappa \nabla u_G^n d\Omega + \eta \int_{\Gamma_c} w_G^n u_G^n d\Gamma =$$

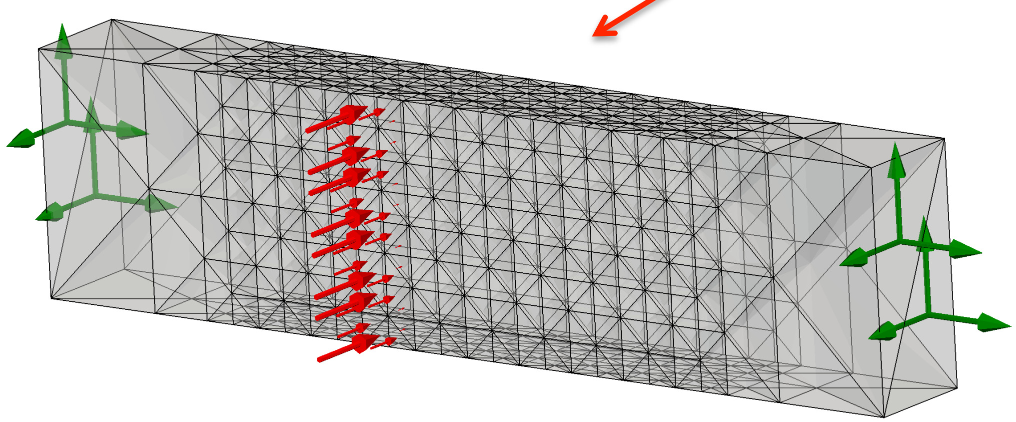
$$\frac{\rho c}{\Delta t}\int_\Omega w_G^n u_G^{n-1} d\Omega + \int_{\Gamma_f} \bar{f}^n w_G^n d\Gamma + \eta \int_{\Gamma_c} \bar{u}^n w_G^n d\Gamma + \int_\Omega Q^n w_G^n d\Omega$$

Global-Local Enrichments for Heat Equation

Let $u_G^n(\boldsymbol{x}) \in \mathbb{S}_G^{GFEM,n}(\Omega)$ be the GFEM solution at time $t = t^n = n\Delta t$

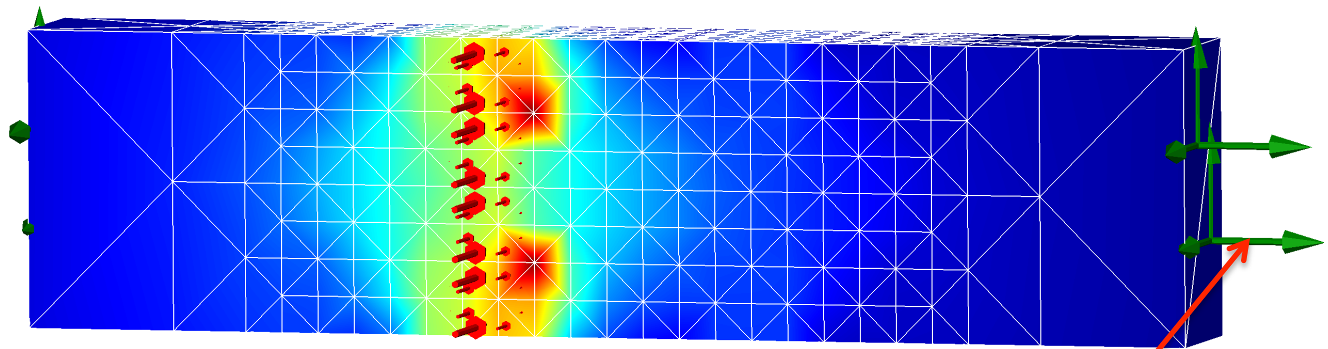


- Define local domain around the laser flux location at time $t = t^{n+1}$



Global-Local Enrichments for Heat Equation

- Solve following *local problem* at time $t = t^{n+1}$ using, e.g., *hp*-GFEM



$$\text{Find } u_L^{n+1} \in \mathbb{S}_L^{\text{GFEM},n+1}(\Omega_L) \text{ such that, } \forall\ w_L^{n+1} \in \mathbb{S}_L^{\text{GFEM},n+1}(\Omega_L)$$

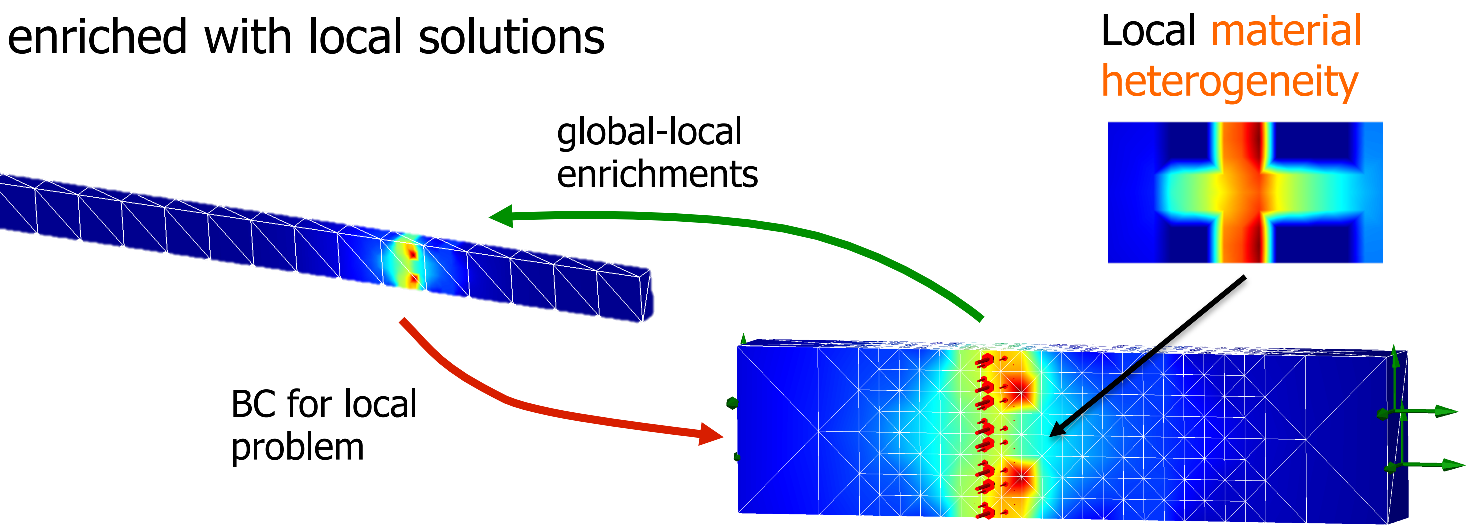
$$\int_{\Omega_L} \left(\nabla w_L^{n+1}\right)^T \kappa \nabla u_L^{n+1} d\Omega + \eta \int_{\partial\Omega_L \backslash (\partial\Omega_L \cap \Gamma_f)} w_L^{n+1} u_L^{n+1} d\Gamma$$

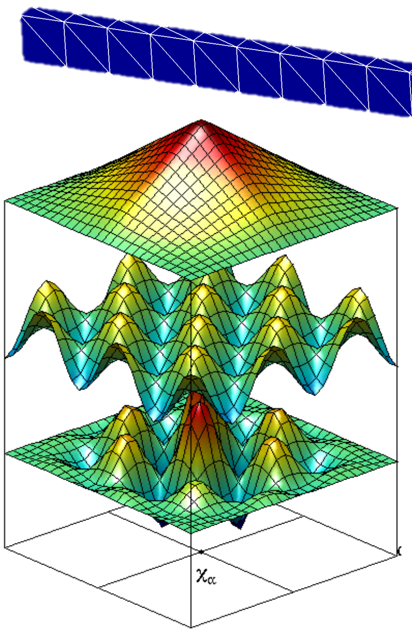
$$= \int_{\Omega_L} Q^{n+1} w_L^{n+1} d\Omega + \int_{\partial\Omega_L \cap \Gamma_f} \bar{f}^{n+1} w_L^{n+1} d\Gamma$$

$$+ \eta \int_{\partial\Omega_L \backslash (\partial\Omega_L \cap \partial\Omega)} w_L^{n+1} u_G^n d\Gamma + \eta \int_{\partial\Omega_L \cap \Gamma_c} \bar{u}^{n+1} w_L^{n+1} d\Gamma$$

Bridging Material and Structural Scales with Global-Local Enrichment

- Global space is enriched with local solutions





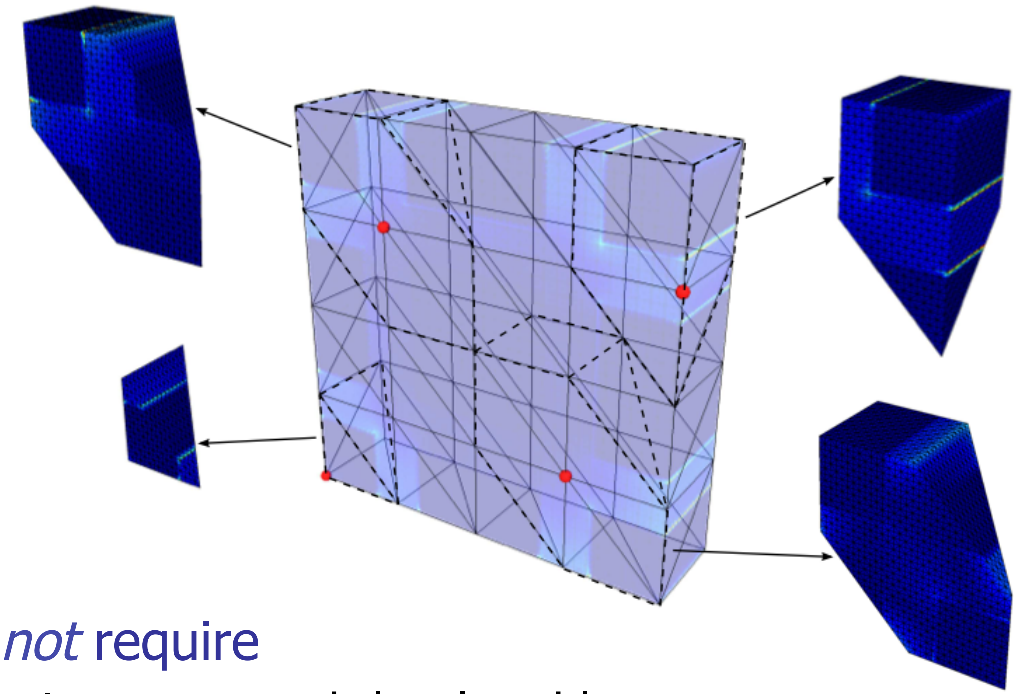
$$\phi_\alpha^{n+1}(\boldsymbol{x}) = \varphi_\alpha(\boldsymbol{x}) u_L^{n+1}(\boldsymbol{x})$$

$$\text{Find } u_G^{n+1}(\boldsymbol{x}) \in \mathbb{S}_G^{\mathrm{GFEM},n+1}(\Omega) = \mathbb{S}_G^{\mathrm{FEM}} + \left\{\varphi_\alpha u_\alpha^{\mathrm{gl},n+1},\ \alpha \in \mathcal{I}^{\mathrm{gl}}\right\}$$
$$\text{where } u_\alpha^{\mathrm{gl},n+1}(\boldsymbol{x}) = \underline{u}_\alpha u_L^{n+1}(\boldsymbol{x}) \in \chi_\alpha^{n+1},\ \underline{u}_\alpha \in \mathbb{R}$$

- Discretization spaces updated on-the-fly with global-local enrichment functions

Parallelization of Fine-Scale Computations

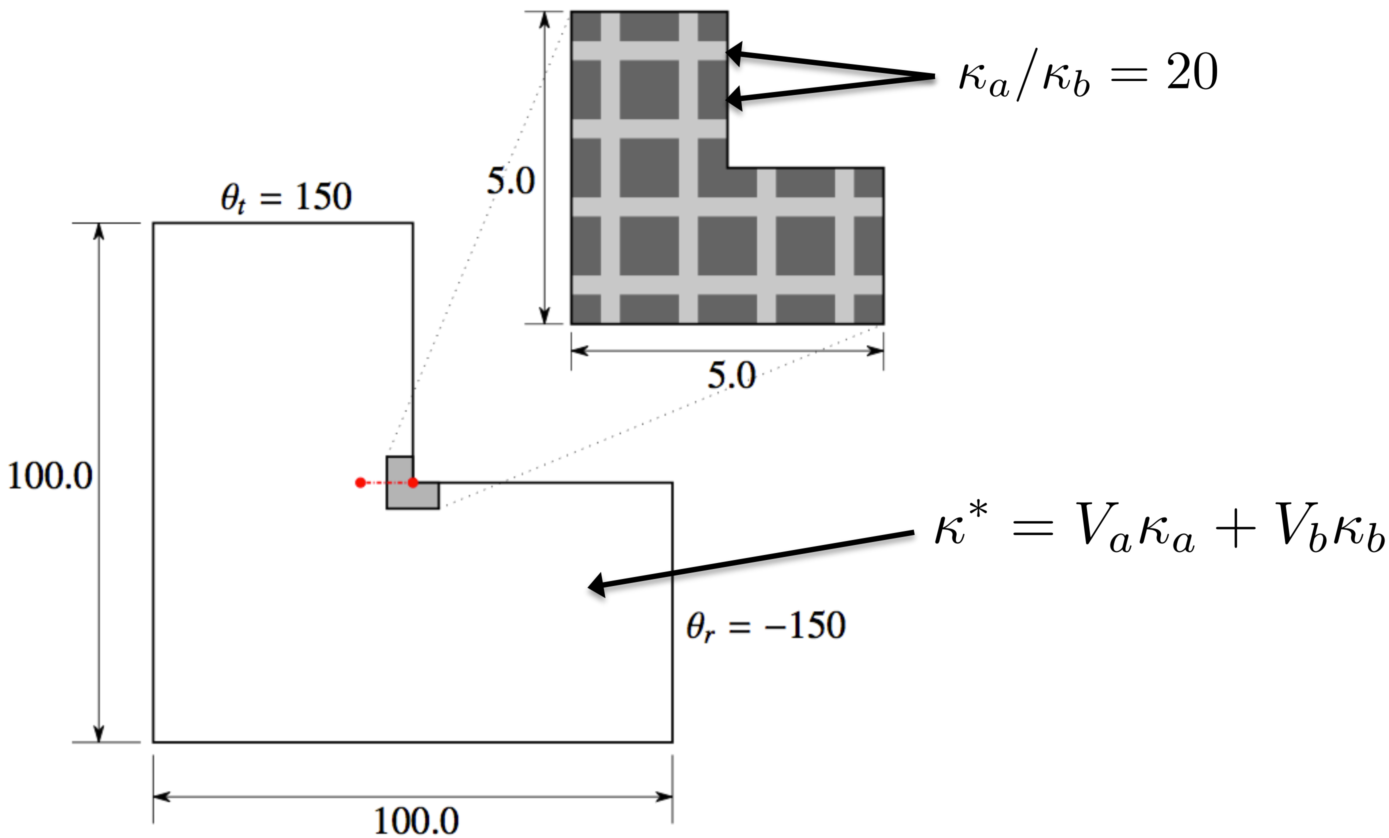
- Subdivide local problem into 'sub-local' domains
  - ✓ Each global patch/cloud (node support) = One sub-local domain



- GFEM^{gl} *does not* require
  - Communication among sub-local problems
  - Continuity across sub-local boundaries
- Analyze fine scale efficiently in parallel [Kim et al. 2010]

Example: Steady-State Heat Transfer on L-Shaped Domain*

- Heat flux singularity *and* material heterogeneity at reentrant corner

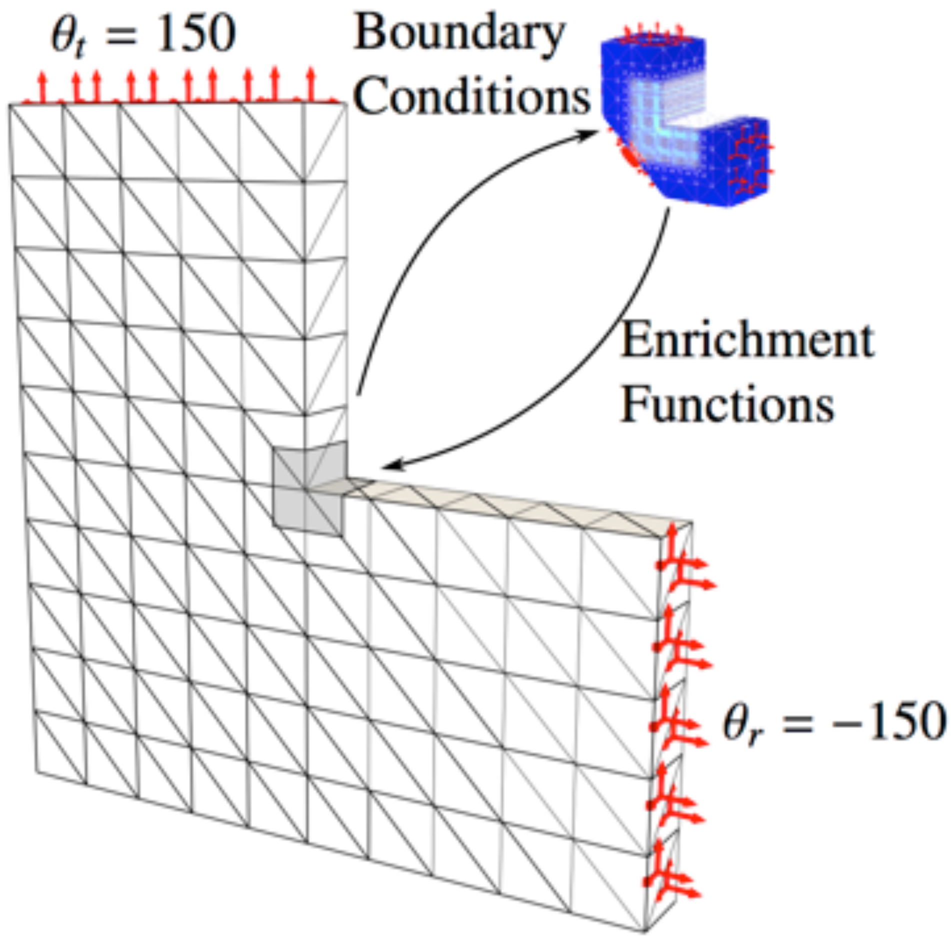


- Homogenization *not valid at corner*
- Adopt GFEM^{gl} to capture interaction between material and global scales

*[Plews and Duarte, 2014]

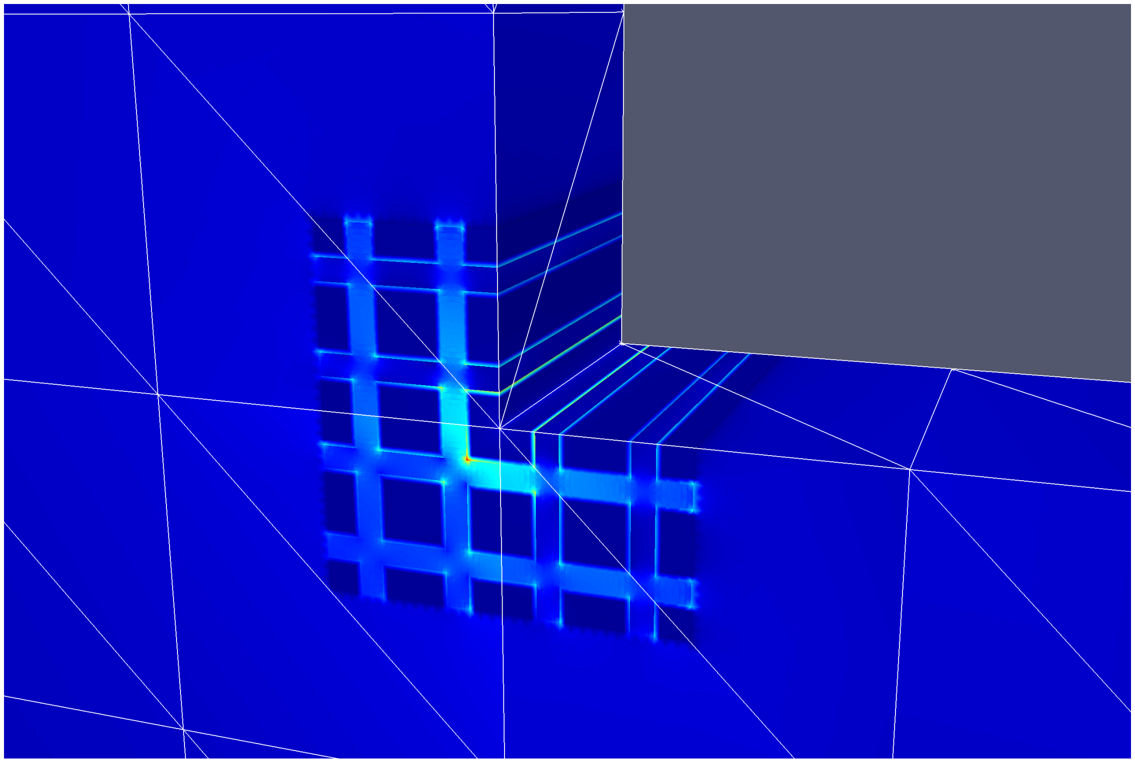
Steady-State Heat Transfer on L-Shaped Domain

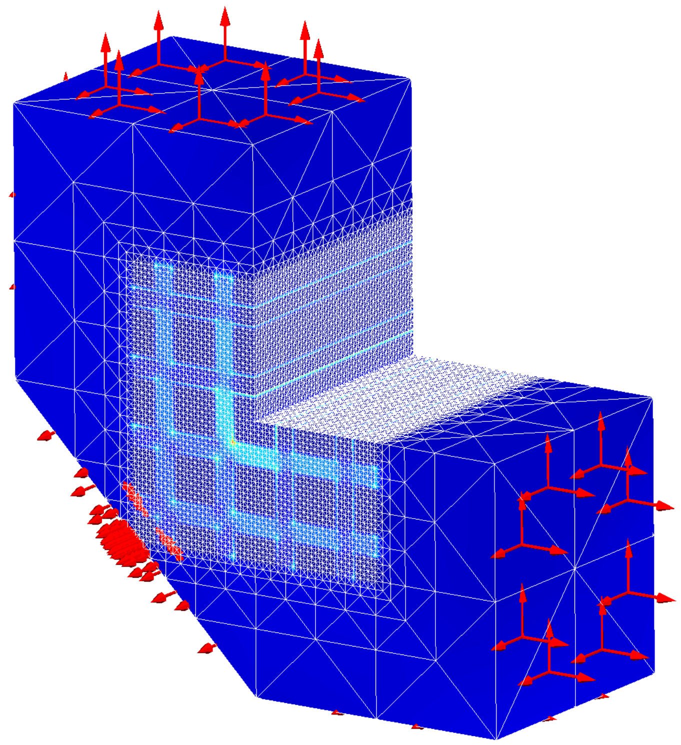
- GFEM^{gl} discretizations
  - Coarse global mesh
  - Refine heavily in sub-*local problems*
  - Solve sub-local problems in parallel
  - Global-local enrichments in neighborhood of corner only, polynomial enrichment elsewhere



Steady-State Heat Transfer on L-Shaped Domain

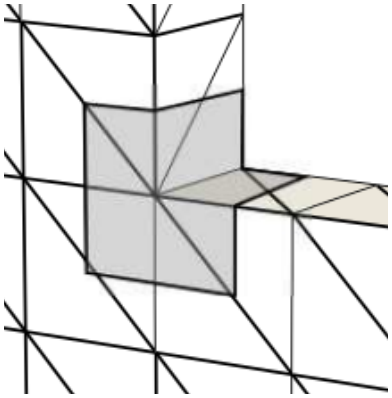
- GFEM^{gl} discretizations: Fine-scale mesh is *non-conforming* with global mesh



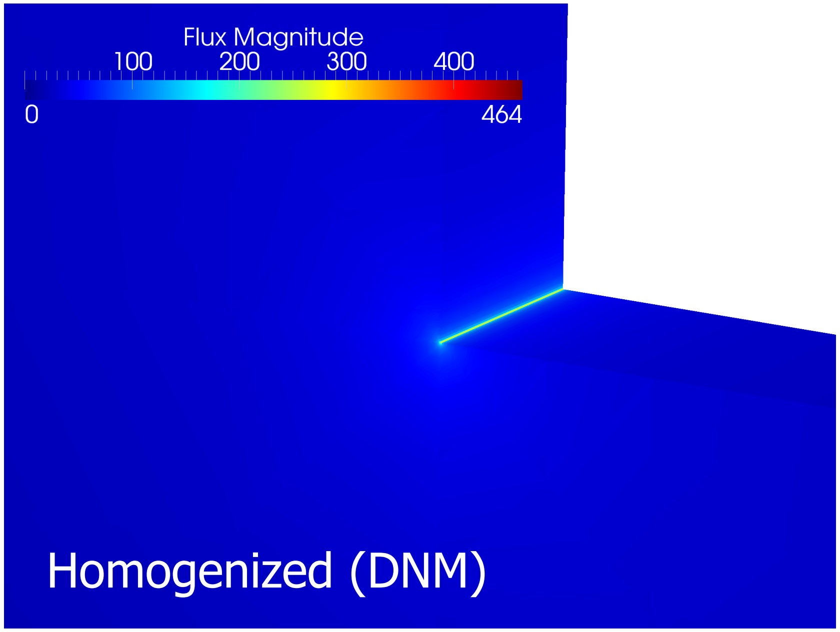


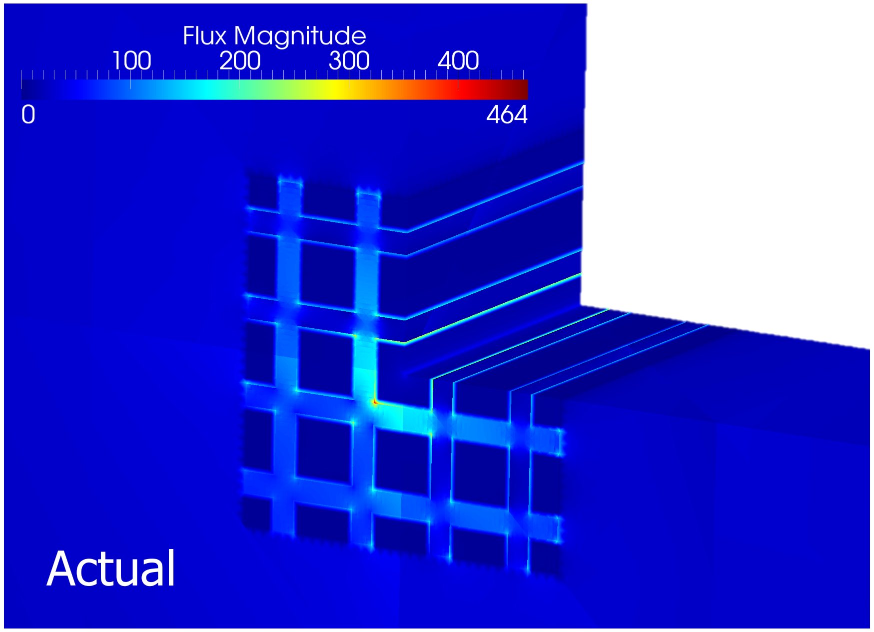
Steady-State Heat Transfer on L-Shaped Domain

- GFEM^{gl} resolves localized gradients and singularities on a coarse global mesh



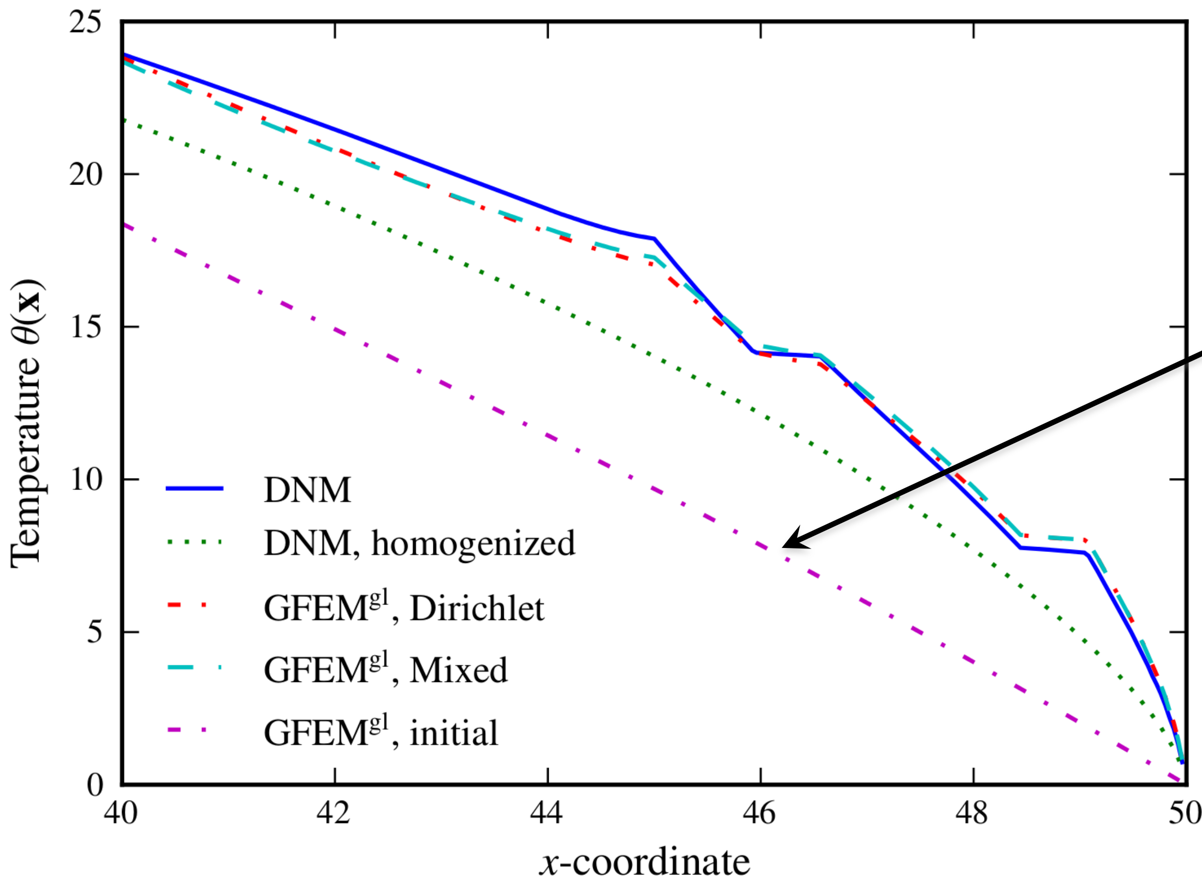
Heat flux at re-entrant corner





Steady-State Heat Transfer on L-Shaped Domain

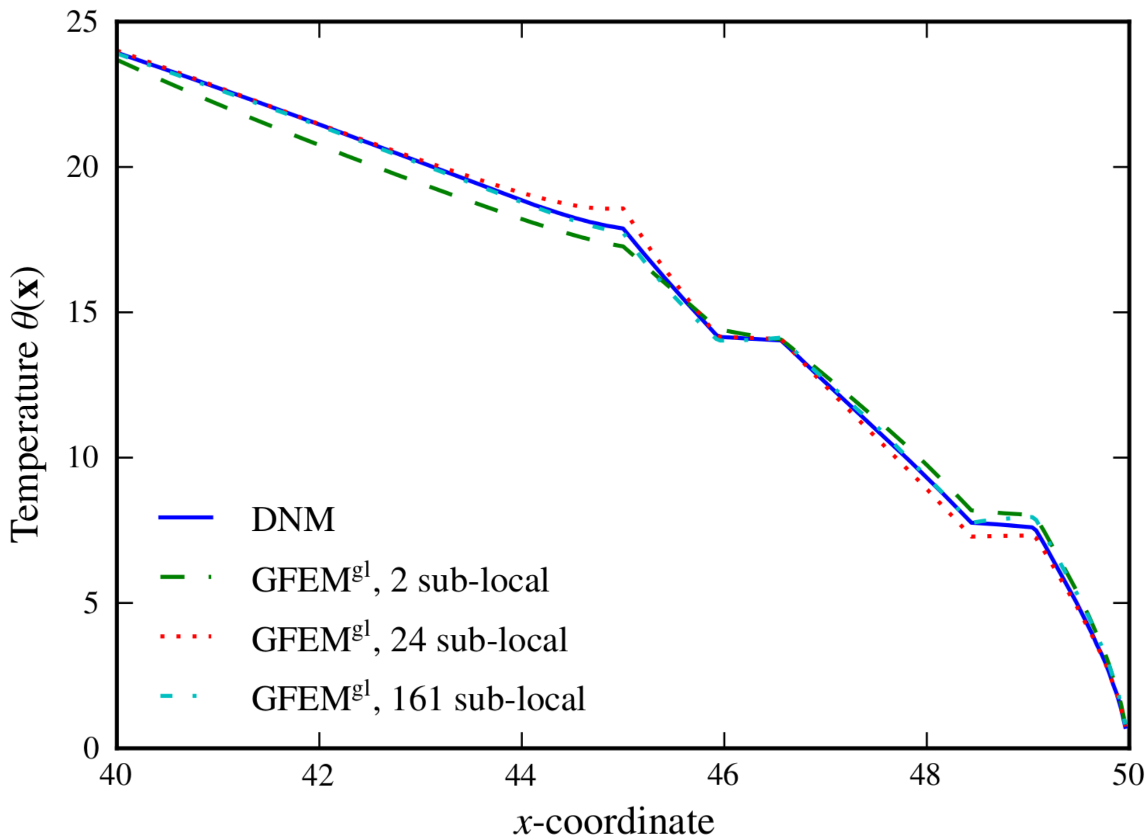
- Solution in neighborhood of reentrant corner: GFEM^{gl} with two sub-local problems



GFEM^{gl} initial global solution on uniform mesh only used to generate boundary conditions for local problems.

Steady-State Heat Transfer on L-Shaped Domain

- Pointwise convergence of $GFEM^{gl}$ solution in neighborhood of reentrant corner



Steady-State Heat Transfer on L-Shaped Domain

- Computational cost and parallelism

Method	Sub-local prob.	Global dofs	Energy ($\times 10^6$)	% difference	Sol. time (s)
DNM (parallel)	–	1,676,652	3.376	–	177.4
GFEM^{gl}	24	880	3.377	0.05%	281.6
	161	1,789	3.377	0.03%	60.8
	864	4,440	3.375	0.01%	30.2

  - Global mesh refined to generate more sub-local problems
  - Identical mesh size maintained in sub-local problems and DNM
  - Server: 24 cores, 2 Intel Xeon E5-2697 v2 2.70GHz processors
  - Pardiso parallel sparse solver adopted
  - Solution time includes assembly, factorization and solve

- Efficiency and accuracy increase with number of sub-local problems

Steady-State Heat Transfer on L-Shaped Domain

- Computational cost and parallelism

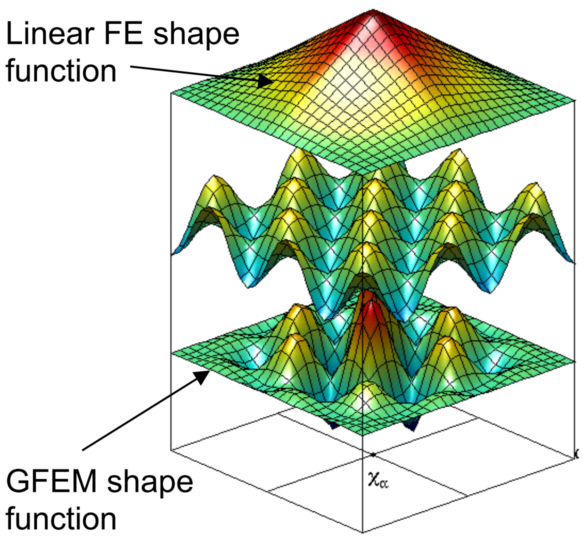
Method	Sub-local prob.	Factorize/solve (s)	Speedup	Efficiency
DNM (serial)	–	1373.1	–	–
DNM (parallel)	–	145.8	9.42	0.393
$GFEM^{gl}$ (parallel)	24	253.0	5.43	0.226
	161	42.0	32.66	1.361
	864	10.3	133.01	5.542

- Speedup and efficiency computed w.r.t. DNM serial solution
- Efficiency over 100% relative to DNM with nearly identical accuracy
- Pardiso parallel sparse solver adopted

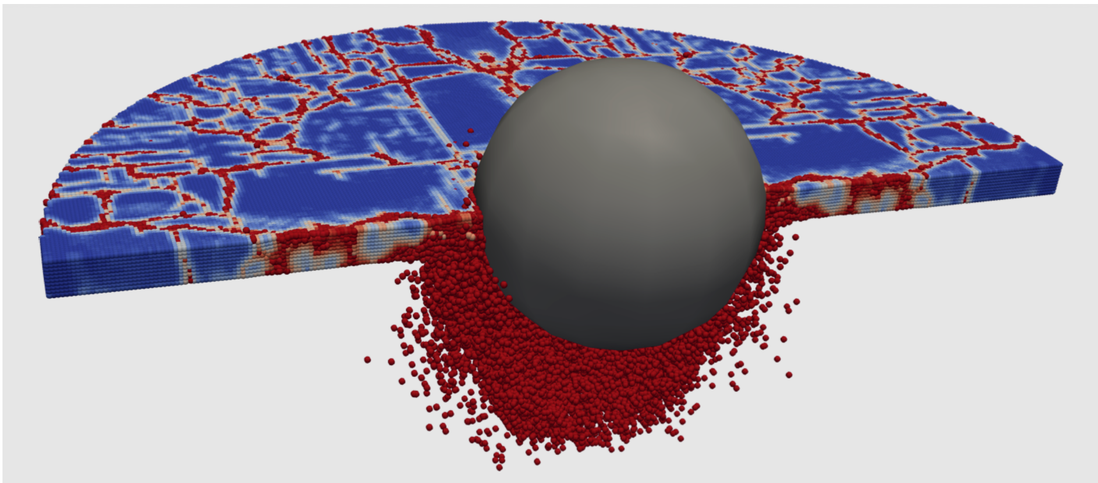
- Efficiency and accuracy increase with number of sub-local problems

GFEM^{gl}: Discretization Method at Fine Scale

- *Enrichment functions can be computed with almost any available discretization method: GFEM, FEM, BEM, Meshfree, Peridynamics, etc.*



Enrichment = Numerical solutions of BVP

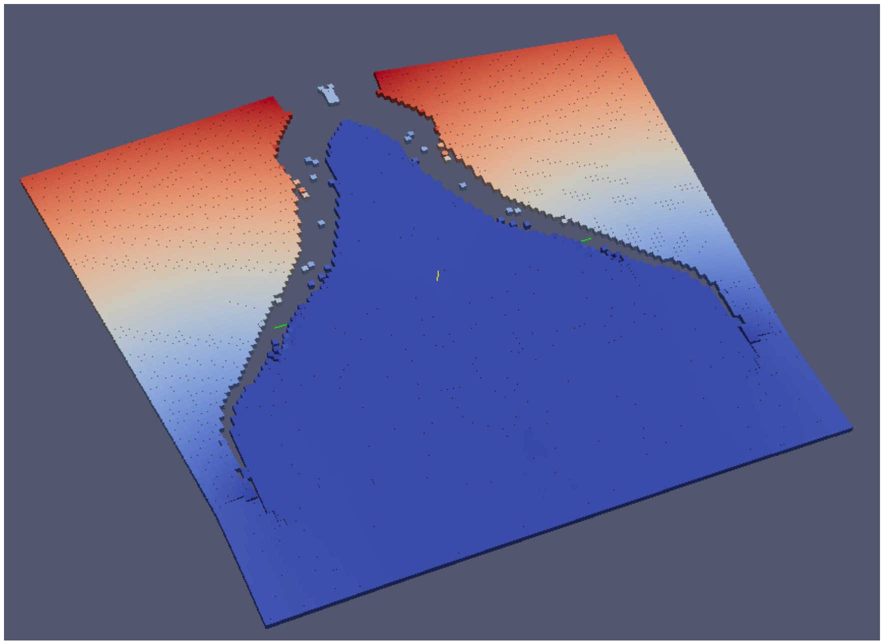


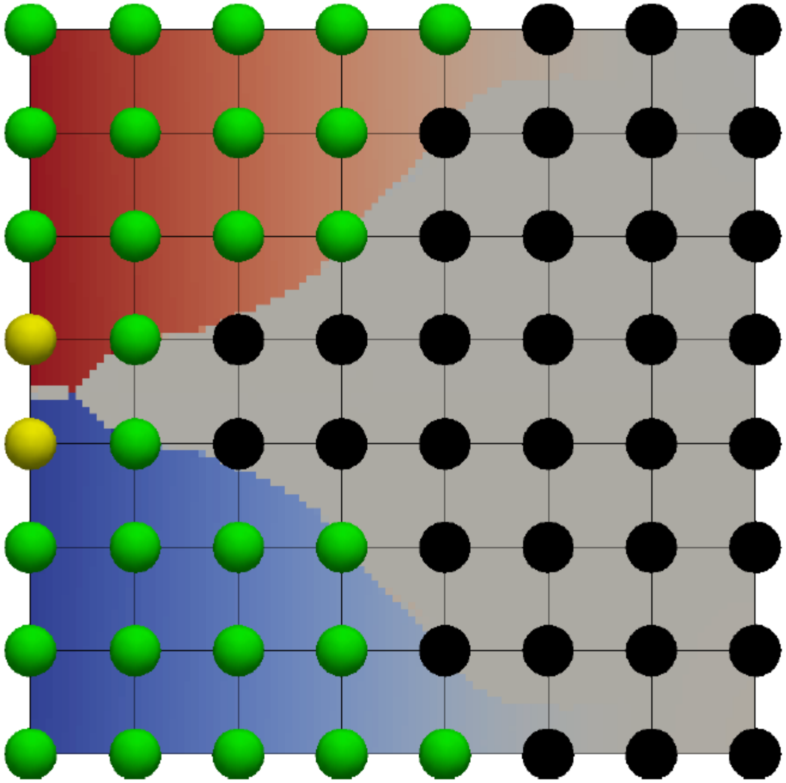
- Simulation of impact and fragmentation using Peridynamics [Sa Wu and Marc A. Schweitzer, Bonn University]*
- Initial conditions for Peridynamics from GFEM sol.
- Use Peridynamics enrichments only where it is needed

* http://schweitzer.ins.uni-bonn.de/people/wu.html

GFEM^{gl}: Discretization Method at Fine Scale

- Peridynamics solution (left) used as enrichment at macro-scale GFEM mesh (right) *





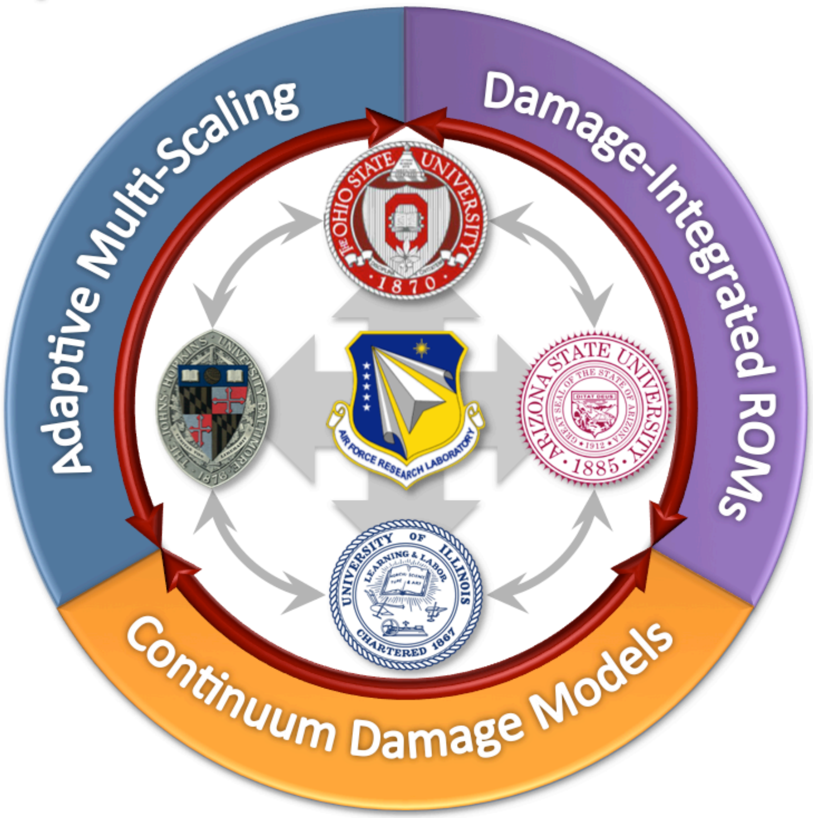
* http://schweitzer.ins.uni-bonn.de/people/wu.html

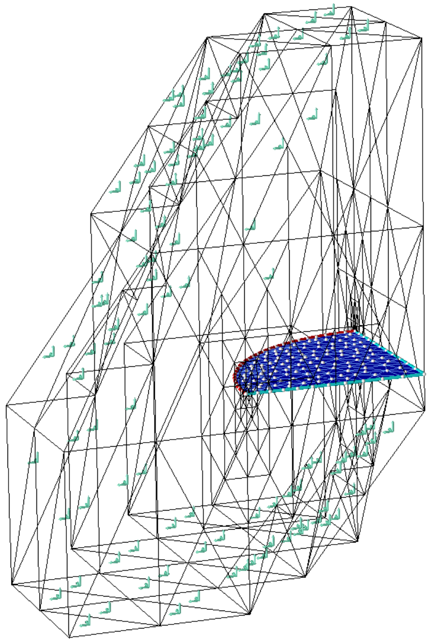
Conclusions and Outlook

- Generalized FEMs offer significant **flexibility** and attractive features
- It enables the solution of problems that are difficult or not practical with the FEM:
  - Multiscale problems:
    - Fine-scale computations are naturally parallelizable
    - Can adopt different discretization methods at each scale without difficulty or introduction of additional fields (LM, mortar, etc.)
  - Coalescence of 3-D fractures: Hydraulic fracturing of oil and gas reservoirs
- Transition to Labs and Industries: Non-intrusive integration with existing FEA software

Acknowledgements

- AFRL-University Collaborative Center in Structural Sciences (C^2S^2)





Questions?

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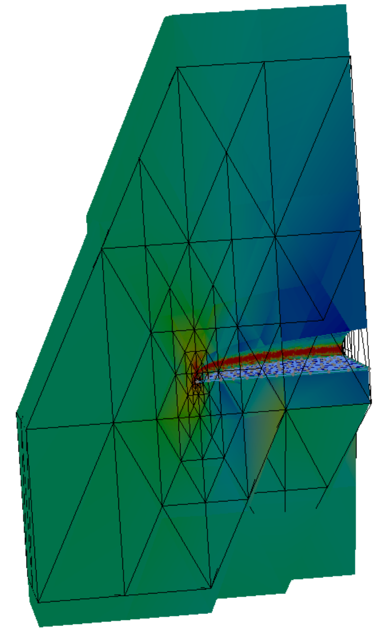

VonMises tetrahedra

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