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Recent Developments in the Generalized Finite Element Method and Applications in 3D Fracture Propagation and Coalescence

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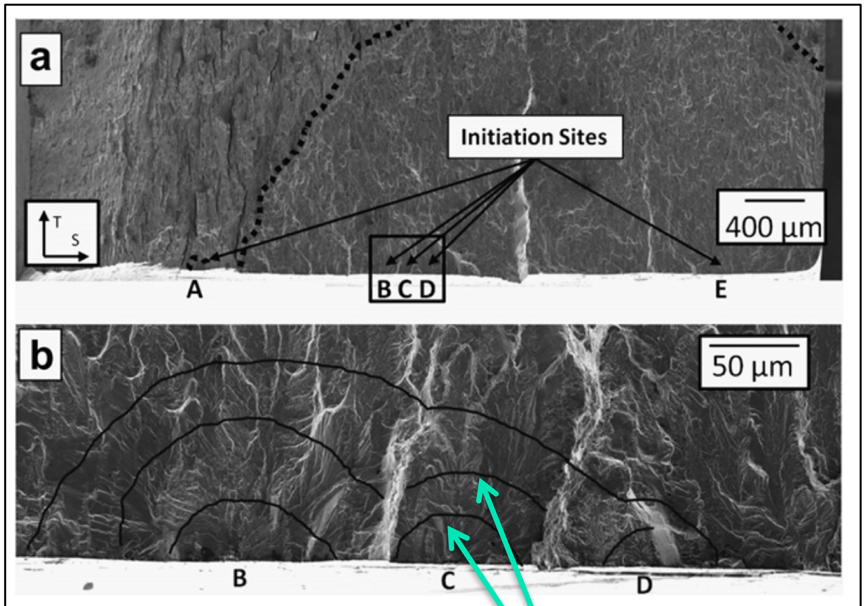
Crack Growth and Coalescence: Motivation

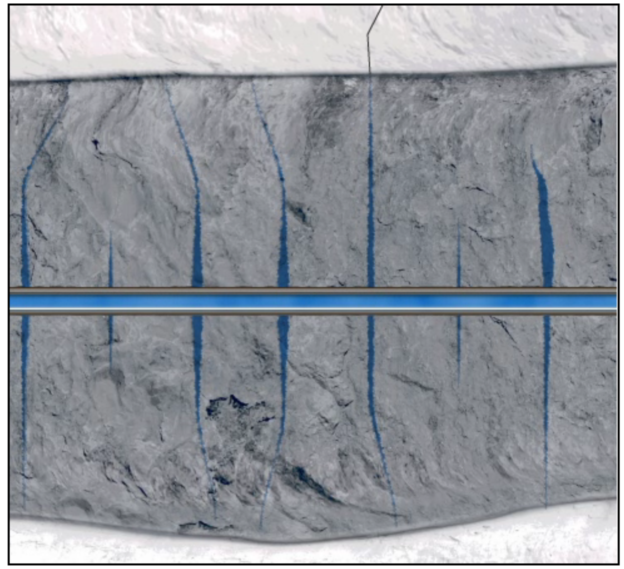
- Crack growth prediction is of great importance in many applications

Coalescence of fatigue micro-cracks



Crack fronts

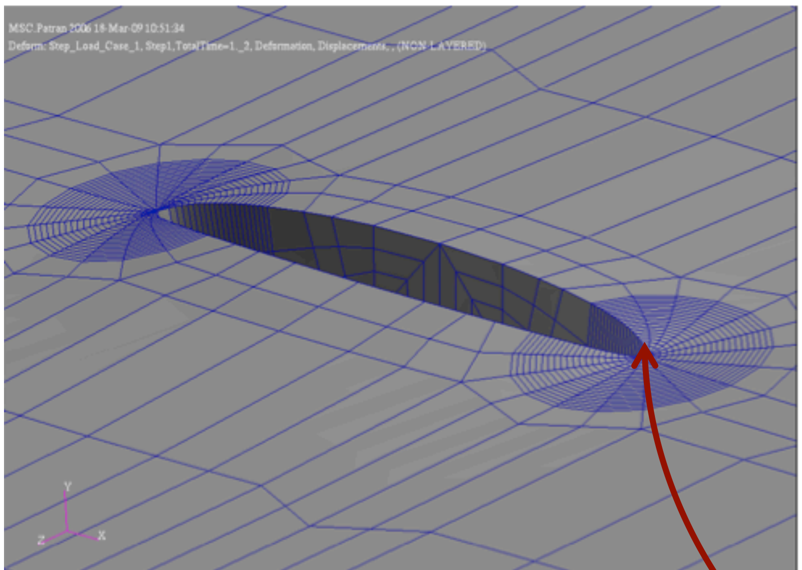
Hydraulic fractures from horizontal well

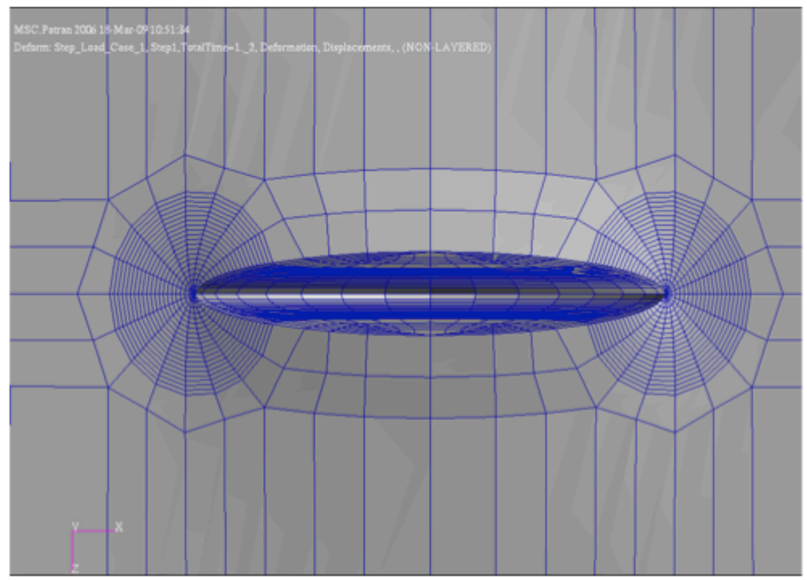


Reflective crack in asphalt overlay

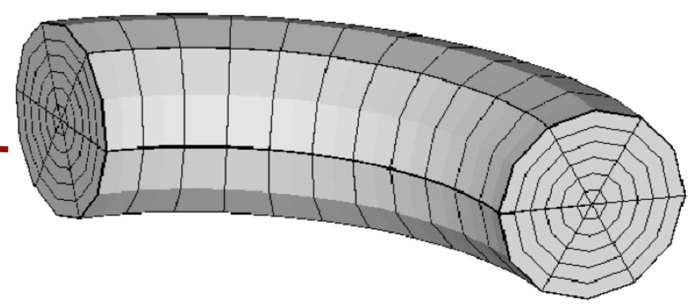
Modeling 3-D Fractures: Limitations of Standard FEM

- It is not "just" fitting the 3-D evolving crack surface
- FEM meshes must satisfy special requirements for acceptable accuracy





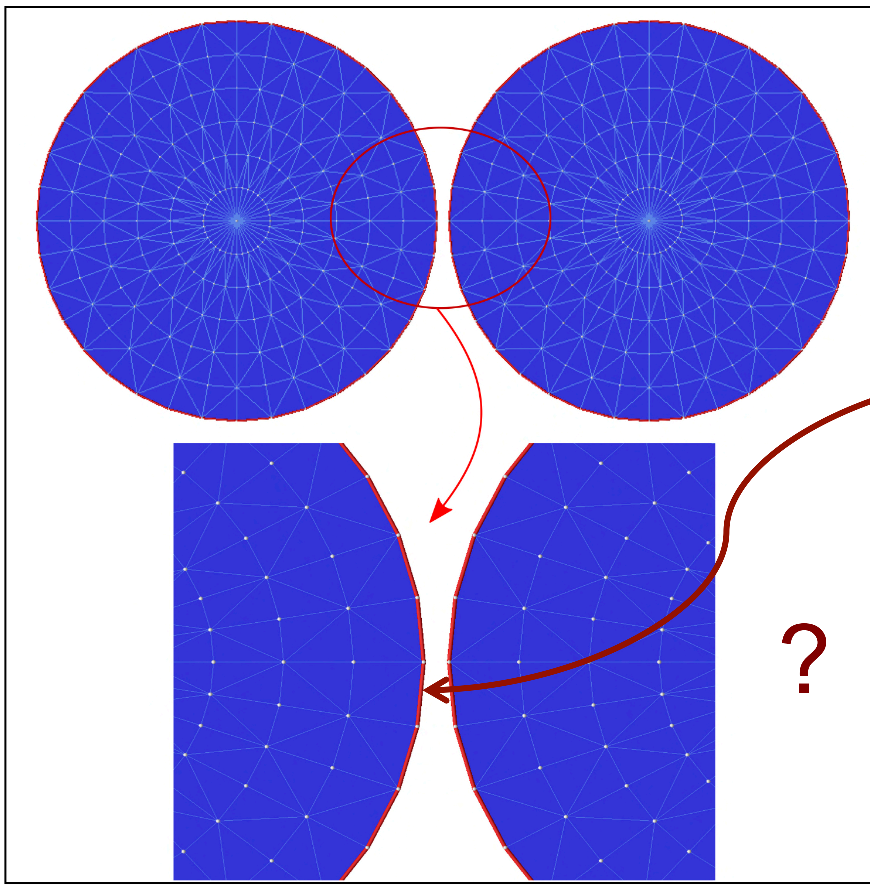
FEM mesh for a surface crack



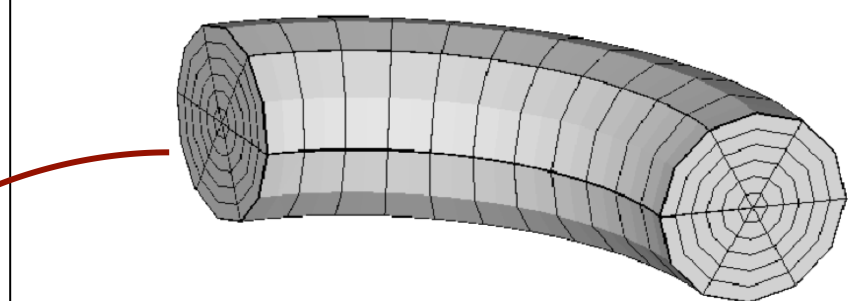
Mesh with quarter-point elements

Limitations of Standard FEM

- Difficulties arise if crack front is close to complex geometrical features
- Crack surfaces with sharp turns
- Coalescence of cracks



Not possible in general to automatically create structured meshes along both crack fronts when they are in close proximity



Even with these crafted meshes and quarter-point elements, convergence rate of std FEM is slow (controlled by singularity at crack front)

Outline

- Motivation
- Basic ideas of GFEM
- GFEM for 3D Cracks
- Applications
  - ✓ Interaction of hydraulic and natural fractures
  - ✓ Coalescence of fractures
- Conclusions

Early Works on Generalized FEMs

- Babuska, Caloz and Osborn, 1994 (Special FEM).
- Duarte and Oden, 1995 (Hp Clouds).
- Babuska and Melenk, 1995 (PUFEM).
- Oden, Duarte and Zienkiewicz, 1996 (Hp Clouds/GFEM).
- Duarte, Babuska and Oden, 1998 (GFEM).
- Belytschko et al., 1999 (Extended FEM).
- Strouboulis, Babuska and Copps, 2000 (GFEM).

- Basic idea:
  - Use a partition of unity to build Finite Element shape functions

- Review paper

  Belytschko T., Gracie R. and Ventura G. A review of extended/generalized finite element methods for material modeling, *Mod. Simul. Matl. Sci. Eng.,* 2009

  "The XFEM and GFEM are basically identical methods: the name generalized finite element method was adopted by the Texas school in 1995–1996 and the name extended finite element method was coined by the Northwestern school in 1999."

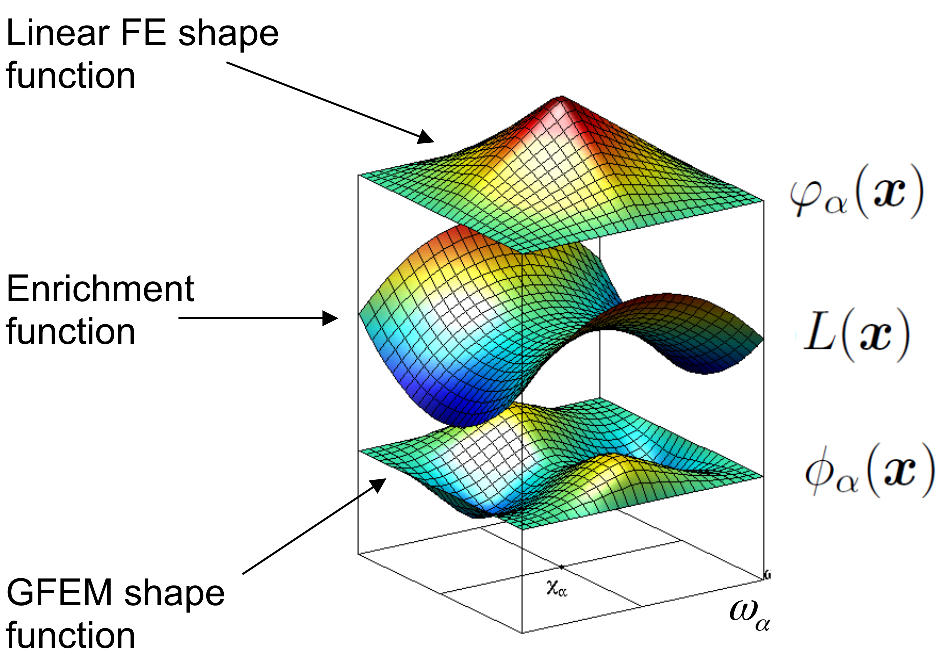
Generalized Finite Element Method

- GFEM can be interpreted as a FEM with shape functions built using the concept of a partition of unity:

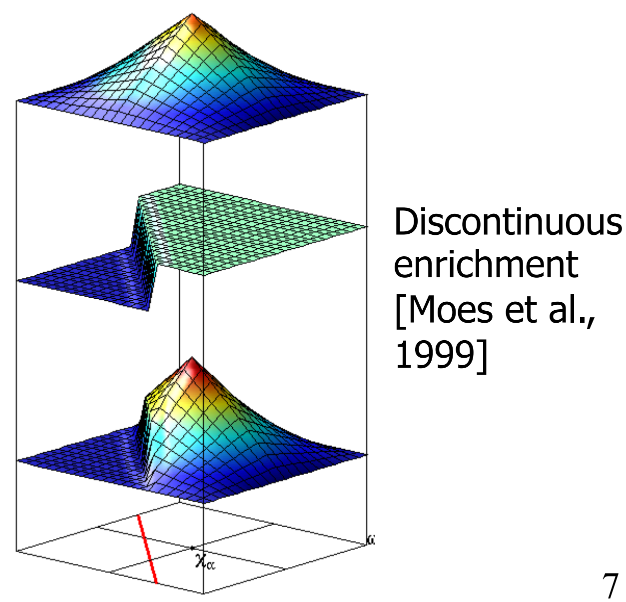
GFEM shape function = FE shape function * enrichment function

$$\boxed{\phi_\alpha(\boldsymbol{x}) = \varphi_\alpha(\boldsymbol{x}) L(\boldsymbol{x})} \qquad \sum_\alpha \varphi_\alpha(\boldsymbol{x}) = 1$$

- Allows construction of shape functions incorporating a-priori knowledge about solution

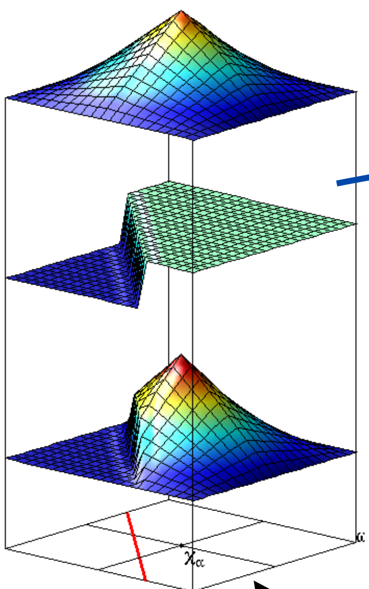


[Oden, Duarte & Zienkiewicz, 1996]



GFEM Approximation for 3-D Cracks

$$\boldsymbol{X}^{hp}(\Omega) = \left\{ \boldsymbol{u} = \sum_{\alpha=1}^{N} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[\underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$

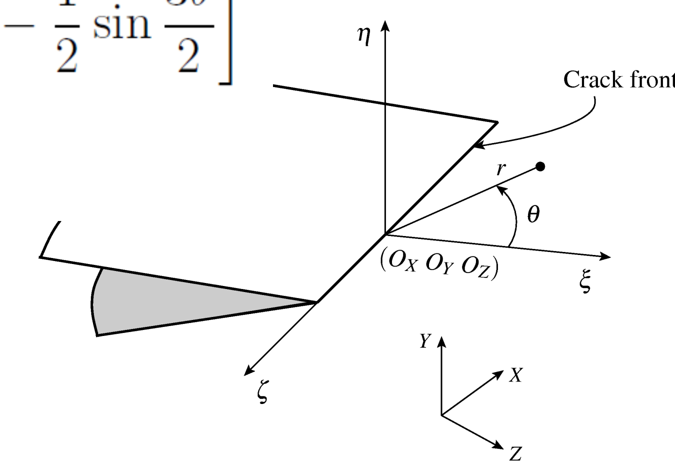


$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

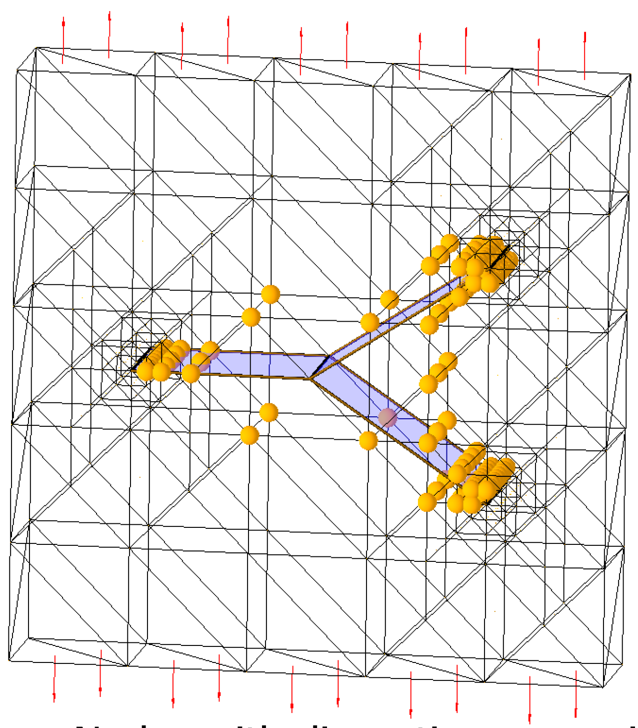
[Duarte and Oden 1996]



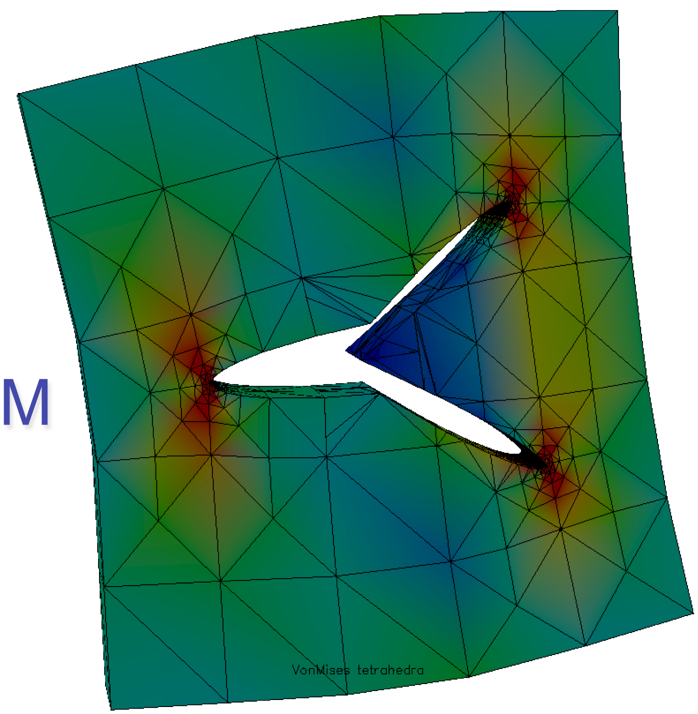
cloud or patch $'\alpha$

Modeling Cracks with hp-GFEM

- Discontinuities modeled via enrichment functions, *not* the FEM mesh
- Mesh refinement *still required* for acceptable accuracy



hp-GFEM



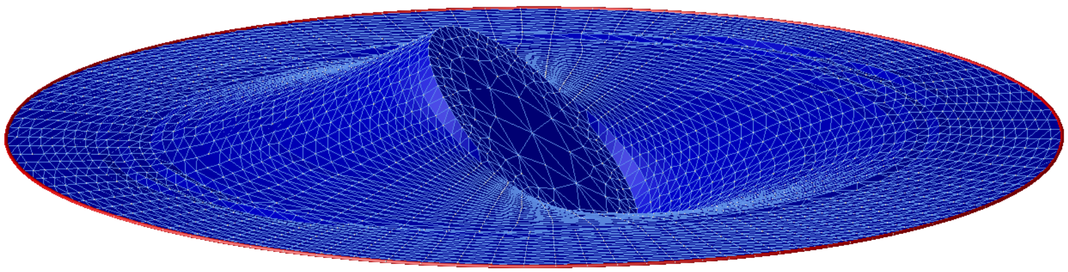
● = Nodes with discontinuous enrichments

Von Mises stress

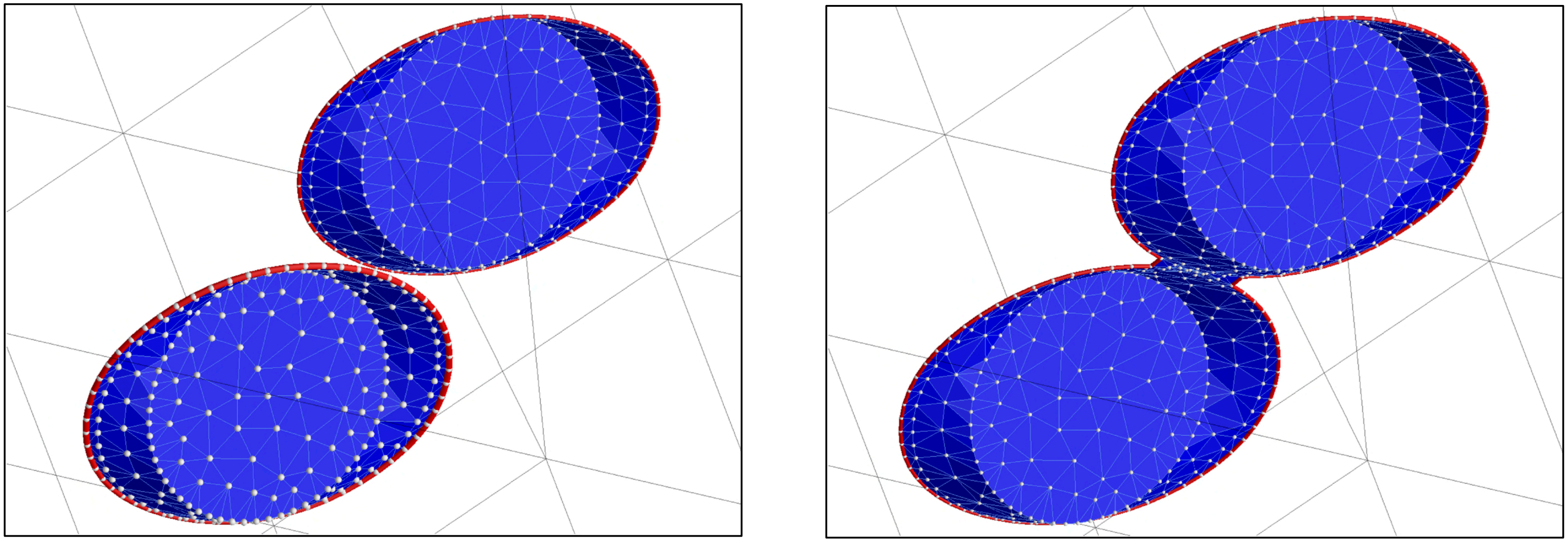
[Duarte et al., International Journal Numerical Methods in Engineering, 2007]

3D Crack Surface Representation

- High-fidelity explicit representation of crack surfaces [Duarte et al., 2001, 2009]



- Coalescence of fractures [Garzon et al., 2014]



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Hydraulic Fracturing of Gas Shale Reservoirs

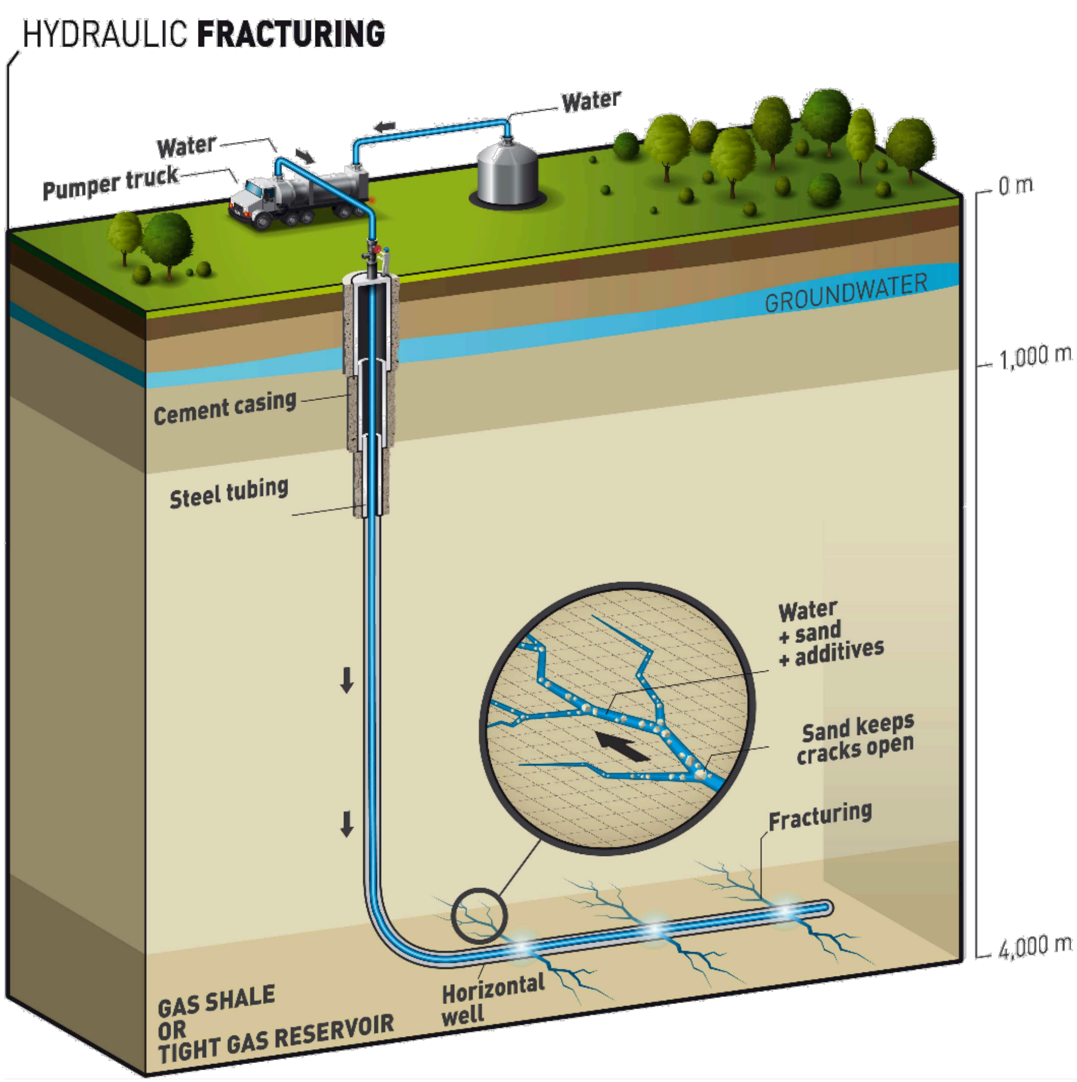
Motivation

- Natural gas production in the US has increased significantly in the past few years thanks to advances in hydraulic fracturing of gas shale reservoirs
- Yet there are concerns about the environmental impact of toxic fluids used in this process

Objectives

- Computational simulations will lead to better designs of hydraulic fracture treatments, thus reducing the amount of toxic fluids used
- Realistic modeling of hydraulic fracturing treatments can evaluate the potential impact of interactions between hydraulic fractures and naturally existing fractures in shale reservoirs

What is Hydraulic Fracturing?



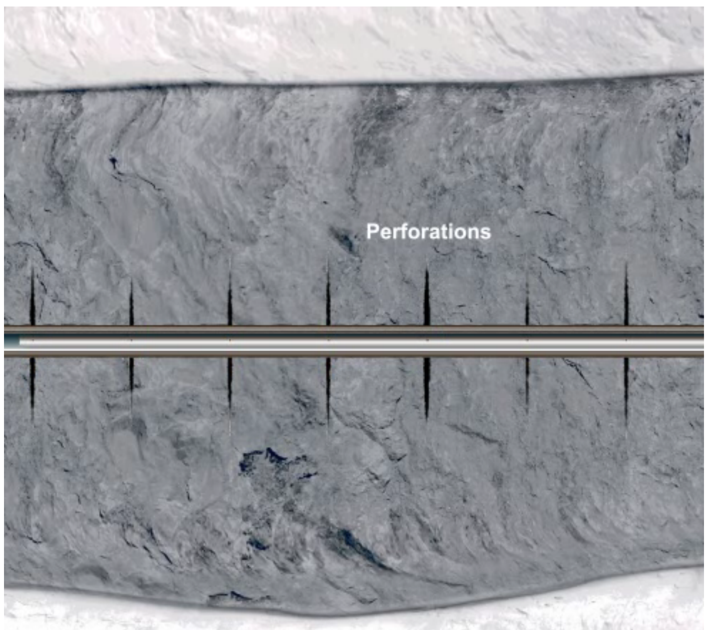
Video

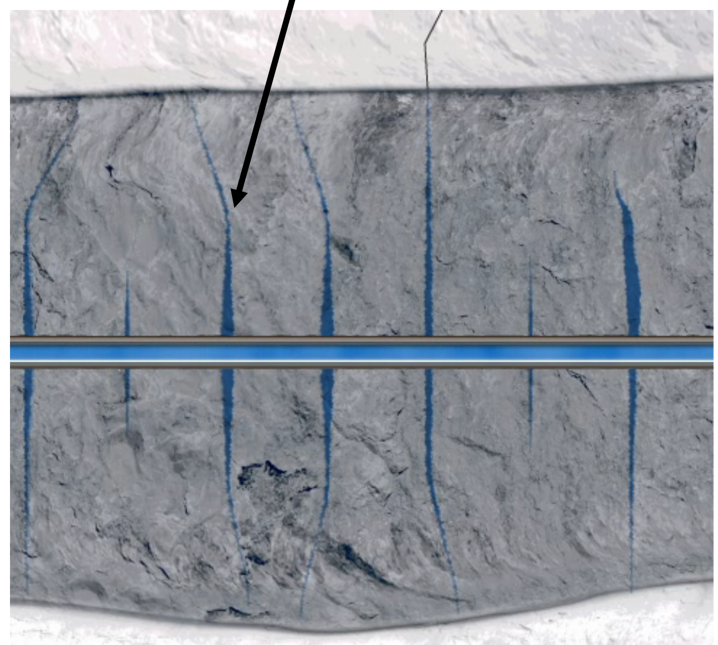
Graham Roberts, New York Times, http://www.nytimes.com/interactive/2011/02/27/us/fracking.html

Hydraulic Fracturing Simulation

Current Focus: 3-D effects not captured by available simulators

- Initial stages of fracture propagation: Fracture re-orientation, interaction and coalescence



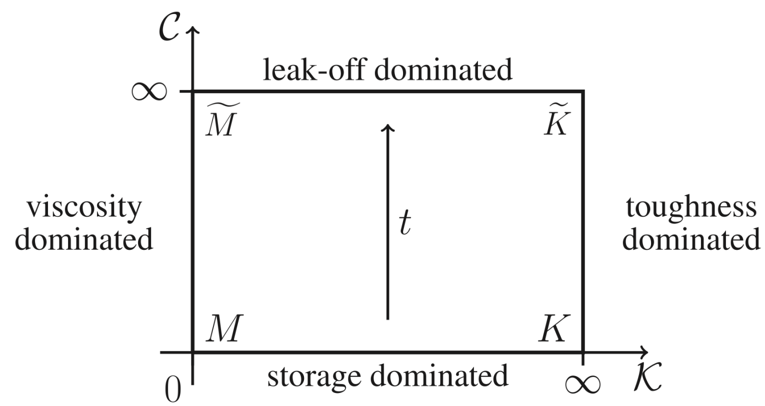


Hydraulic Fracturing Regimes

- Fracture propagation is governed by
  - two competing energy dissipation mechanisms: Viscous flow and fracturing process;
  - two competing storage mechanisms: In the fracture and in the porous matrix

Dimensionless toughness $$\mathcal{K} = \frac{4K_{Ic}}{\sqrt{\pi}} \left(\frac{1}{3Q_0 E'^3 \mu} \right)^{1/4}$$

Leak-off coefficient $$\mathcal{C} = 2C_L \left(\frac{E't}{12\mu Q_0^3} \right)^{1/6}$$



Hydraulic fracture parametric space*

Current Focus: Storage-toughness dominated regime

- Low permeability reservoirs: Neglect flow of hydraulic fluid across crack faces:
  - Storage dominated regime
- High confining stress and low viscosity fluid (water):
  - Constant pressure distribution in fracture; Toughness dominated regime
- Brittle elastic material

*[Carrier & Granet, EFM, 2013]

Weak Form at Propagation Step k

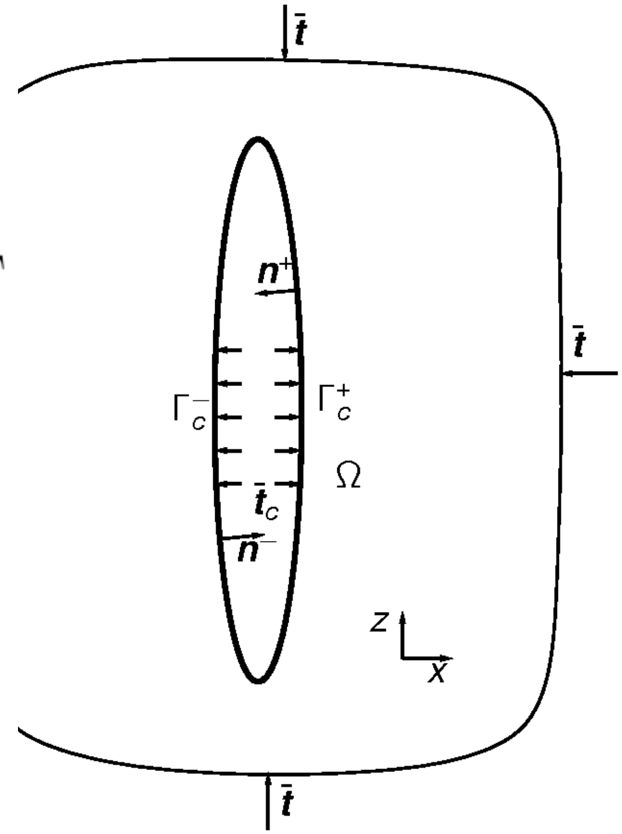
Find $\boldsymbol{u}^k \in H^1(\Omega)$, such that $\forall\ \boldsymbol{v}^k \in H^1(\Omega)$

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}^k) : \boldsymbol{\varepsilon}(\boldsymbol{v}^k) d\Omega$$

$$= \int_{\Omega} \boldsymbol{b} \cdot \boldsymbol{v}^k d\Omega + \int_{\partial\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{v}^k d\Gamma + \int_{\Gamma_c^{k+}} \bar{\boldsymbol{t}}_c^{k+} \cdot [\![\boldsymbol{v}^k]\!] d\Gamma$$

where $[\![\boldsymbol{v}^k]\!]$ is the virtual displacement jump across the crack surface Γ^k at propagation step k and

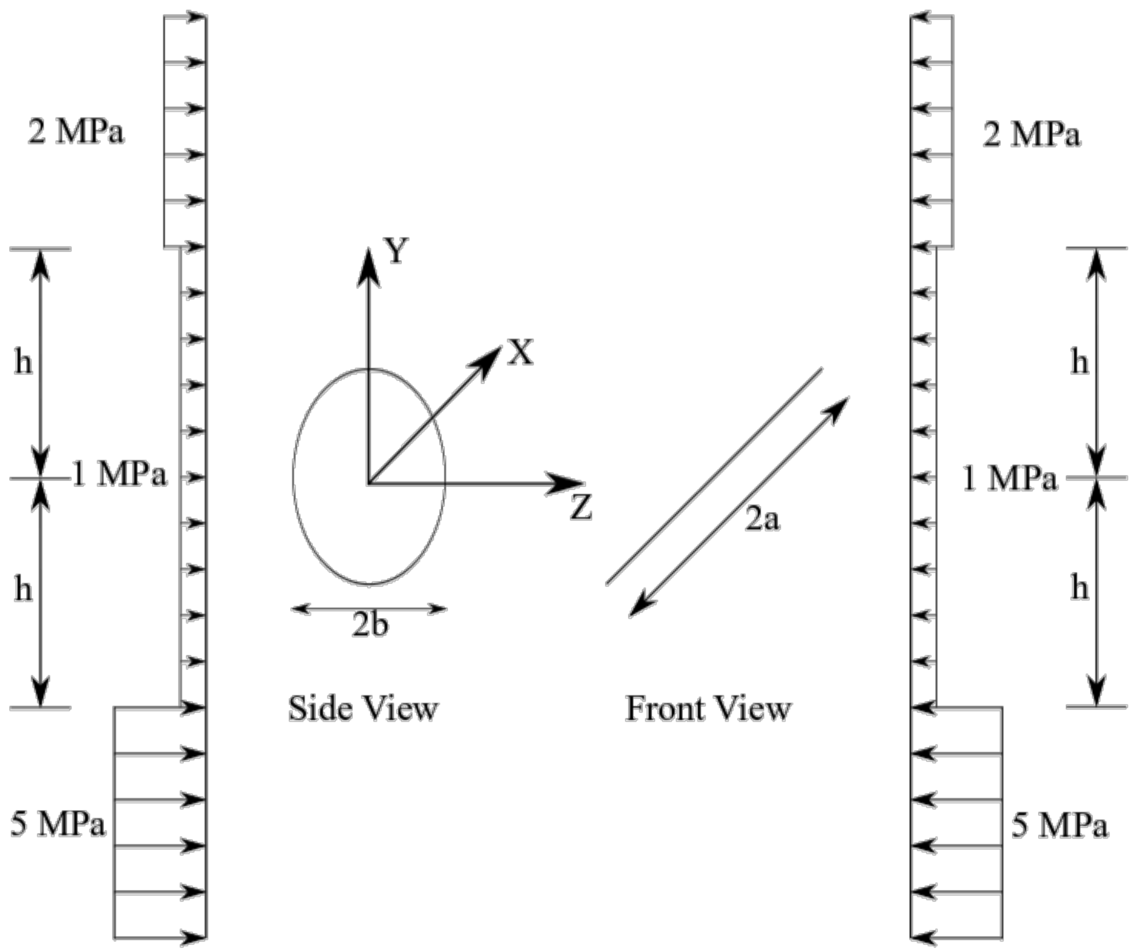
$$\bar{\boldsymbol{t}}_c^{k+} = -p^k \boldsymbol{n}^{k+} = p^k \boldsymbol{n}^{k-}$$

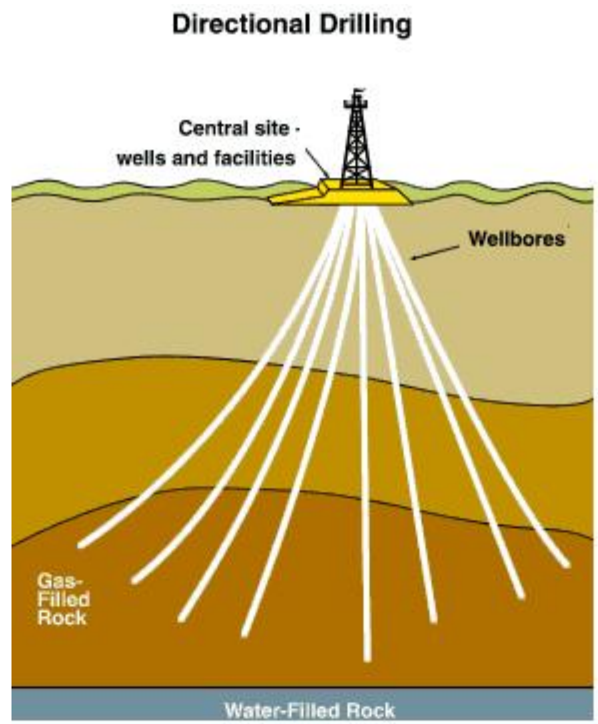


Cross section of fracture

Application: Non-Planar Fracture Growth

- Propagation from a horizontal or deviated well
- Misalignment of fracture and confining in-situ stresses





a = 10m
b = 5m
h = 15m
p = 3.5 MPa

Fracture Propagation Model

- Crack increment at front vertex i according to Mear-Wheeler* Model

$$\Delta a^i = \begin{cases} 0, & \text{if } K^i_{I,eq} < K_{Ic} \\ \Delta a_{max} \left(\frac{K^i_{I,eq} - K_{Ic}}{K^{max}_{I,eq} - K_{Ic}} \right)^m, & \text{if } K^i_{I,eq} > K_{Ic} \end{cases}$$

$\Delta a_{\max}$ and m are model constants

$K^i_{I,eq} = \text{Mode I equivalent SIF (Shollmann's criterium)}$

$K_{Ic} = \text{Fracture toughness}$

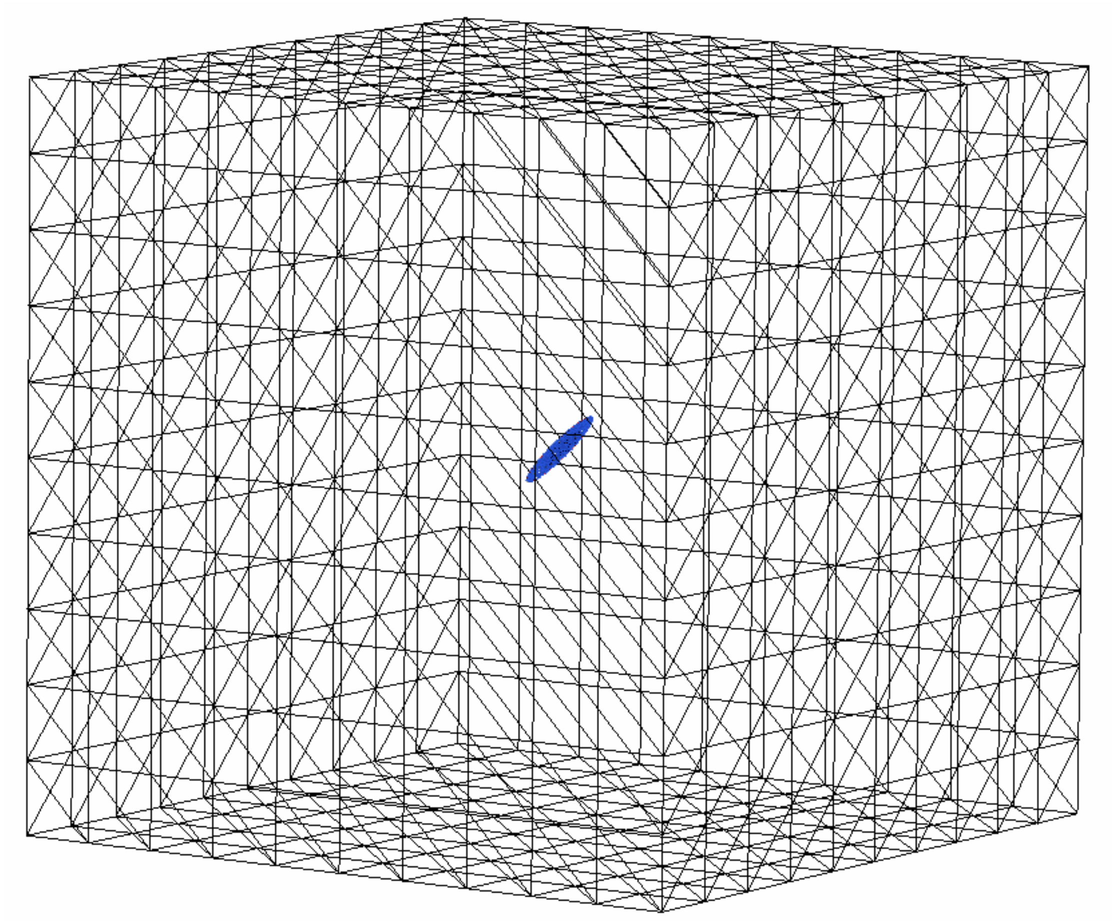
$$K_{Ic} = 0.894\,\text{MPa}\sqrt{\text{m}}$$

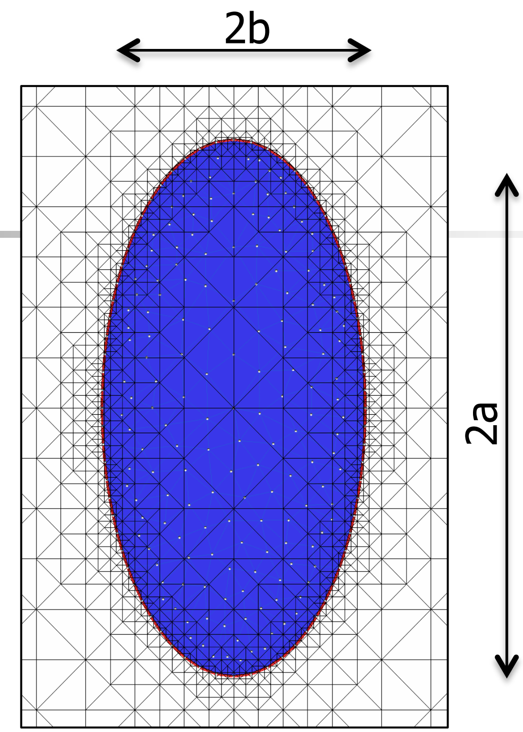
$$m = 1 \qquad \Delta a^{max} = 0.5\,m$$

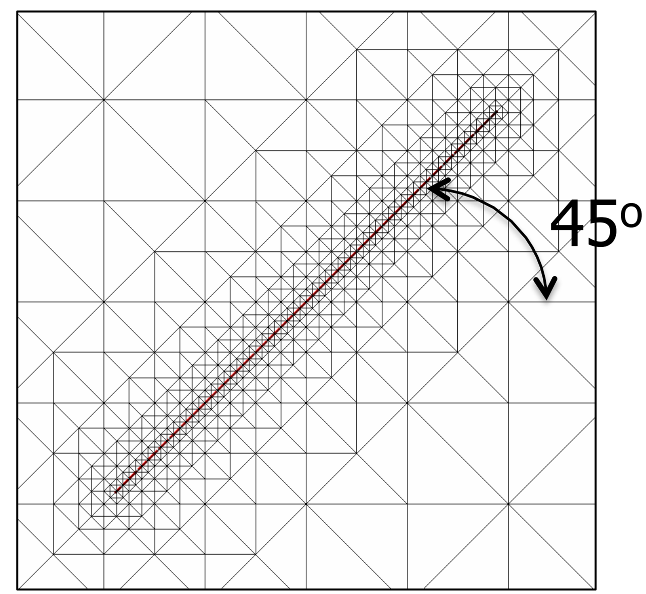
$$E = 5\,\text{GPa} \qquad \nu = 0.3$$

*[Rungamornrat et al., 2005]

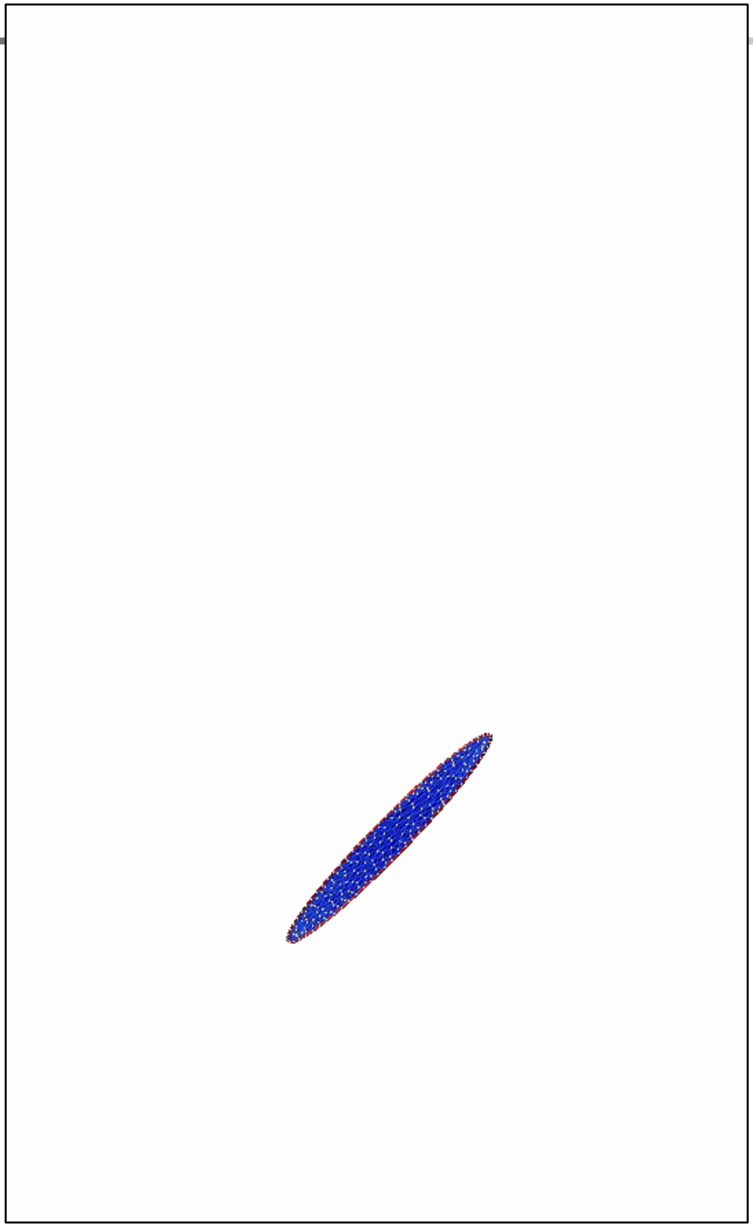
Inclined elliptical crack



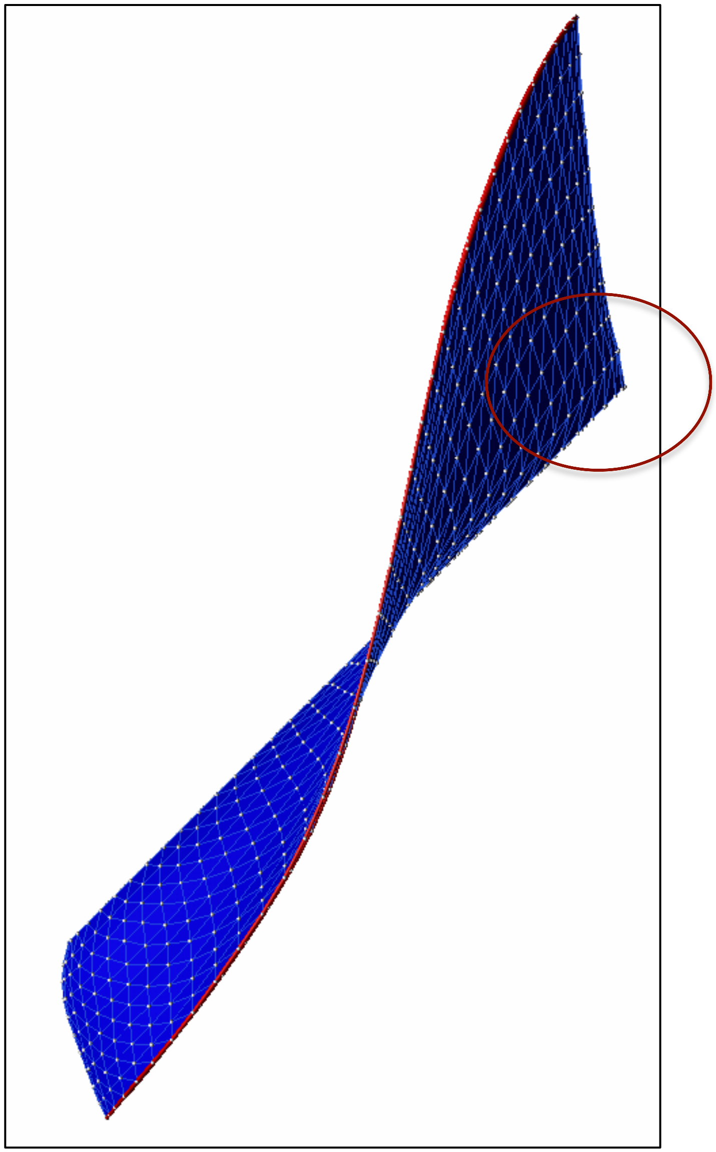


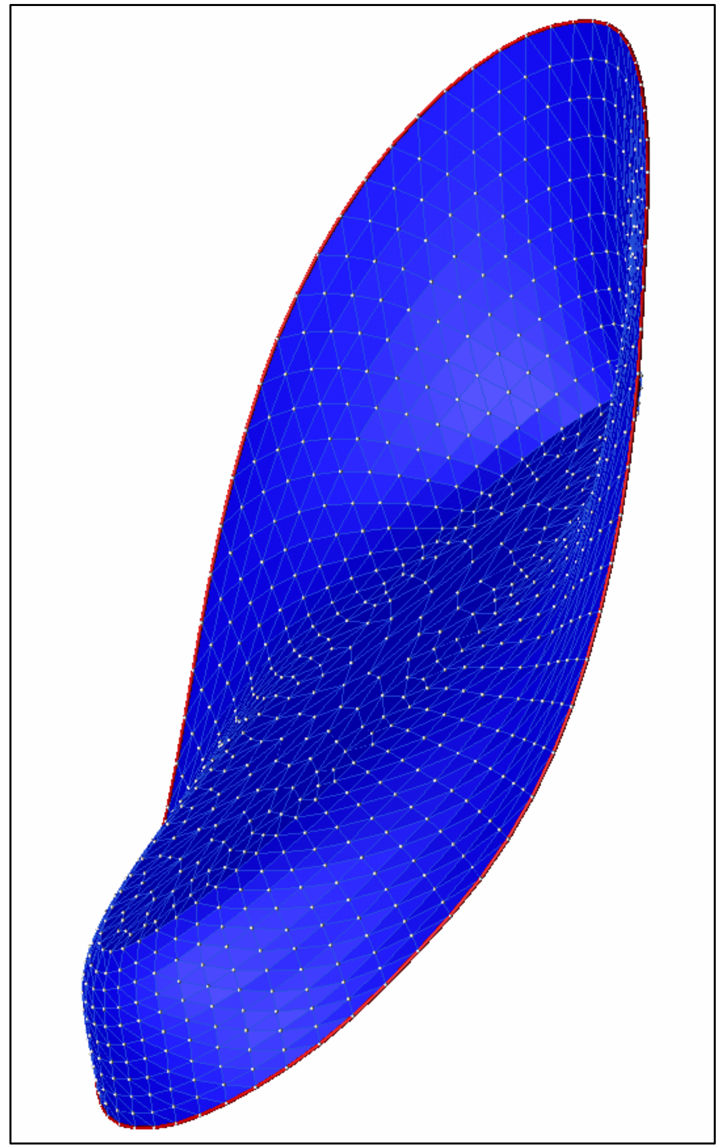


Inclined elliptical crack

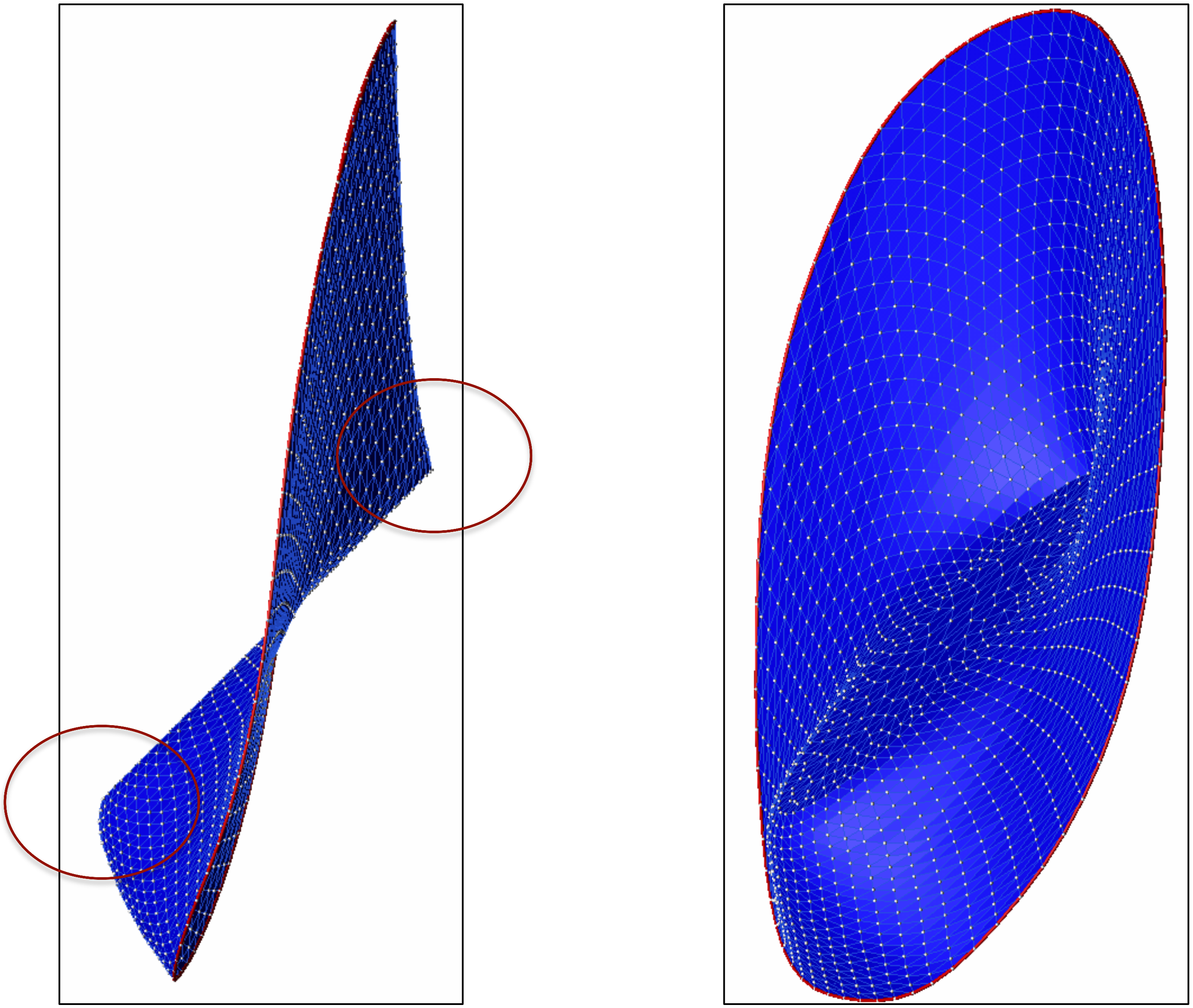


Inclined elliptical crack: Step 10

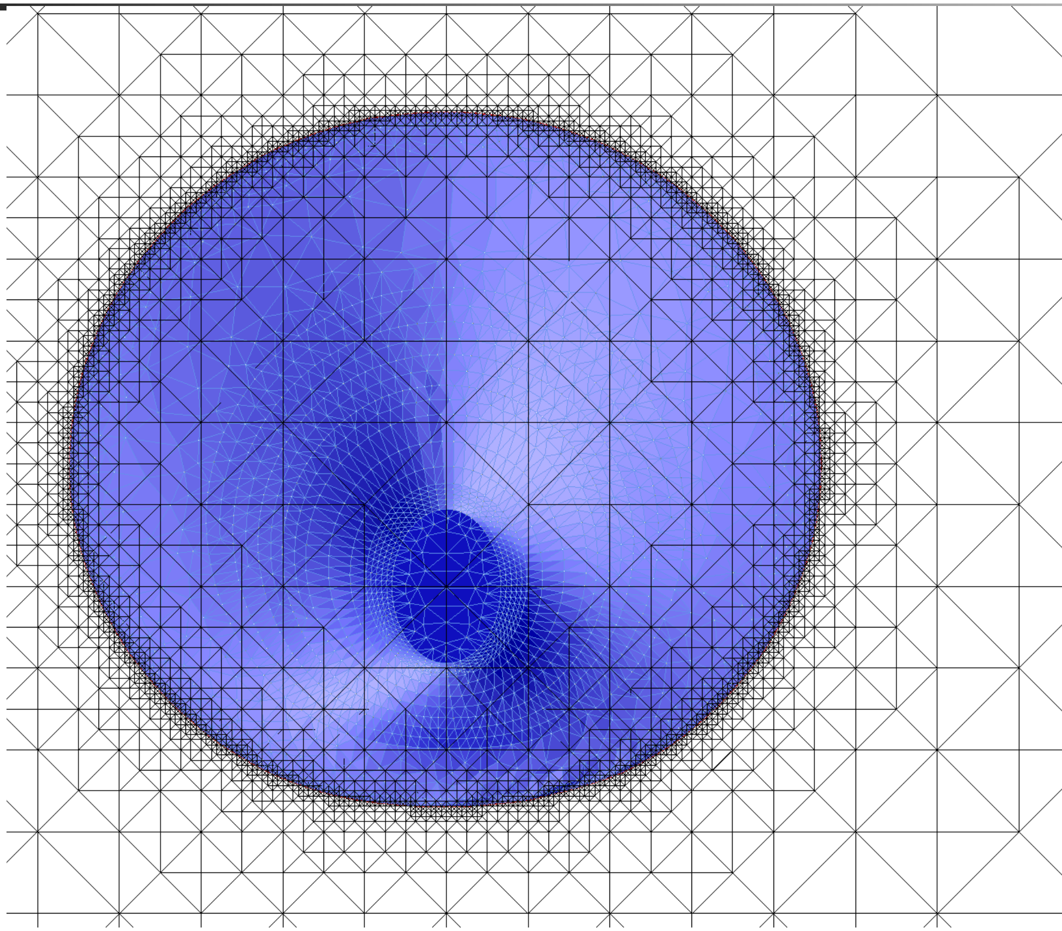


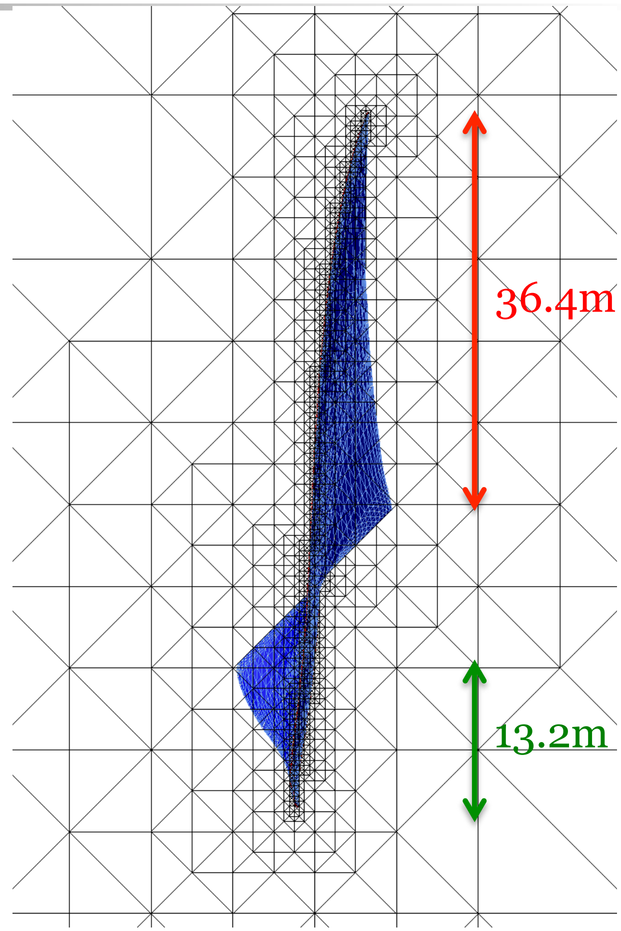


Inclined elliptical crack: Step 20



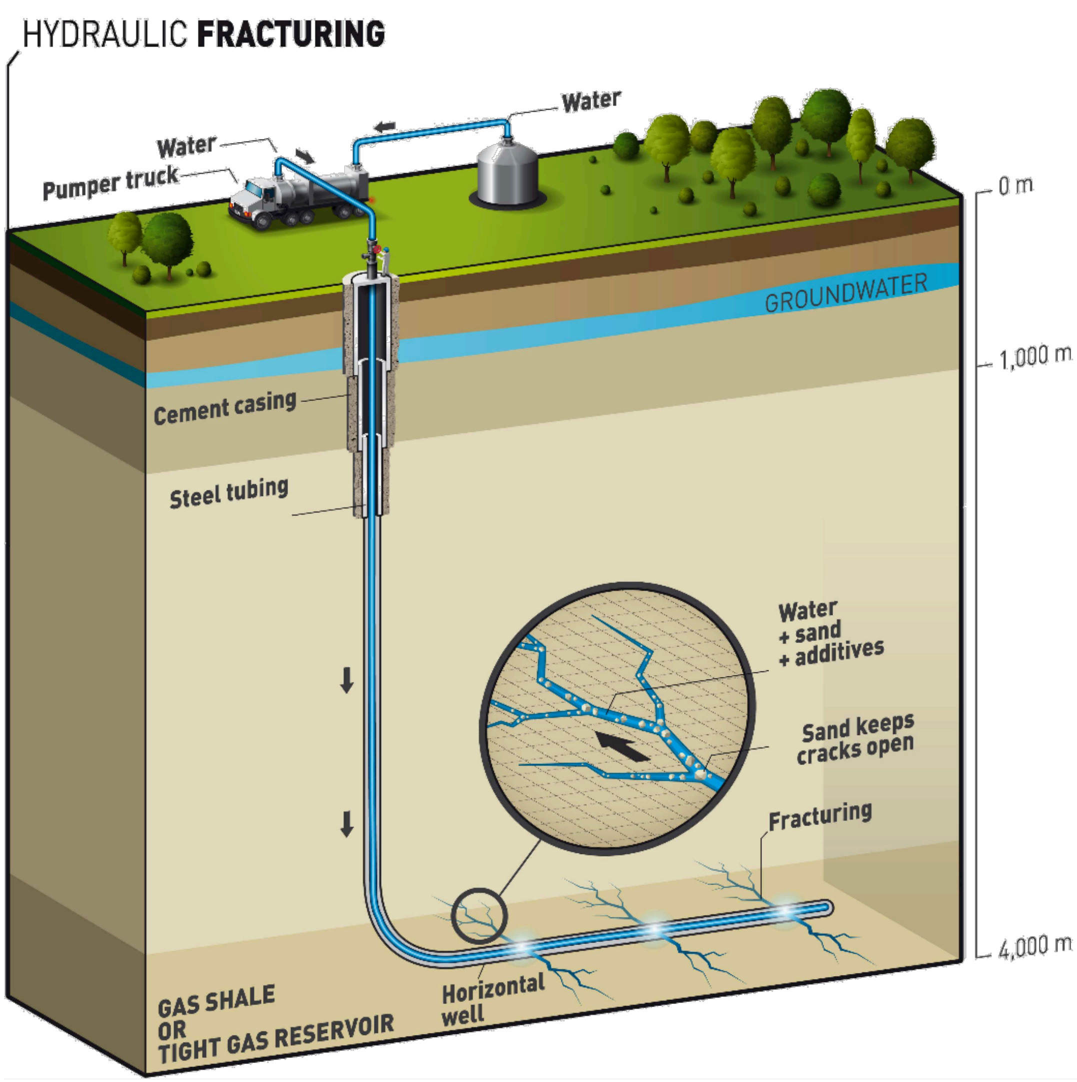
Non-Planar Fracture Growth





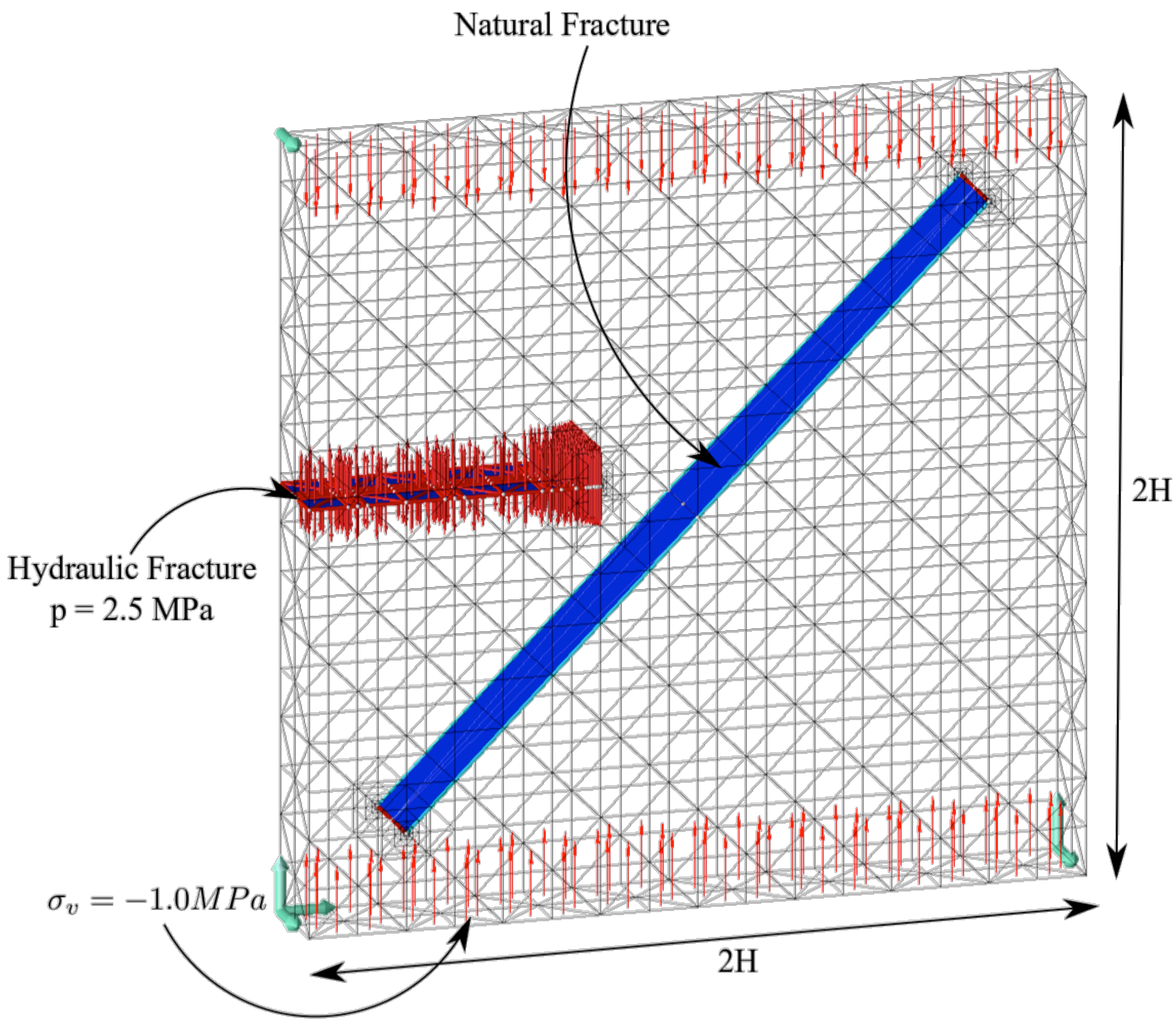
- Adaptive refinement along the crack front
- Sharp features are preserved
- High fidelity of crack surface

Hydraulic Fracturing: Interaction with Natural Fractures



Graham Roberts, New York Times, http://www.nytimes.com/interactive/2011/02/27/us/fracking.html

Interaction with Natural Fractures



Material Properties

$E = 5\ GPa$
$\nu = 0.30$

Domain size

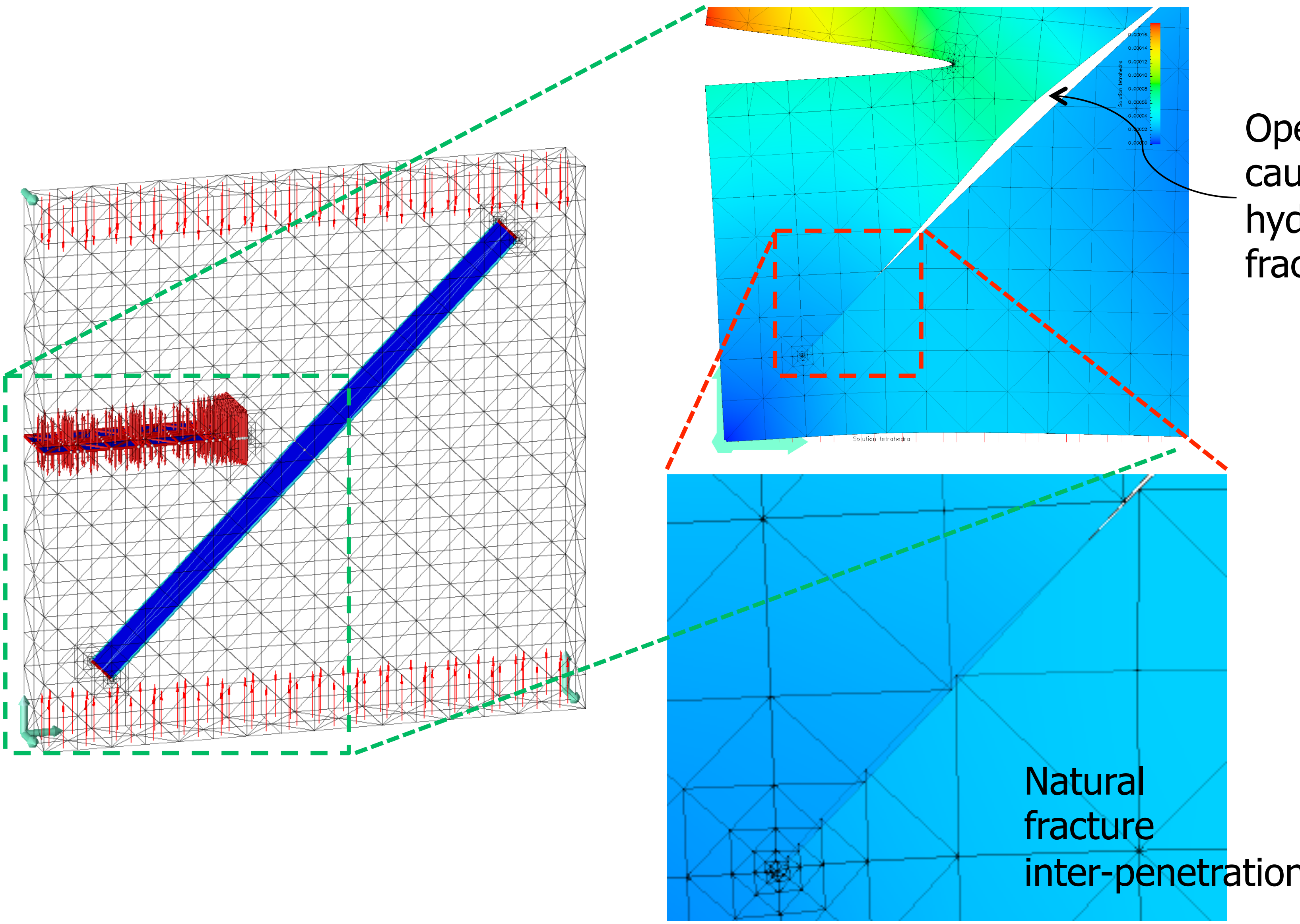
$2m \times 2m \times 0.25m$

Crack Sizes

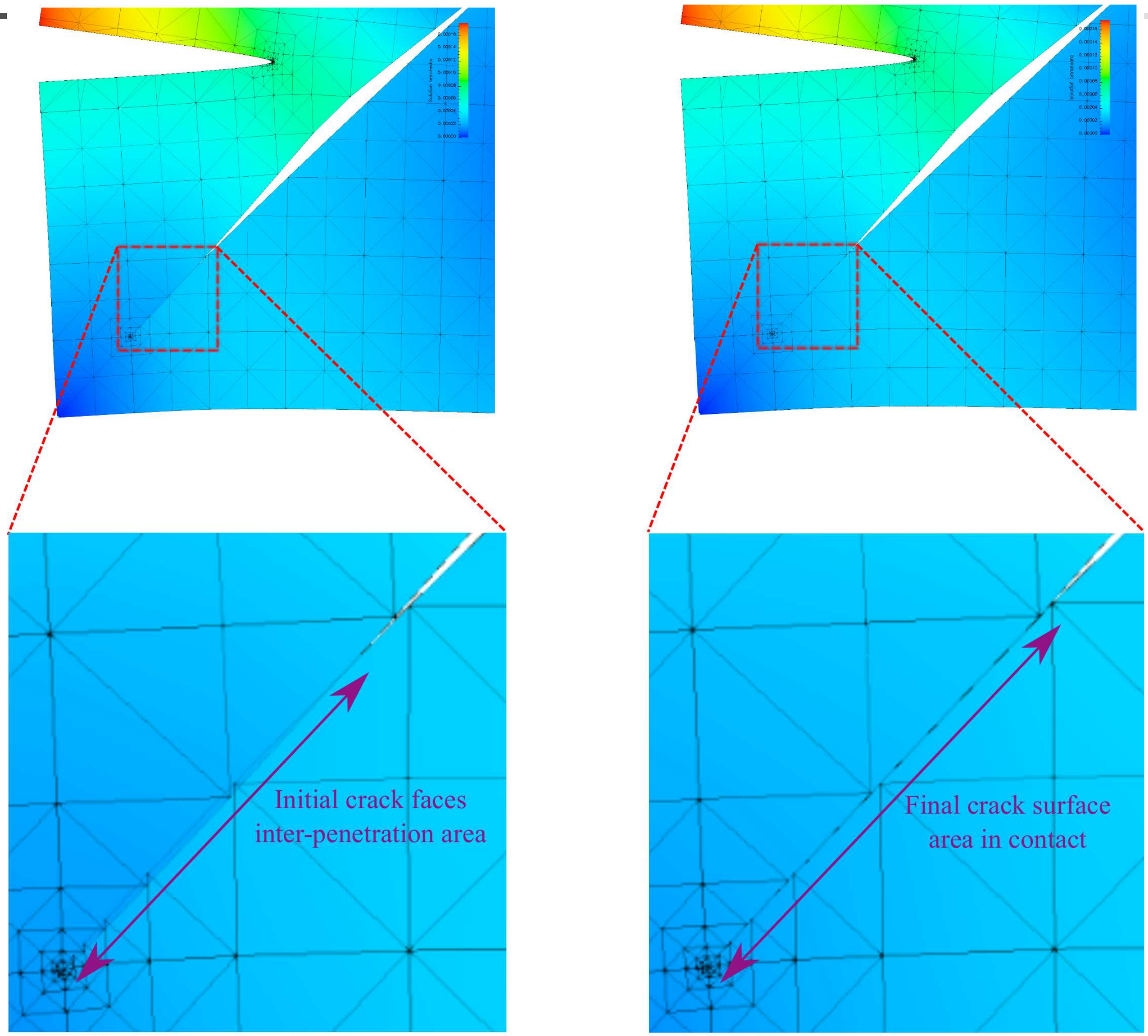
Natural Fracture $= 2.12m$

Hydraulic Fracture $= 0.75m$

Interaction with Natural Fractures



Interaction with Natural Fractures



Without contact constraints

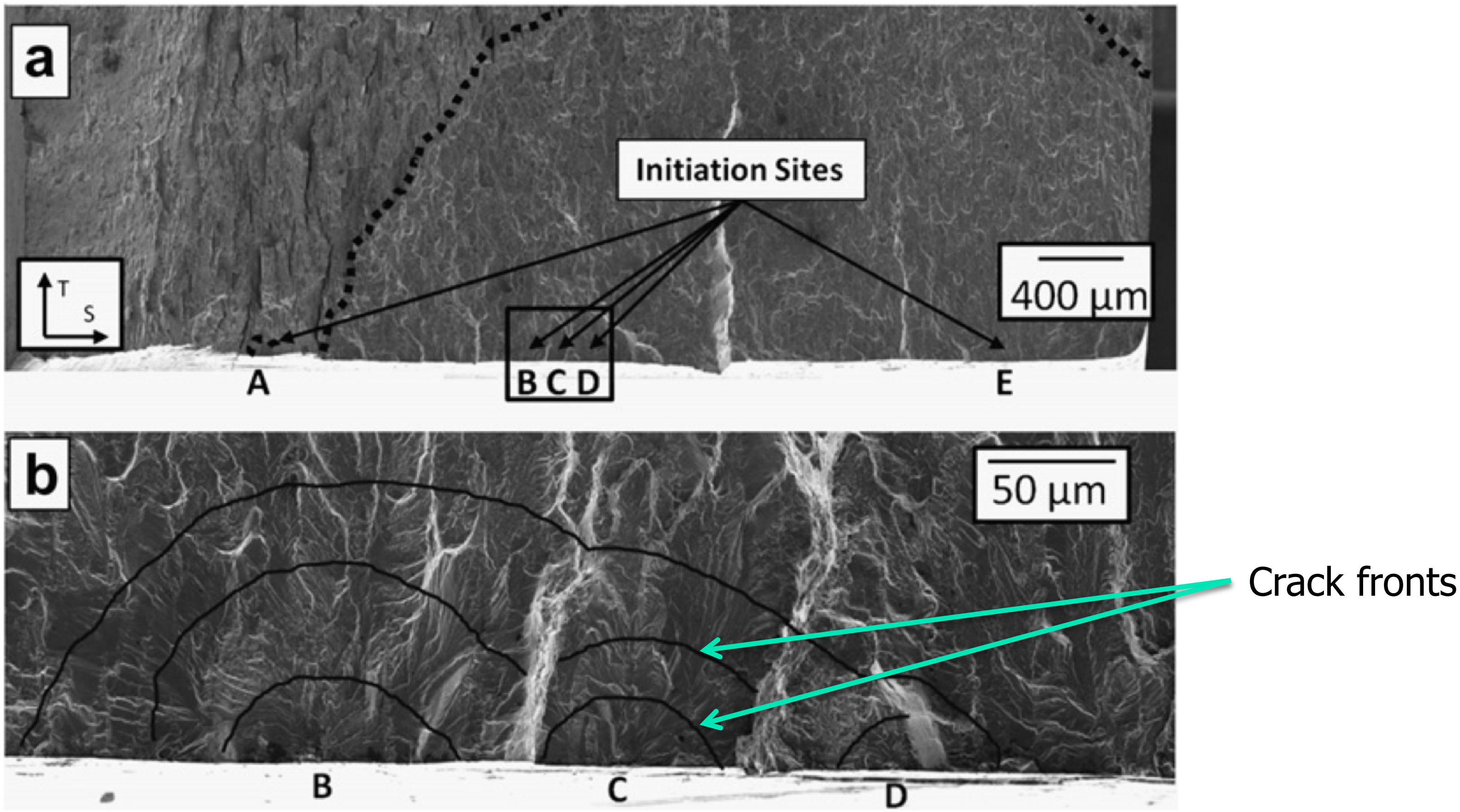
With contact constraints

Outline

- Motivation
- Basic ideas of GFEM
- Analysis of 3D Cracks with the GFEM
- Applications
  - Interaction of hydraulic and natural fractures
  - Coalescence of cracks
- Conclusions

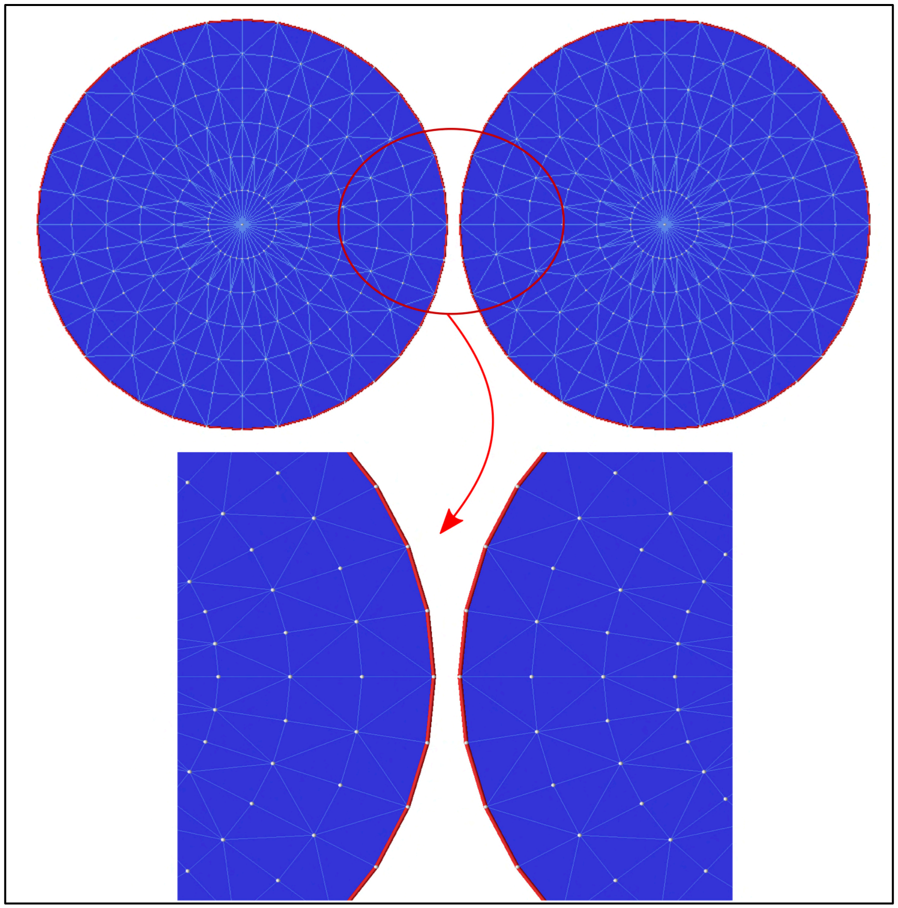
Coalescence of Cracks

- Early stages of fatigue crack growth and hydraulic fracturing involve coalescence of multiple fractures



Scanning electron fractographs showing coalescence of fatigue micro-cracks in aluminum 7075-T651 [Burns et al., IJF 2012]

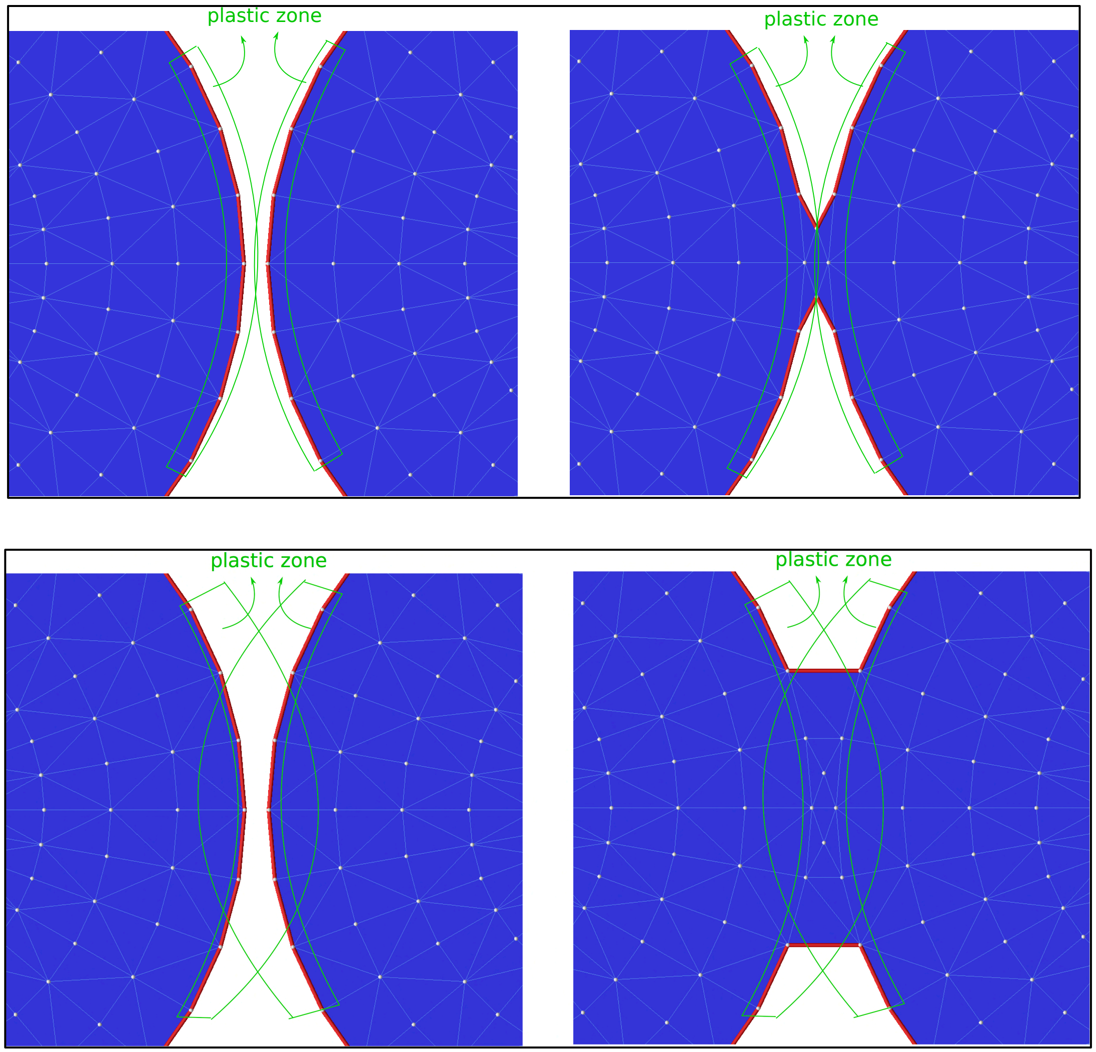
Coalescence of Crack Surfaces*



- Crack surfaces interact and coalesce
- When coalescence happens?
- What is the size of the coalesced zone or minimum distance for coalescence?

* J. Garzon et ali, 2014

Coalescence Criterion



Size of coalesced zone or minimum distance for coalescence given by, e.g., size of process zone [1]

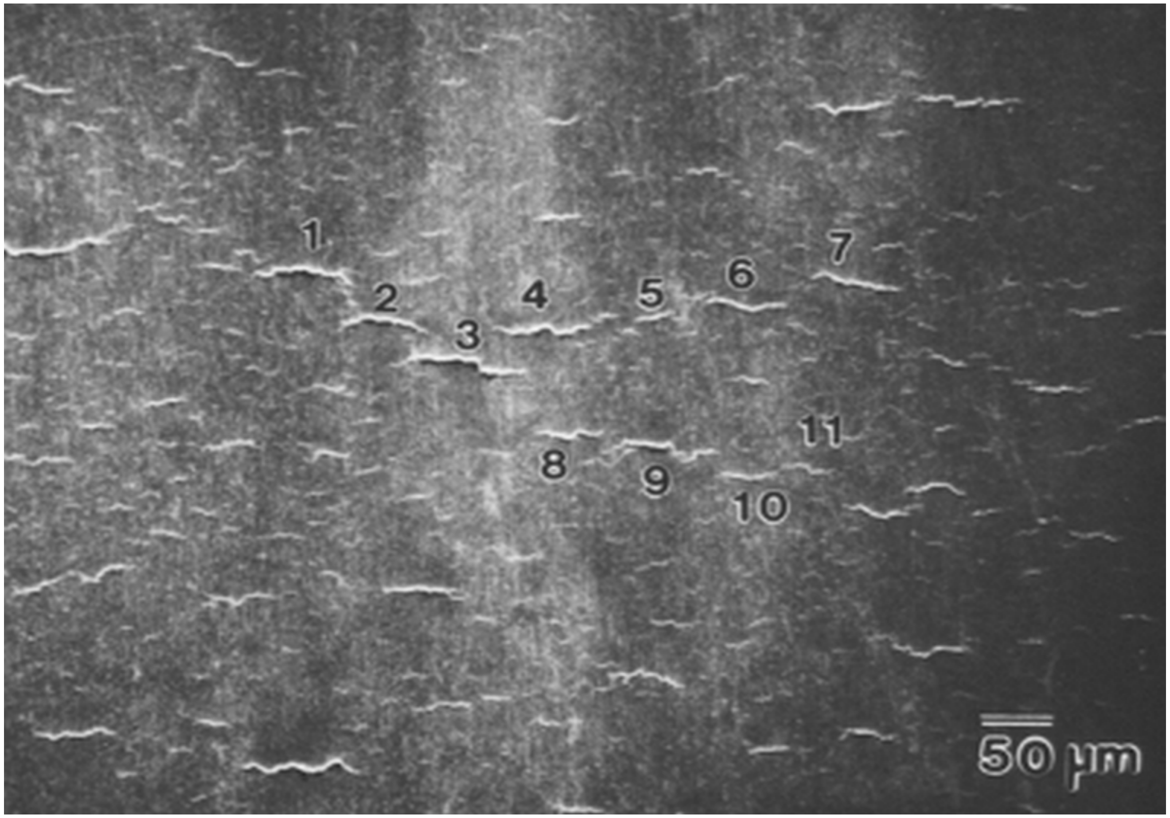
$$r_p = \frac{(K_{eq})^2}{\pi \sigma_y^2}$$

[1] Swift T. Damage tolerance capability. Fatigue of Aircraft Materials, 1992. Delft University Press.

* J. Garzon and P. O'Hara (AFRL)

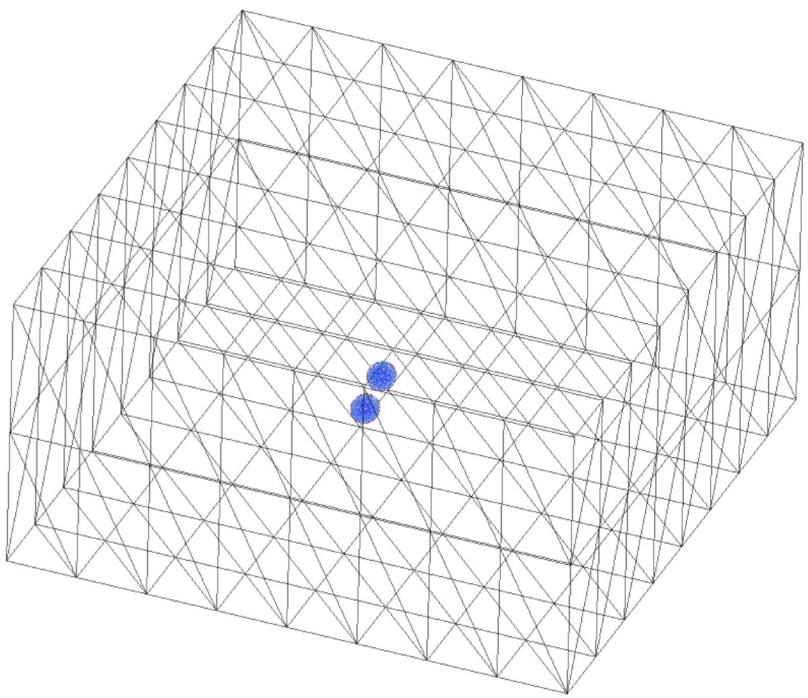
Coalescence of Micro-Cracks

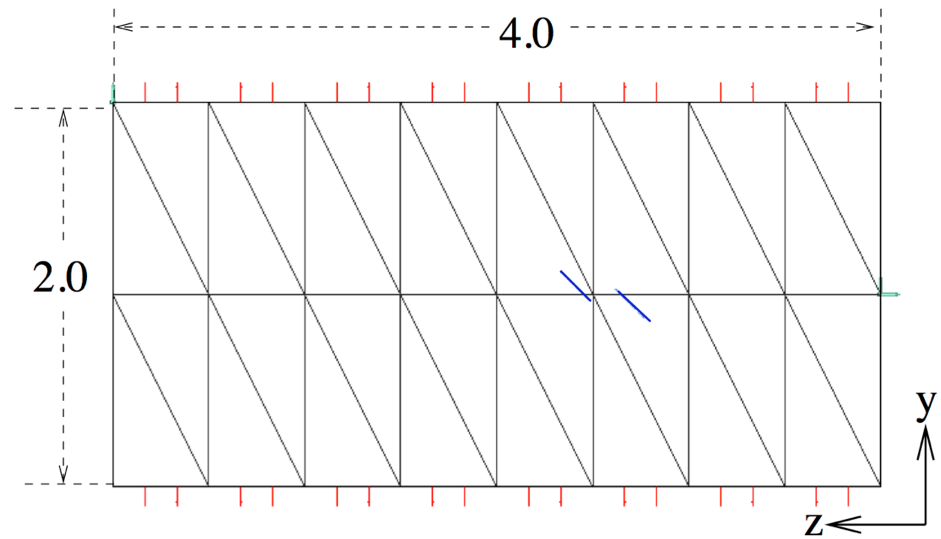
- Challenge: Coalesced cracks are in general non-planar

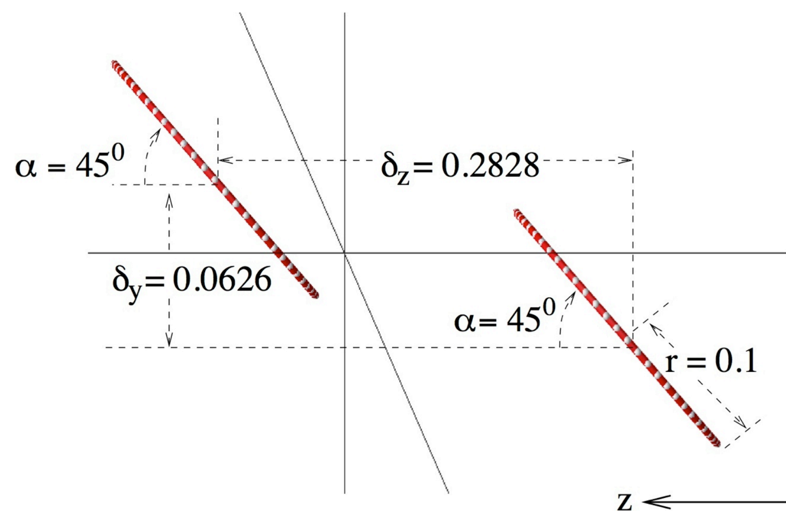


Coalescence of surface micro-cracks [L. Lawson, 2005]

Coalescence of Non-Planar Cracks*

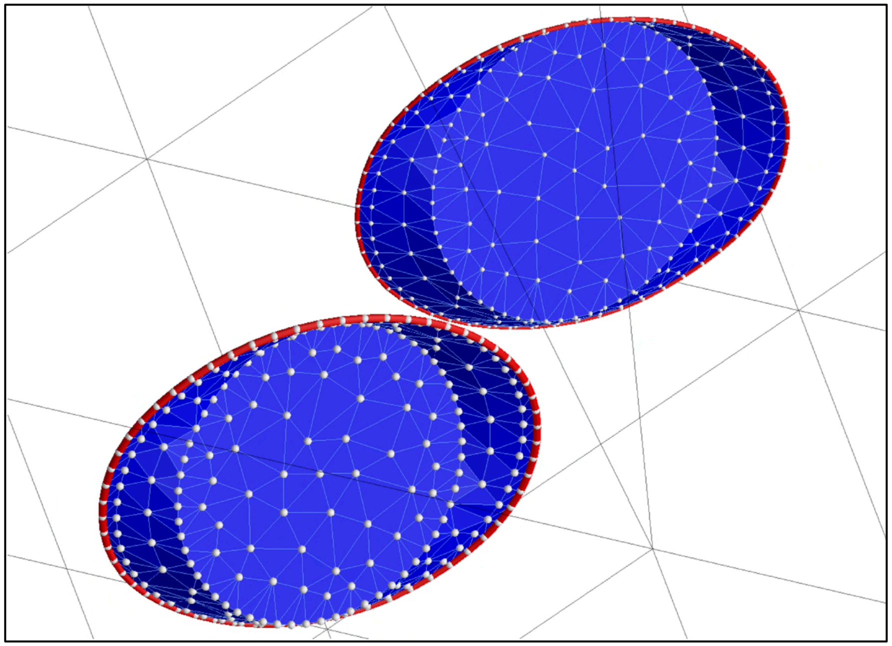




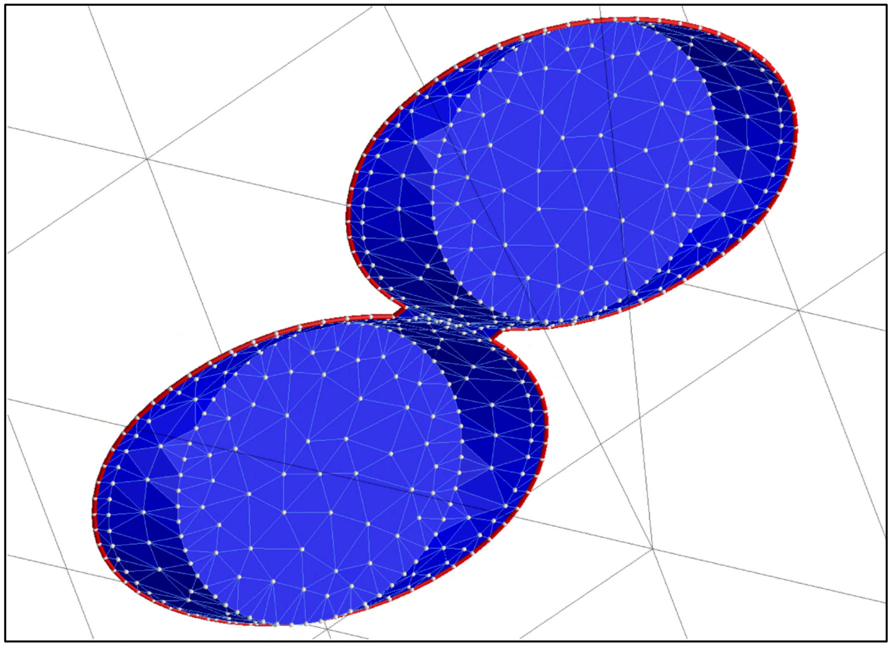


* J. Garzon et ali, 2014

Coalescence of Non-Planar Cracks

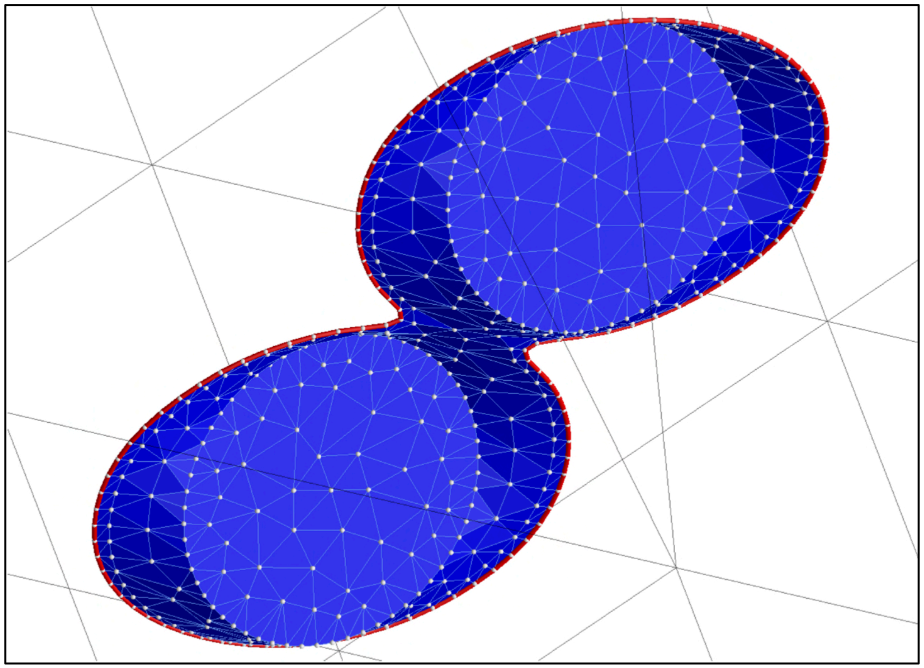


Propagation step prior to coalescence

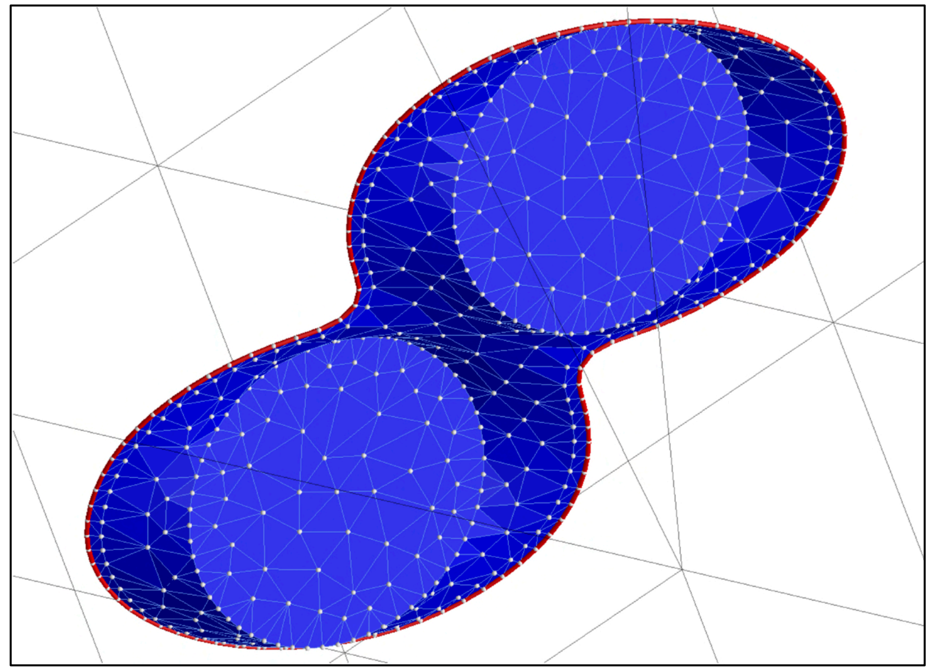


Propagation step just after coalescence

Coalescence of Non-Planar Cracks

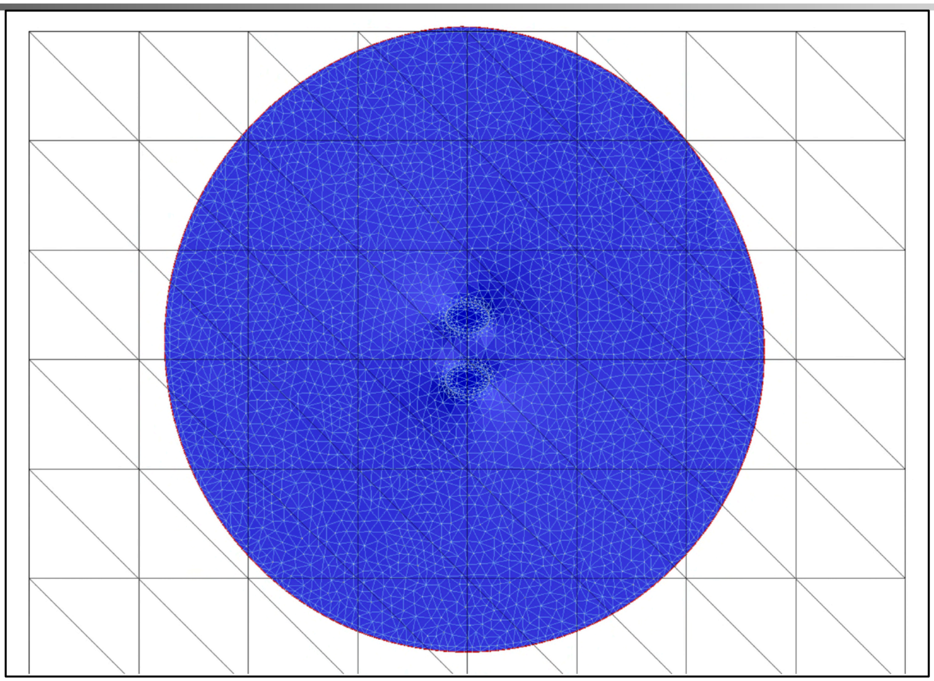


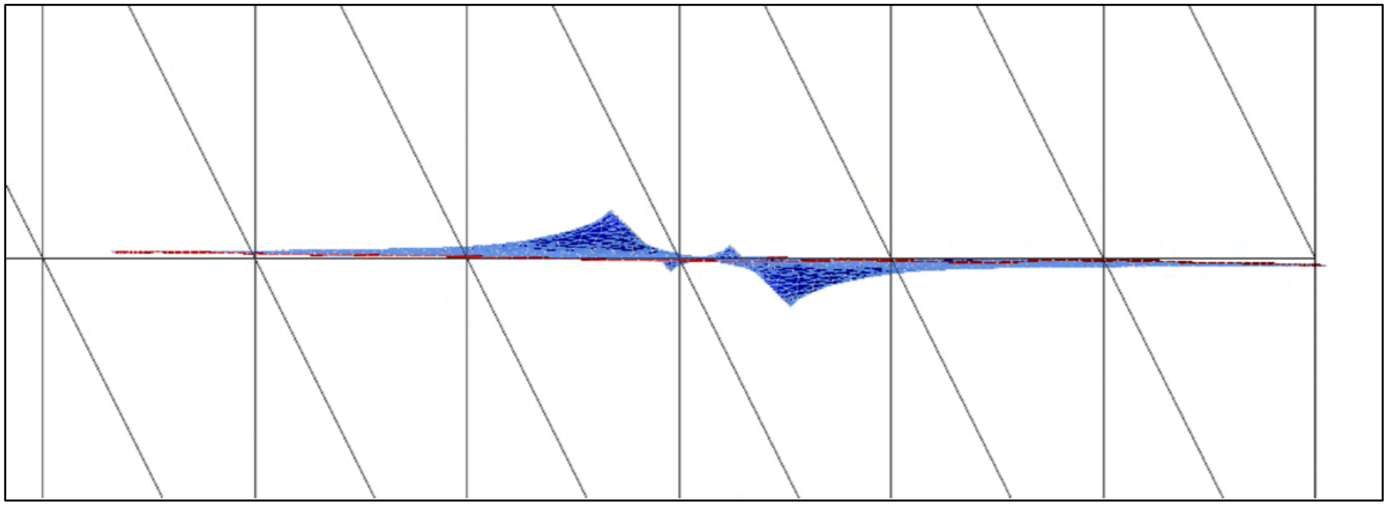
Two propagation steps after coalescence



Five propagation steps after coalescence

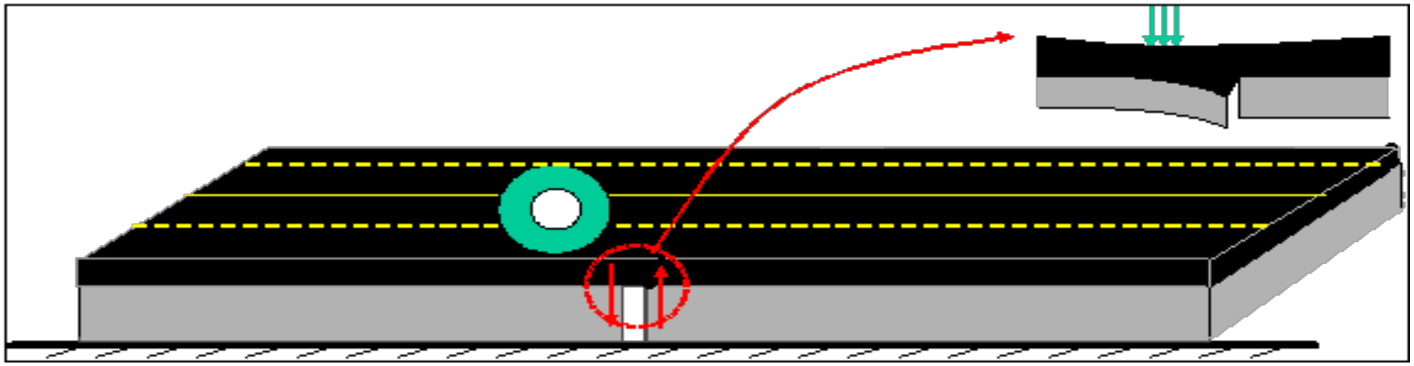
Coalescence of Non-Planar Cracks





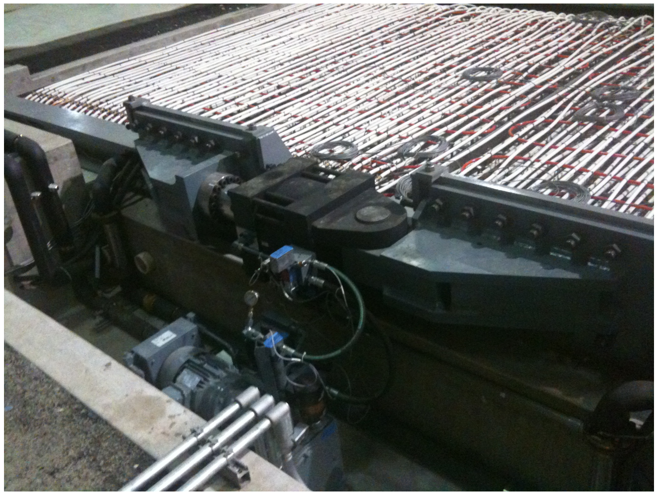
Application: Reflective Crack Growth in Pavements

- Cracks and joints in a pavement with asphalt concrete overlay "reflect" up to the surface, propagating through overlay

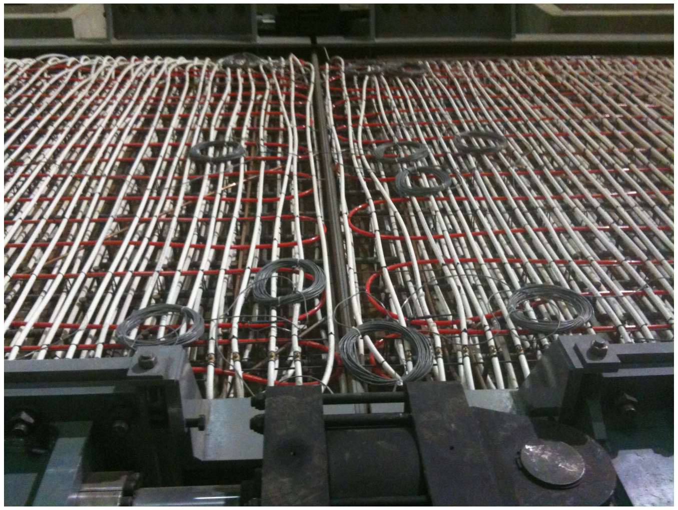


Application: Reflective Crack Growth in Pavements

- Reflective crack testing at FAA – NAPTF – Simulation and life prediction



Frame and Actuator (350 Tons)



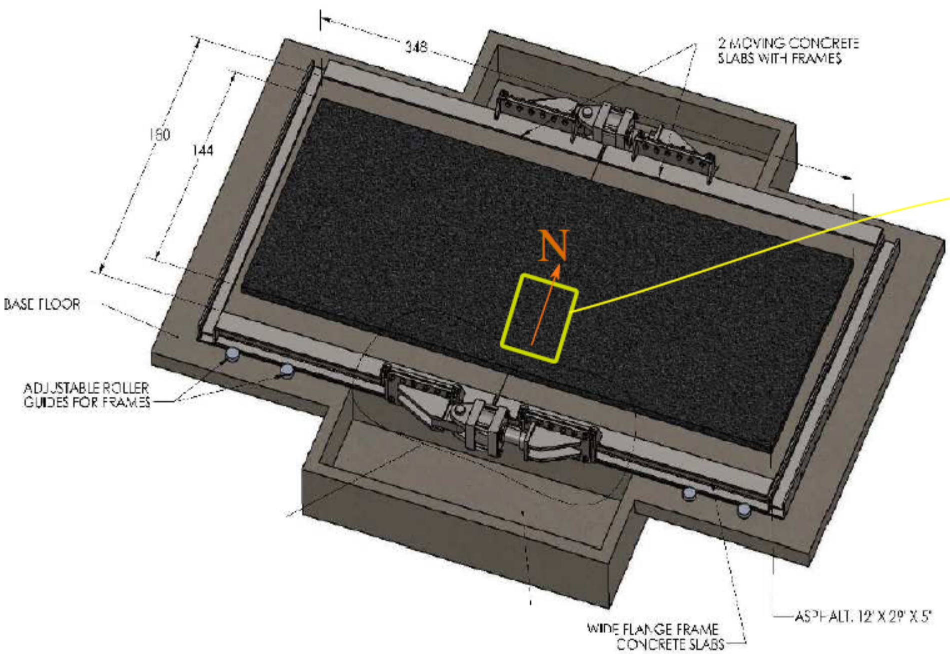
Joint – Hydronic Cooling

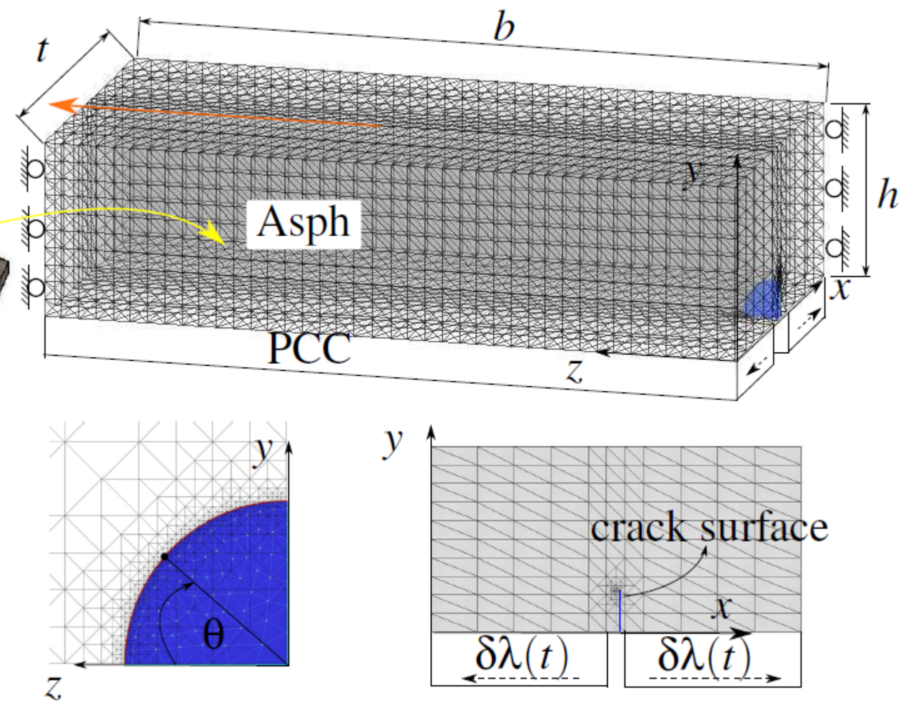
Application: Reflective Crack Growth in Pavements

- Computational challenges
  - Strong 3-D effects: Crack channeling
  - RC surface change in size by orders of magnitude
  - Fatigue cracking with thousands of cycles
  - Coalescence of 3-D cracks significantly affects life of pavement

Reflective Cracking Simulation

- Reflective crack testing at FAA – NAPTF – Simulation and life estimate





m	G_m (ksi)	K_m (ksi)	ρ_m (s)
1	72.463	157.00	6.298e-7
2	399.45	865.47	7.679
3	447.50	969.56	63.669
4	533.56	1156.0	1.9782e3
5	183.20	396.94	2.908e4

$G_0 = 1635.74$ and $K_0 = 3544.11$

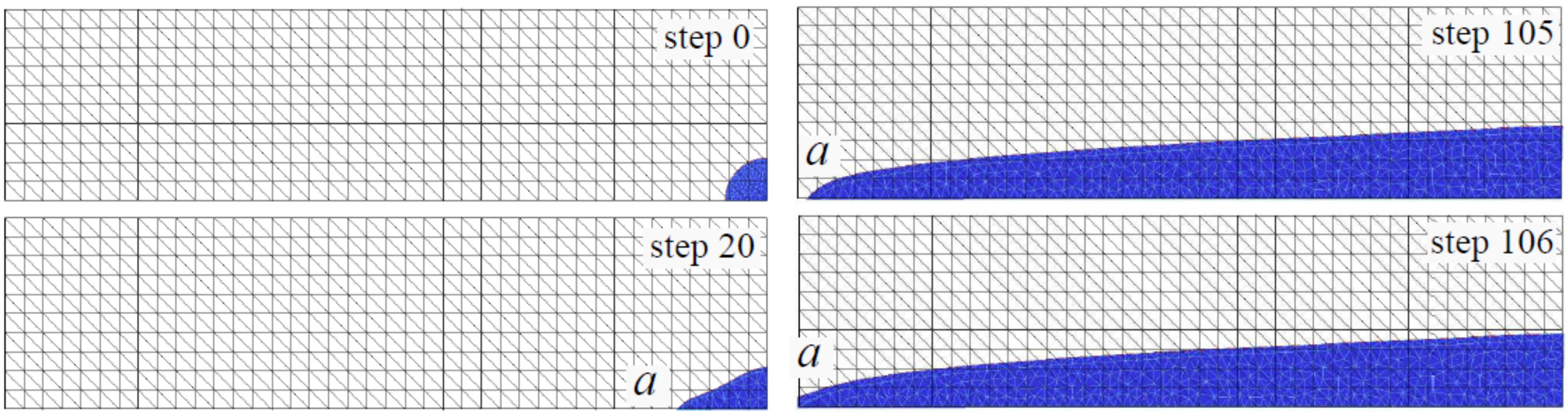
→ Material Prop. from Laboratory Evaluation of FAA Cracking Rig Materials report.

Cycles @ 0.10 mil/sec

Video

Reflective Cracking Simulation

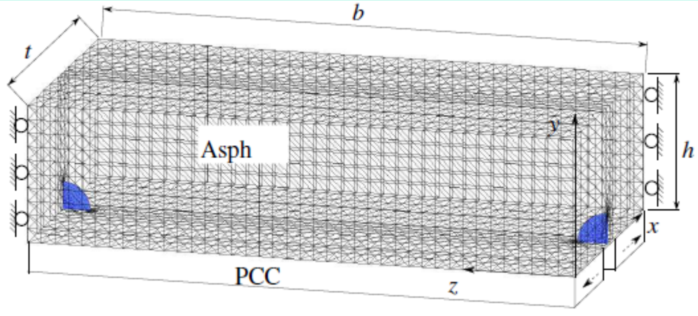
- Reflective crack testing at FAA – NAPTF – Simulation and life estimate



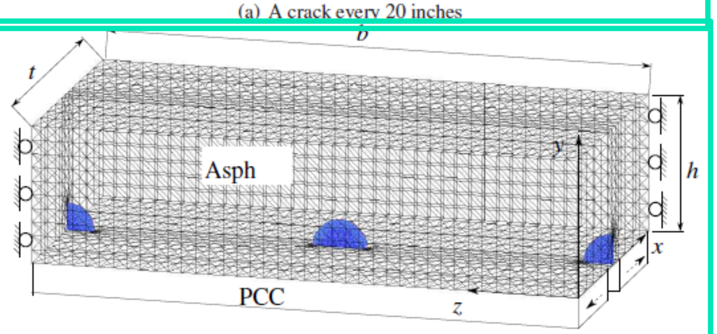
- Strong channeling effect: Requires solving a 3-D model
- RC surface grows by orders of magnitude
- Crack front speeds varies significantly along the front
- Interactions with domain boundary:
  - Difficult to automatically create structured mesh around crack front as required by FEM
- Considered only ONE crack
  - In reality, there may be several cracks growing and coalescing…..
  - How the number of "seed cracks" affects the life of the pavement?

Coalescence of 3D Reflective Cracks

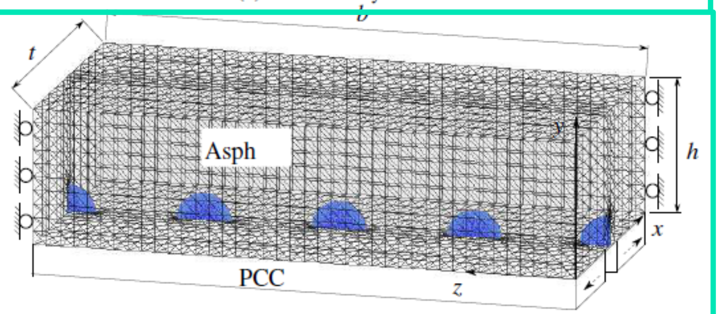
- Reflective crack testing at FAA – NAPTF – Simulation and life estimates



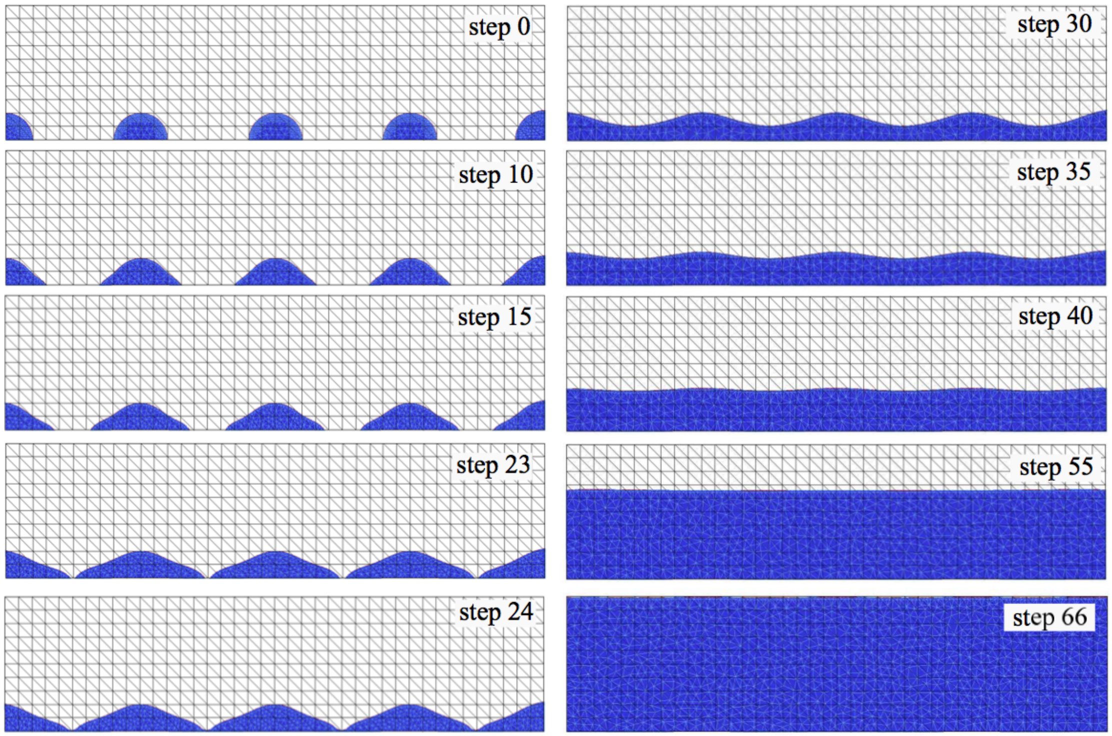
(a) A crack every 20 inches



(b) A crack every 10 inches



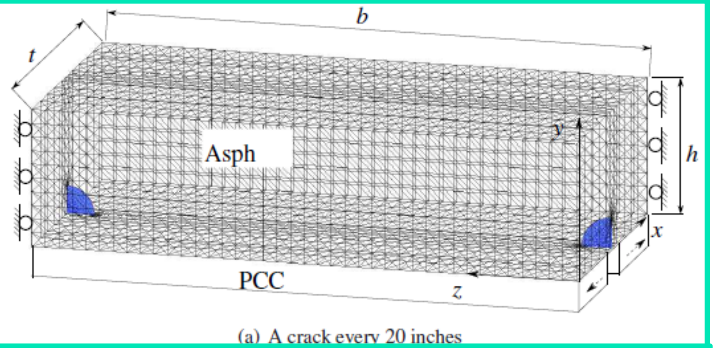
(c) A crack every 5 inches



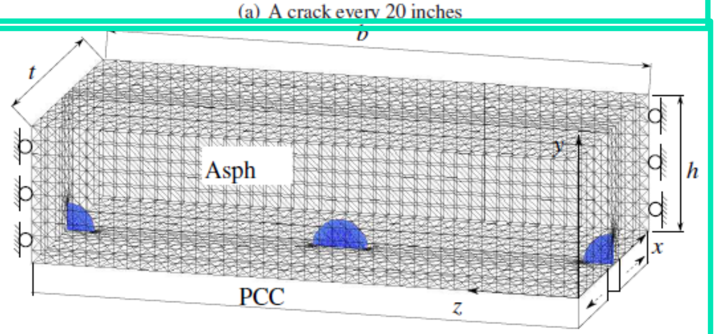
Video

Coalescence of 3D Reflective Cracks

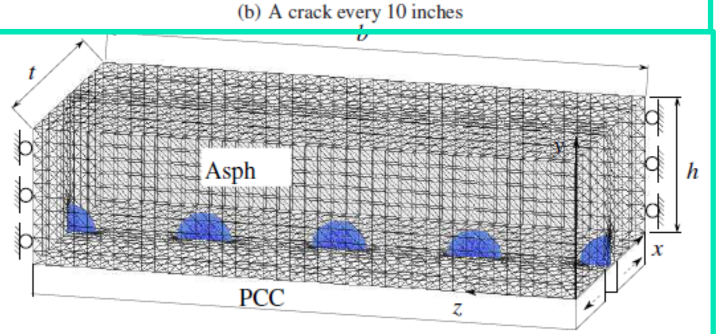
- Reflective crack testing at FAA – NAPTF – Simulation and life estimates



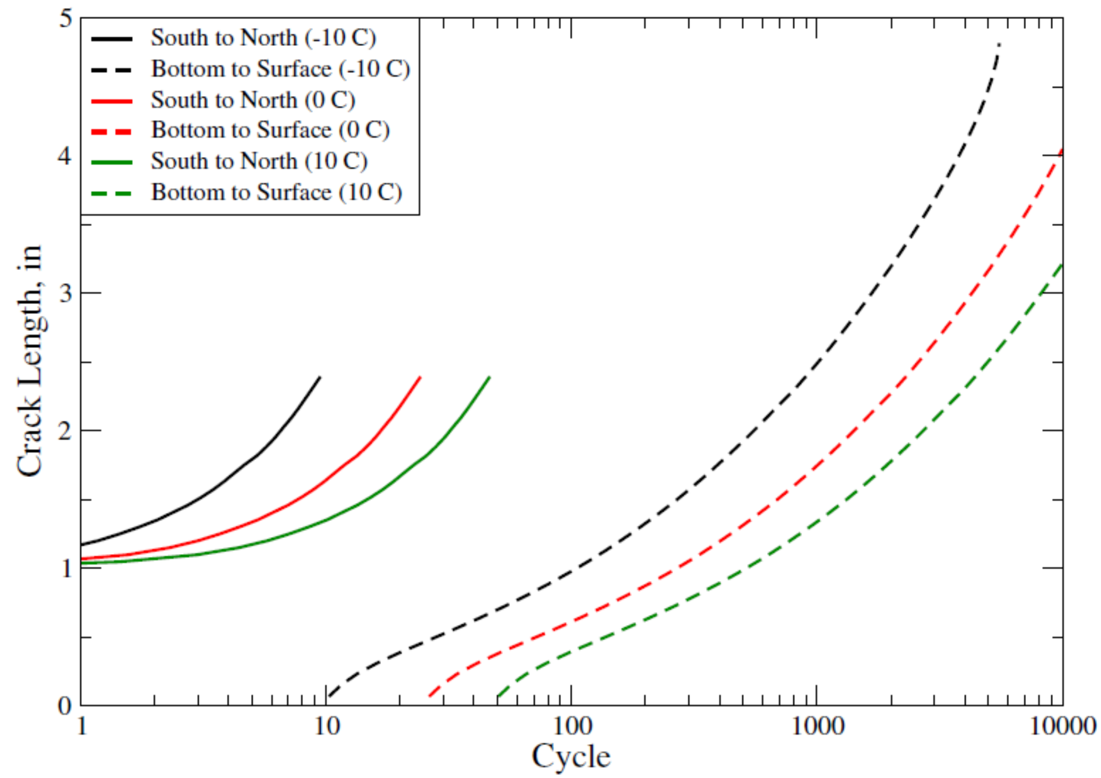
(a) A crack every 20 inches



(b) A crack every 10 inches

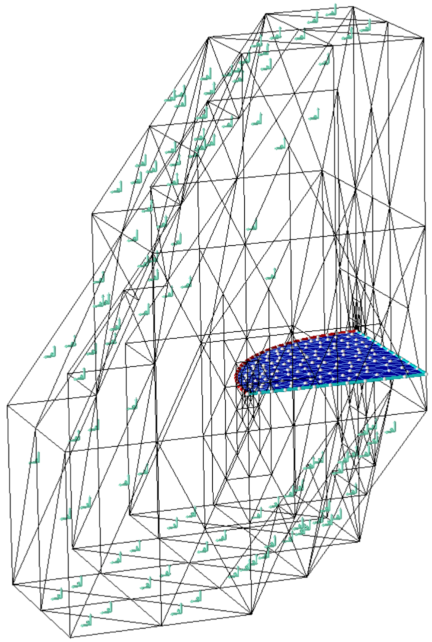


(c) A crack every 5 inches



Conclusions

- Generalized/Extended FEM removes several limitations of FEM
- It enables the solution of problems that are difficult or not practical with the FEM
- This is the case of three-dimensional fracture problems involving
  - ✓ Complex crack surfaces
  - ✓ Fluid-induced fracturing
  - ✓ Coalescence of 3-D fractures, etc.
- Open issues under investigation include
  - ✓ Numerical stability (Stable GFEM)
  - ✓ Non-intrusive integration with existing FEA software



Questions?

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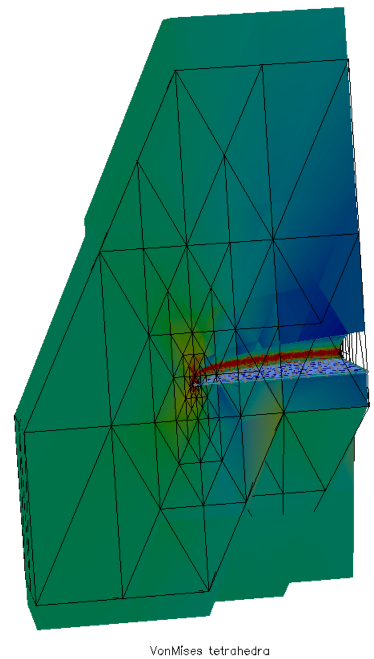
http://gfem.cee.illinois.edu/



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