# Simulation of Non-Planar Three-Dimensional Hydraulic Fracture Propagation

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# Hydraulic Fracturing of Gas Shale Reservoirs

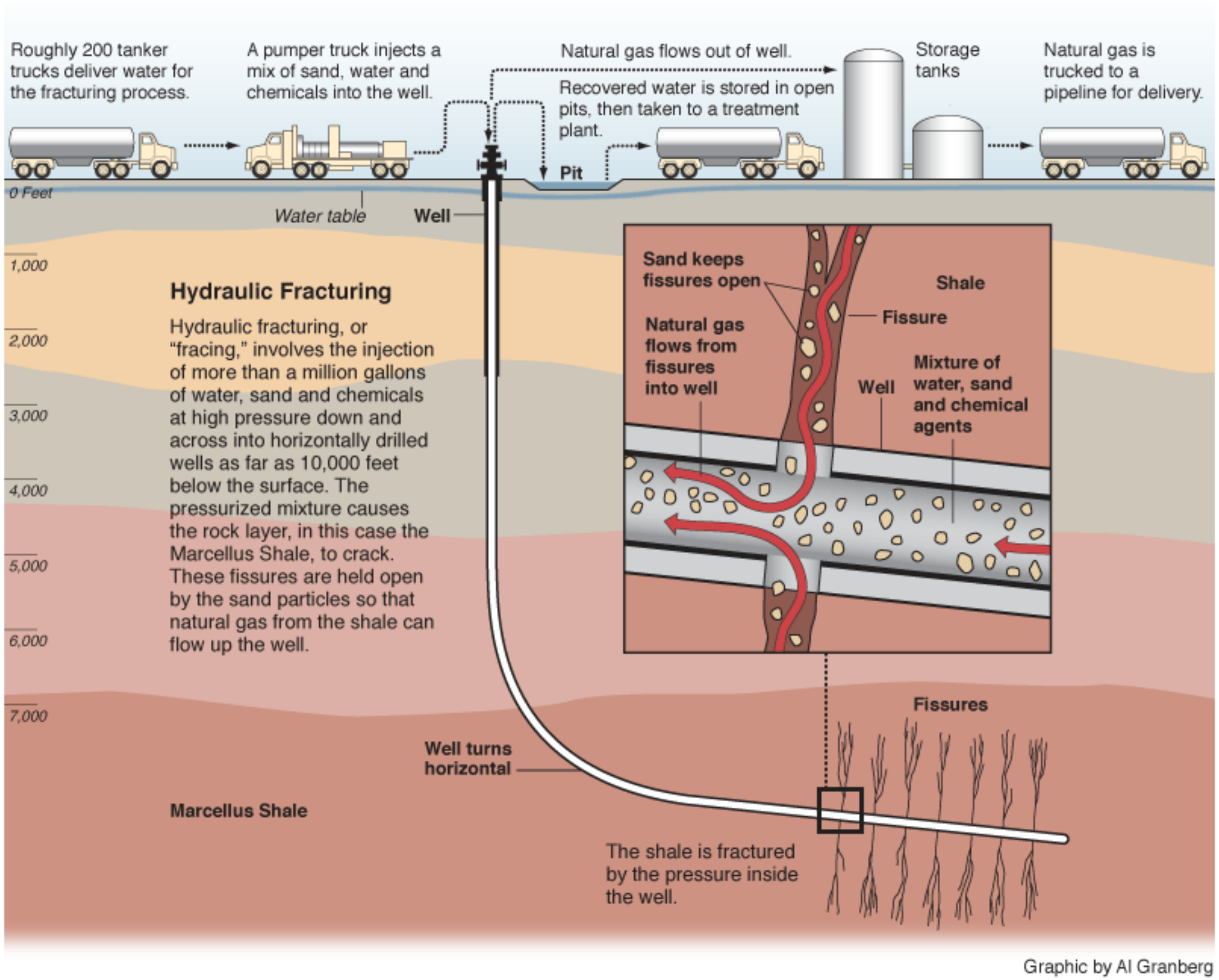
## Motivation

- Natural gas production in the US has increased significantly in the past few years thanks to advances in hydraulic fracturing of gas shale reservoirs
- Yet there are concerns about the environmental impact of toxic fluids used in this process


## Objectives

- Computational simulations will lead to better designs of hydraulic fracture treatments, thus reducing the amount of toxic fluids used
- Realistic modeling of hydraulic fracturing treatments can evaluate the potential impact of interactions between hydraulic fractures and naturally existing fractures in shale reservoirs

# What is Hydraulic Fracturing Anyway?

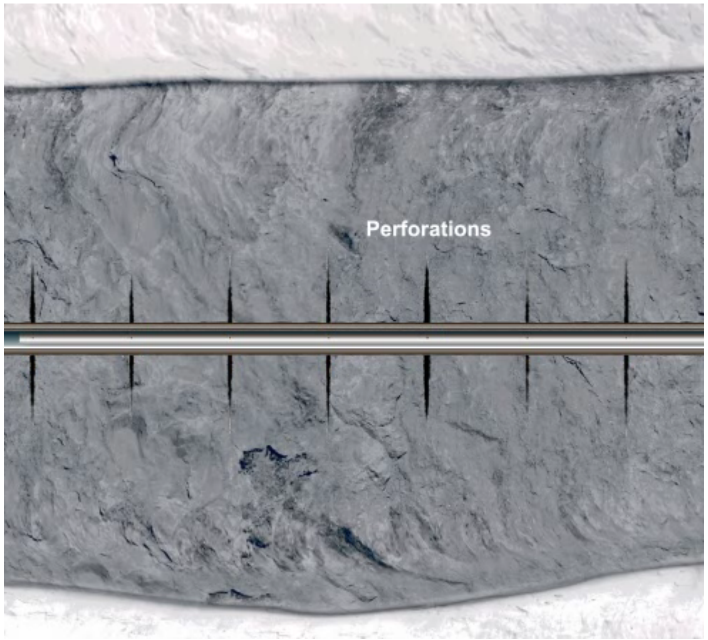


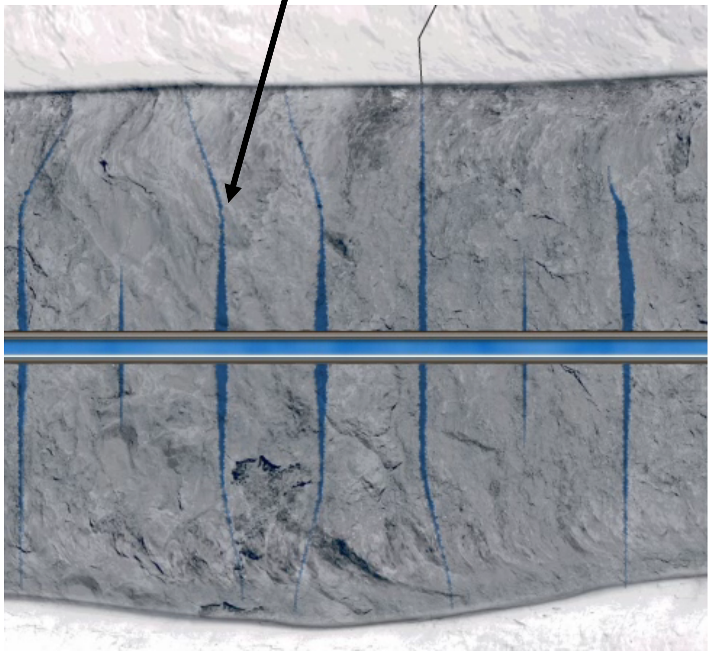

**Movie**

# Hydraulic Fracturing Simulation

## Current Focus: 3-D effects not captured by available simulators

- Initial stages of fracture propagation: Fracture re-orientation



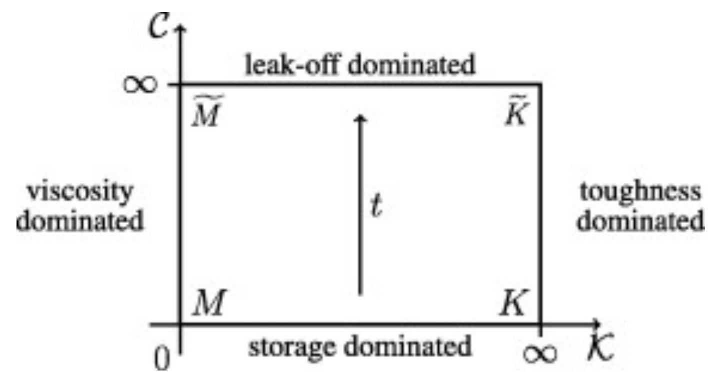



# Hydraulic Fracturing Regimes

- Fracture propagation is governed by
  - two competing energy dissipation mechanisms: Viscous flow and fracturing process;
  - two competing storage mechanisms: In the fracture and in the porous matrix

Dimensionless toughness
$$\mathcal{K} = \frac{4K_{Ic}}{\sqrt{\pi}}\left(\frac{1}{3Q_0E'^3\mu}\right)^{1/4}$$

Leak-off coefficient
$$\mathcal{C} = 2C_L\left(\frac{E't}{12\mu Q_0^3}\right)^{1/6}$$




Hydraulic fracture parametric space*

## Current Focus: Storage-toughness dominated regime

- Low permeability reservoirs: Neglect flow of hydraulic fluid across crack faces
- High confining stress:
  - Fluid lag in frac. << frac. size ↔ const press. in frac. ⟷ toughness dom.
- Brittle elastic material
- Early-time solution → storage dominated

*[Carrier & Granet, EFM, 2013]

# Hydraulic Fracturing Regimes

$$\mathcal{K} = \frac{4K_{Ic}}{\sqrt{\pi}} \left( \frac{1}{3Q_0 E'^3 \mu} \right)^{1/4} \qquad \mathcal{C} = 2C_L \left( \frac{E' t}{12 \mu Q_0^3} \right)^{1/6}$$

$\mathcal{K} =$ Dimensionless toughness

$\mathcal{C} =$ Dimensionless leak-off coefficient

$K_{Ic} =$ Mode-I fracture toughness

$Q_0 =$ Injection rate

$E' =$ Plane strain Young's modulus

$\mu =$ Fluid viscosity

$C_L =$ Leak-off coefficient

$t =$ time

# *Weak Form at Propagation Step k*

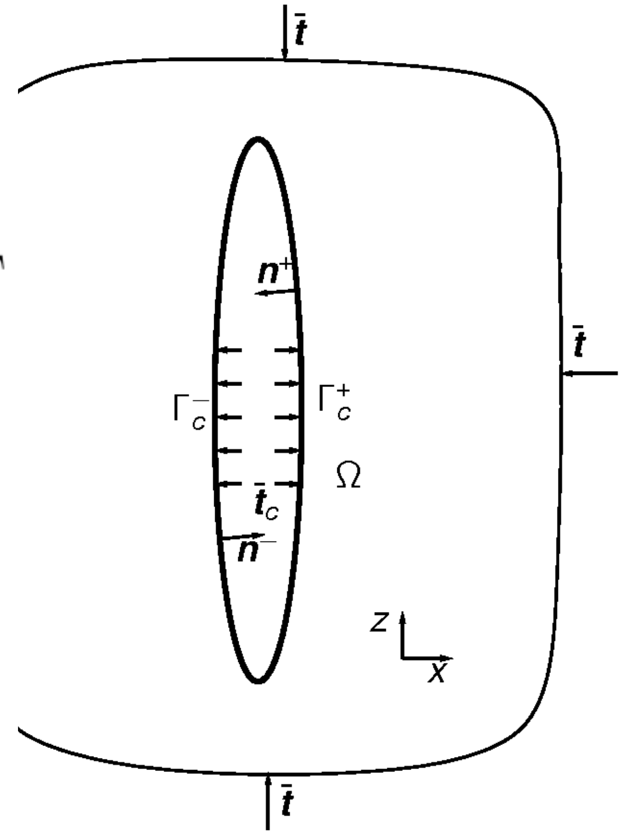
Find $\boldsymbol{u}^k \in H^1(\Omega)$, such that $\forall\ \boldsymbol{v}^k \in H^1(\Omega)$

$$\int_{\Omega} \boldsymbol{\sigma}(\boldsymbol{u}^k) : \boldsymbol{\varepsilon}(\boldsymbol{v}^k) d\Omega$$

$$= \int_{\Omega} \boldsymbol{b} \cdot \boldsymbol{v}^k d\Omega + \int_{\partial\Omega} \bar{\boldsymbol{t}} \cdot \boldsymbol{v}^k d\Gamma + \int_{\Gamma_c^{k+}} \bar{\boldsymbol{t}}_c^{k+} \cdot [\![\boldsymbol{v}^k]\!] d\Gamma$$

where $[\![\boldsymbol{v}^k]\!]$ is the virtual displacement jump across the crack surface $\Gamma^k$ at propagation step $k$ and

$$\bar{\boldsymbol{t}}_c^{k+} = -p^k \boldsymbol{n}^{k+} = p^k \boldsymbol{n}^{k-}$$




Cross section of fracture

# Outline

- Motivation and problem set up
- Generalized FEM for 3-D hydraulic fractures
- Examples
- Closing remarks

# Early works on Generalized FEMs

- Babuska, Caloz and Osborn, 1994 (Special FEM).
- Duarte and Oden, 1995 (Hp Clouds).
- Babuska and Melenk, 1995 (PUFEM).
- Oden, Duarte and Zienkiewicz, 1996 (Hp Clouds/GFEM).
- Duarte, Babuska and Oden, 1998 (GFEM).
- Belytschko et al., 1999 (Extended FEM).
- Strouboulis, Babuska and Copps, 2000 (GFEM).

- Basic idea:
  - Use a partition of unity to build Finite Element shape functions
- Review paper

  Belytschko T., Gracie R. and Ventura G. A review of extended/generalized finite element methods for material modeling, *Mod. Simul. Matl. Sci. Eng.,* 2009

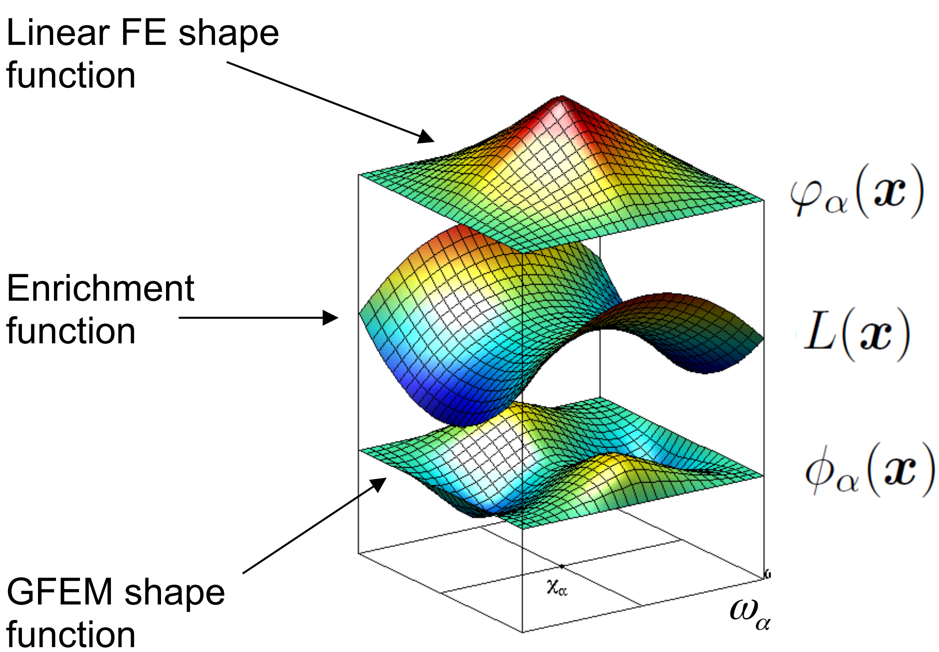
  “The XFEM and GFEM are basically <u>identical</u> methods: the name generalized finite element method was adopted by the Texas school in 1995–1996 and the name extended finite element method was coined by the Northwestern school in 1999.”

# Generalized Finite Element Method

- GFEM can be interpreted as a FEM with shape functions built using the concept of a partition of unity:

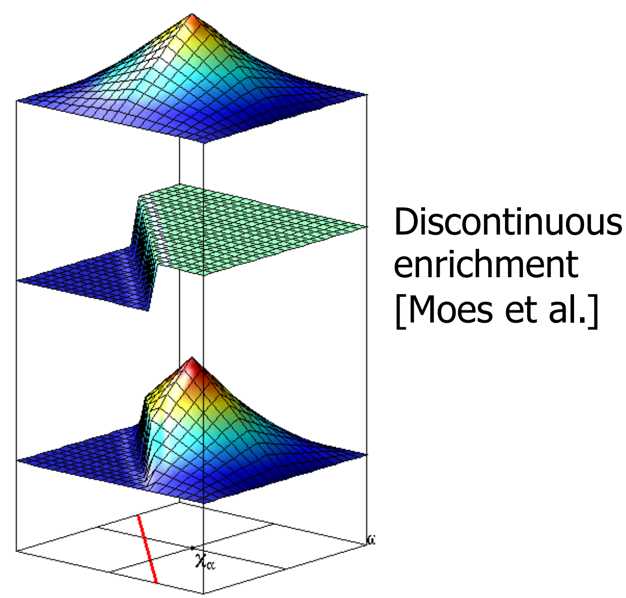
GFEM shape function = FE shape function * enrichment function

$$\boxed{\phi_\alpha(\boldsymbol{x}) = \varphi_\alpha(\boldsymbol{x}) L(\boldsymbol{x})}$$



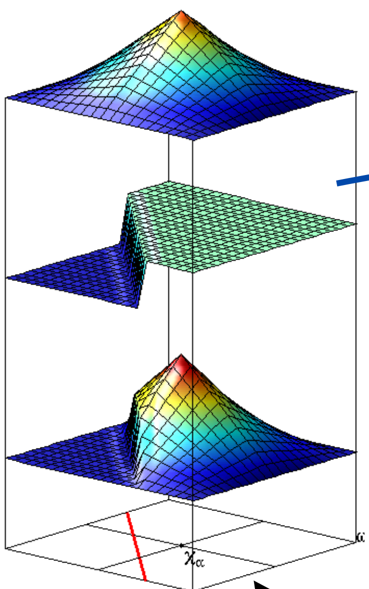

[Oden, Duarte & Zienkiewicz, 1996]

- Allows construction of shape functions incorporating a-priori knowledge about solution



# GFEM Approximation for 3-D Cracks

$$\boldsymbol{X}^{hp}(\Omega) = \left\{ \boldsymbol{u} = \sum_{\alpha=1}^{N} \underbrace{\varphi_\alpha(\boldsymbol{x})}_{\text{PoU}} \left[ \underbrace{\hat{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{polynomial}} + \underbrace{\mathcal{H}\tilde{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{discontinuous}} + \underbrace{\breve{\boldsymbol{u}}_\alpha(\boldsymbol{x})}_{\text{singular}} \right] \right\}$$



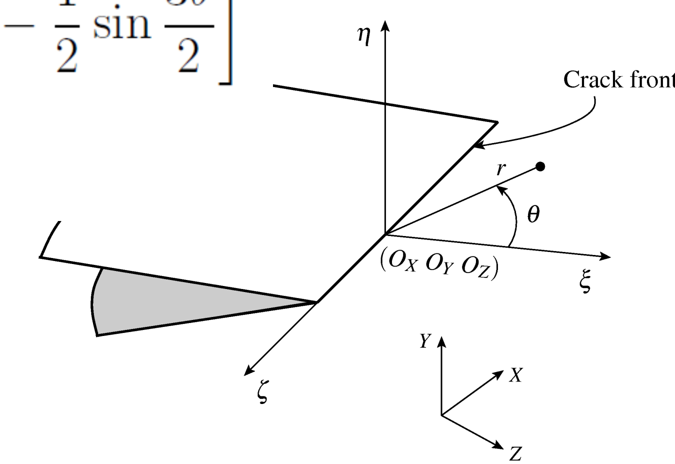
$$\breve{L}^{\xi}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa - \frac{1}{2}\right)\cos\frac{\theta}{2} - \frac{1}{2}\cos\frac{3\theta}{2}\right]$$

$$\breve{L}^{\eta}_{\alpha 1}(r,\theta) = \sqrt{r}\left[\left(\kappa + \frac{1}{2}\right)\sin\frac{\theta}{2} - \frac{1}{2}\sin\frac{3\theta}{2}\right]$$

$$\breve{L}^{\zeta}_{\alpha 1}(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}$$

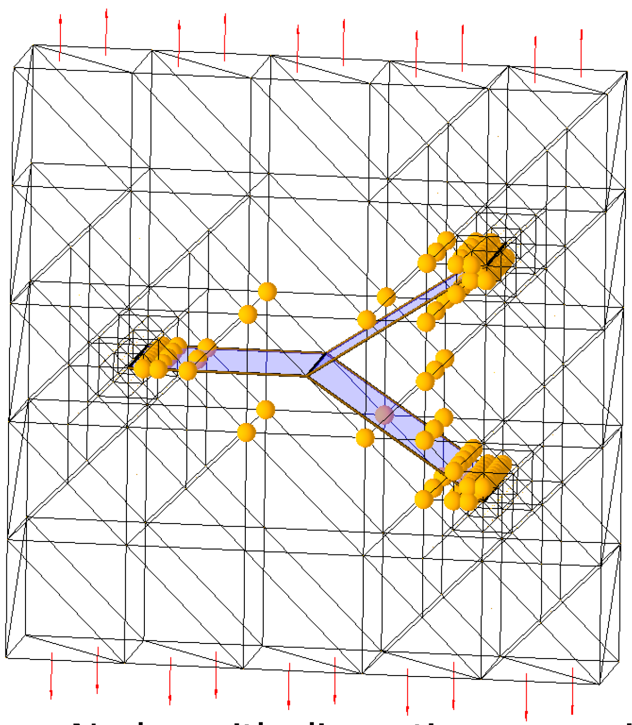
[Duarte and Oden 1996]

cloud or patch $\omega_\alpha$

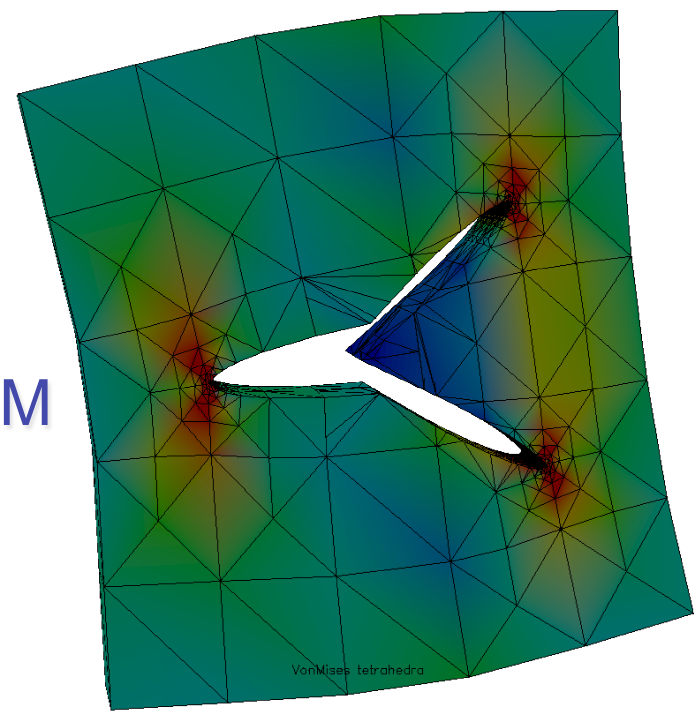


# Modeling Cracks with hp-GFEM

- Discontinuities modeled via enrichment functions, *not* the FEM mesh
- Mesh refinement *still required* for acceptable accuracy



*hp*-GFEM



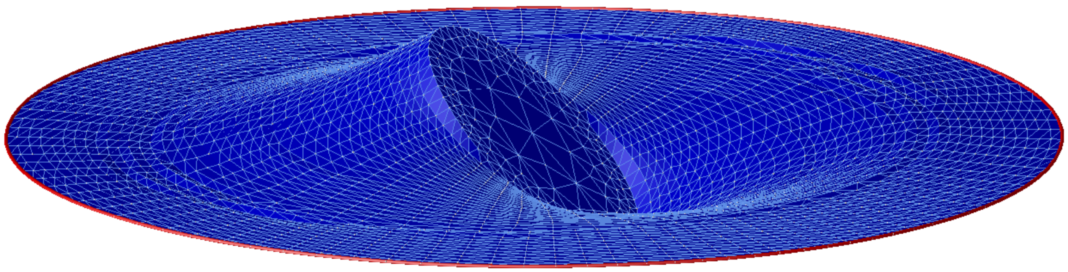

● = Nodes with discontinuous enrichments

Von Mises stress

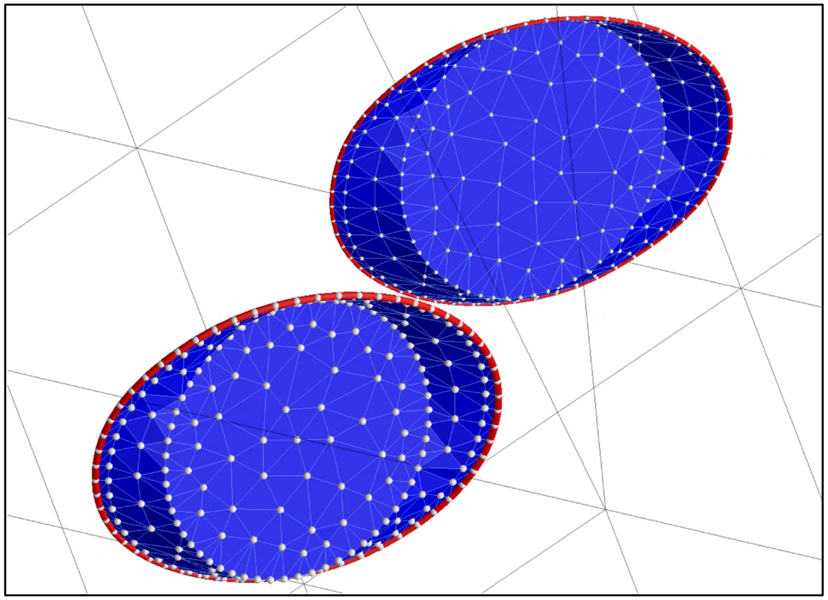
[Duarte et al., International Journal Numerical Methods in Engineering, 2007]

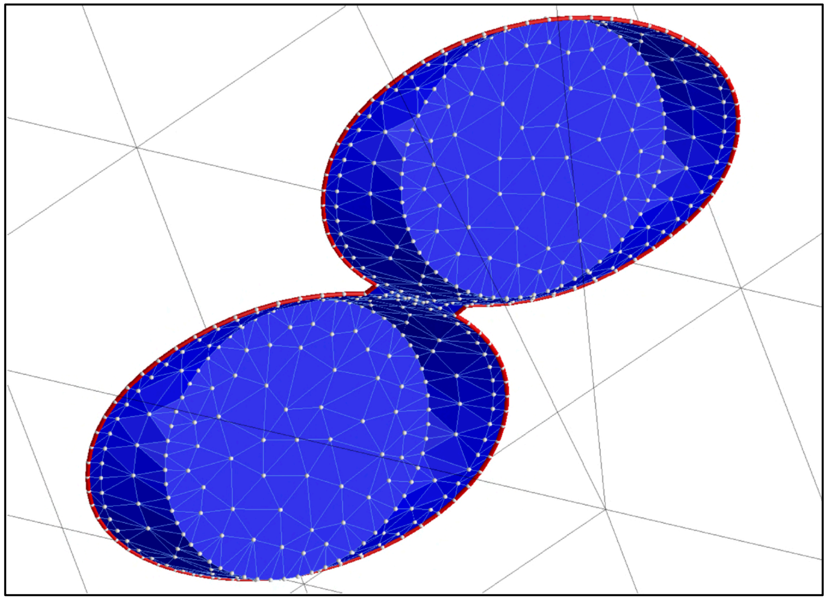
# *3-D Crack Surface Representation*

- High-fidelity explicit representation of crack surfaces [Duarte et al., 2001, 2009]

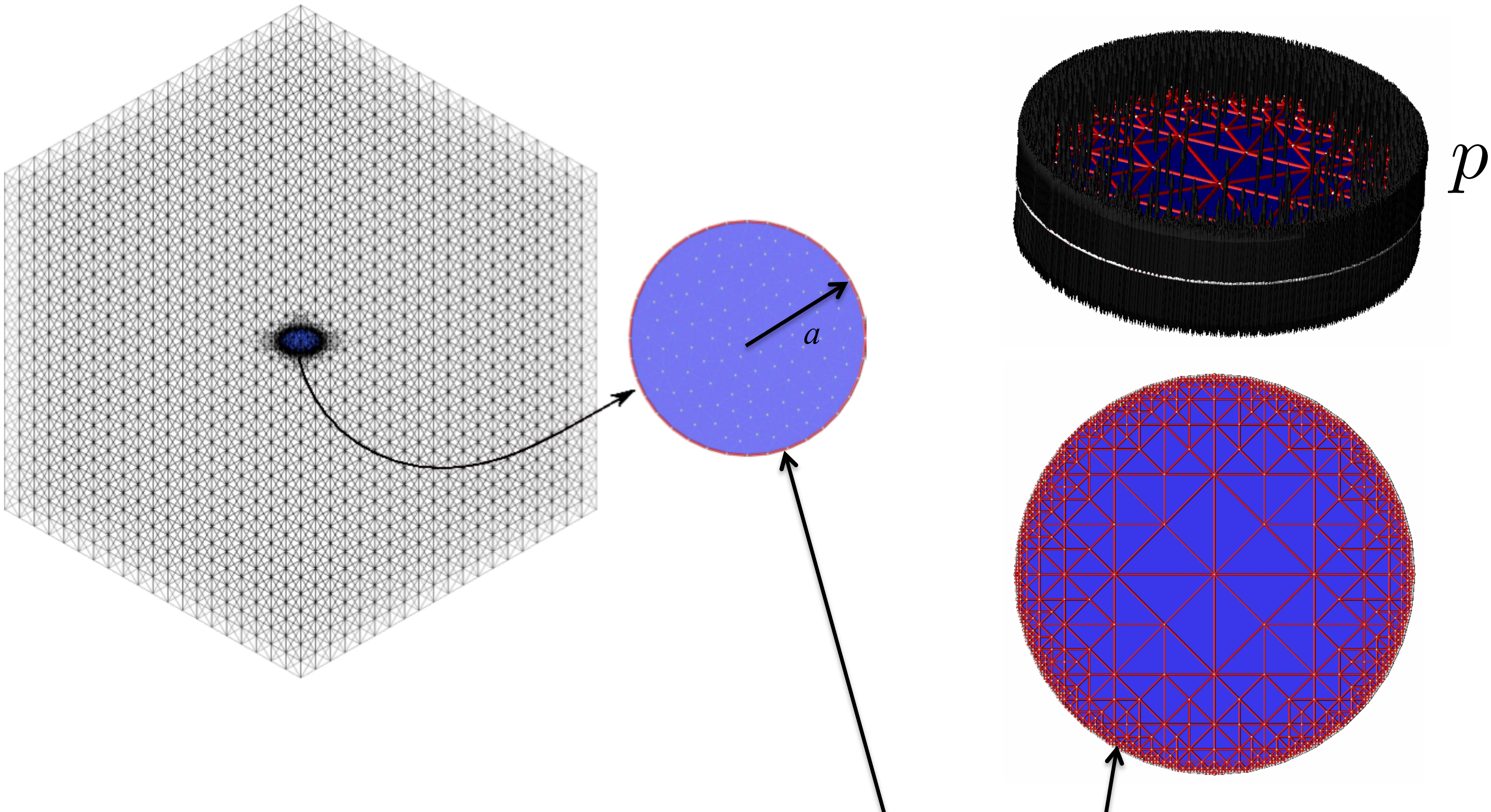


- Coalescence of fractures [Garzon et al., 2013]



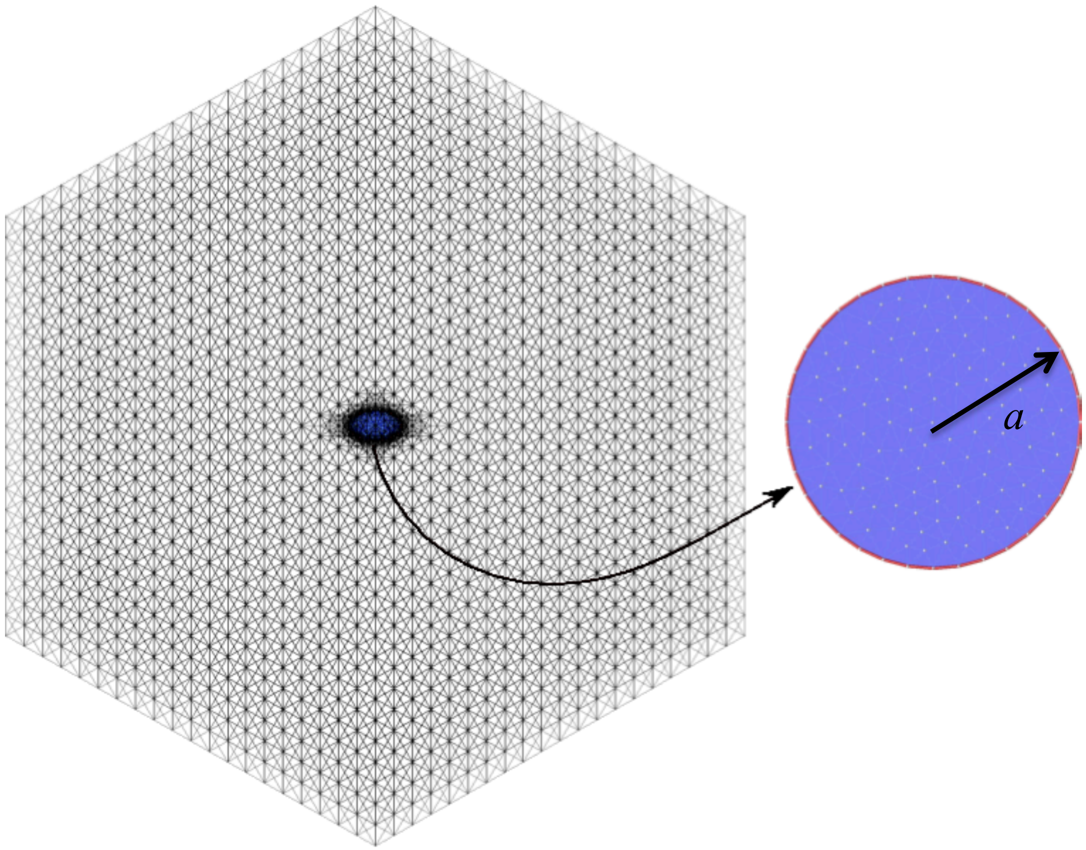


# Verification: Propagation of Penny Shaped Crack




Geometrical and Computational crack surface loaded with fluid pressure *p*

# Verification: Propagation of Penny Shaped Crack




Critical pressure

$$p_c(a) = \left( \frac{E^* G_c \pi}{4a} \right)^{1/2}$$

Adopt [Bourdin et al. 2012]:

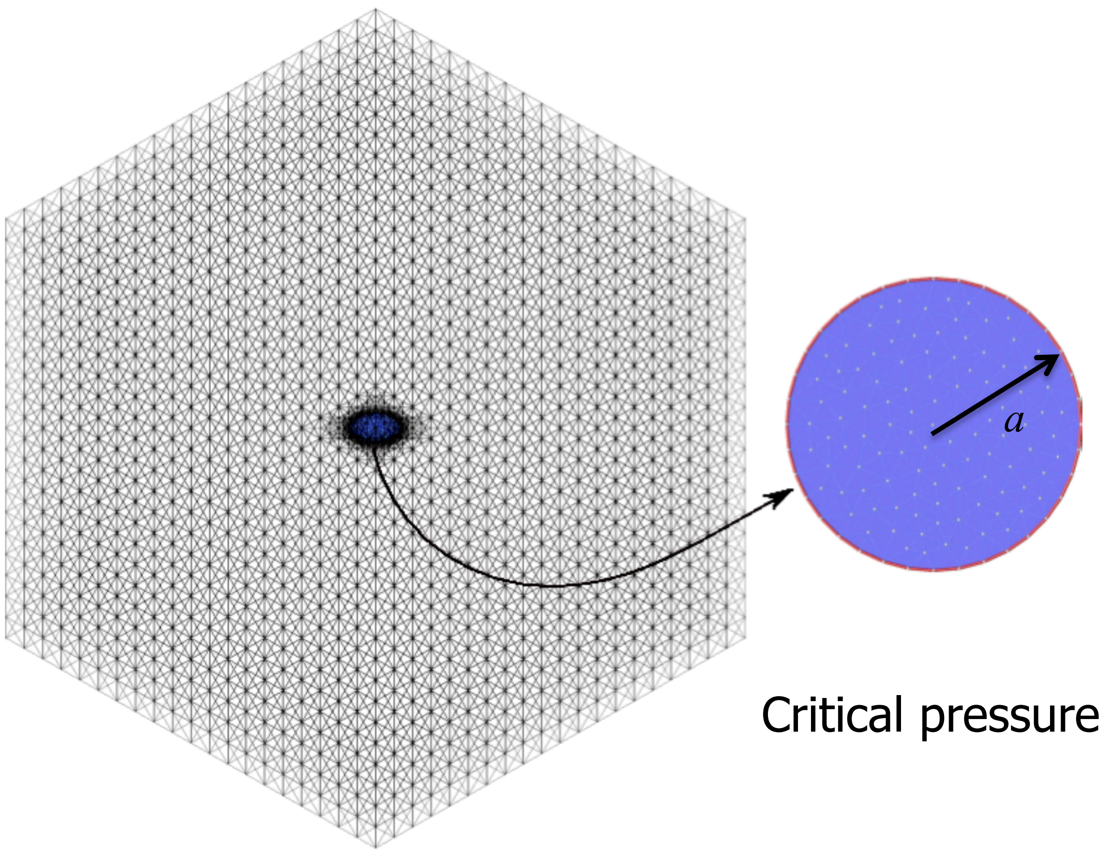
$$E^* = 1$$

$$G_c = 1.91 \times 10^{-9}$$

$$a = 0.5$$

$$p_c(0.5) = 5.477 \times 10^{-5}$$

# Propagation of Penny Shaped Crack




**GFEM Model**

$$h_{\min}/a = 0.016$$

$$h_{\max}/a = 0.027$$

$$p\text{-}order = 2$$

$$N = 215\,376 \ \ dofs$$

$$T = 5.25 \ \ min$$

Critical pressure

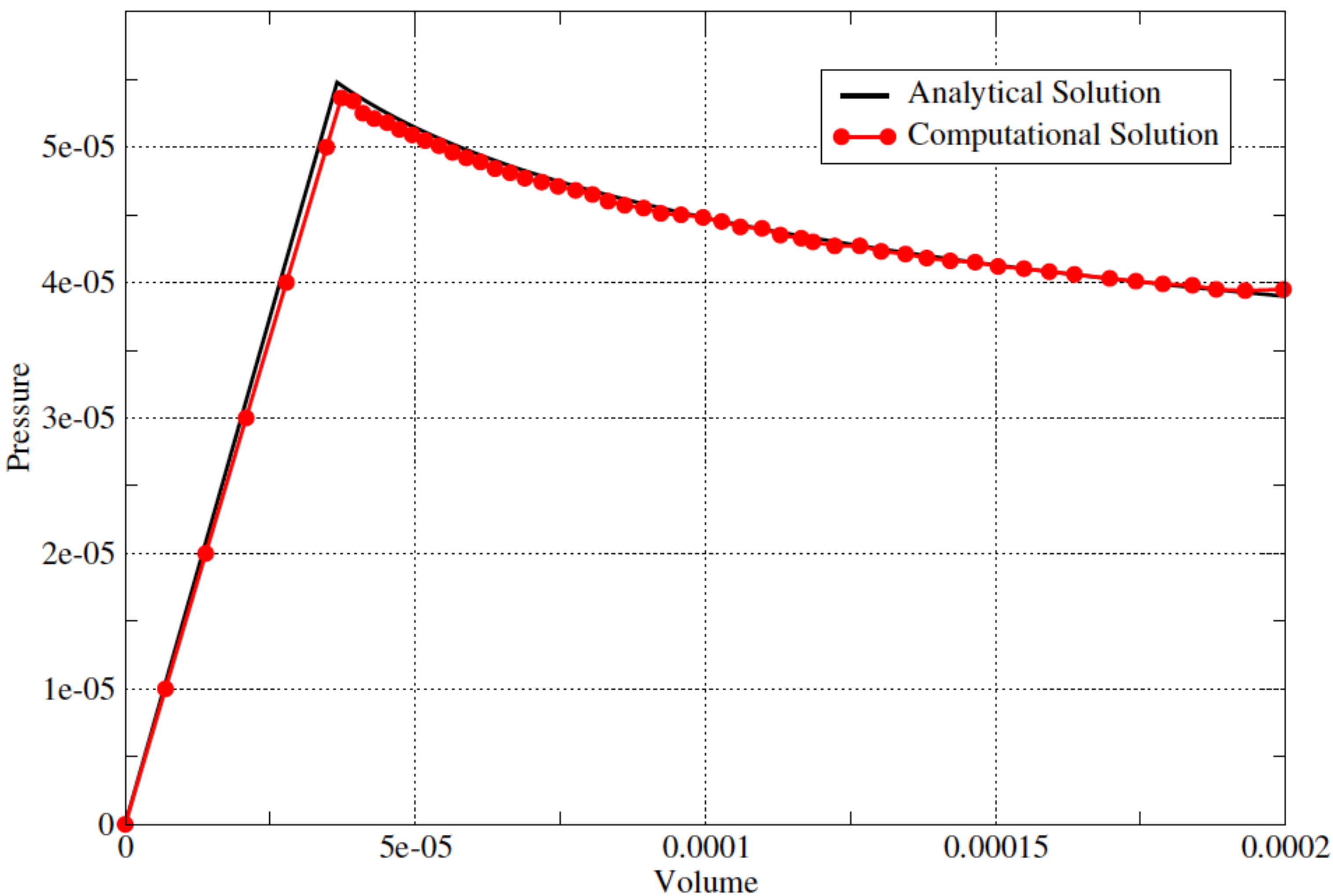
$$p_c^h(a) = \frac{K_c}{K(a)} p$$

$$p_c^h(0.5) = 5.415 \times 10^{-5}$$

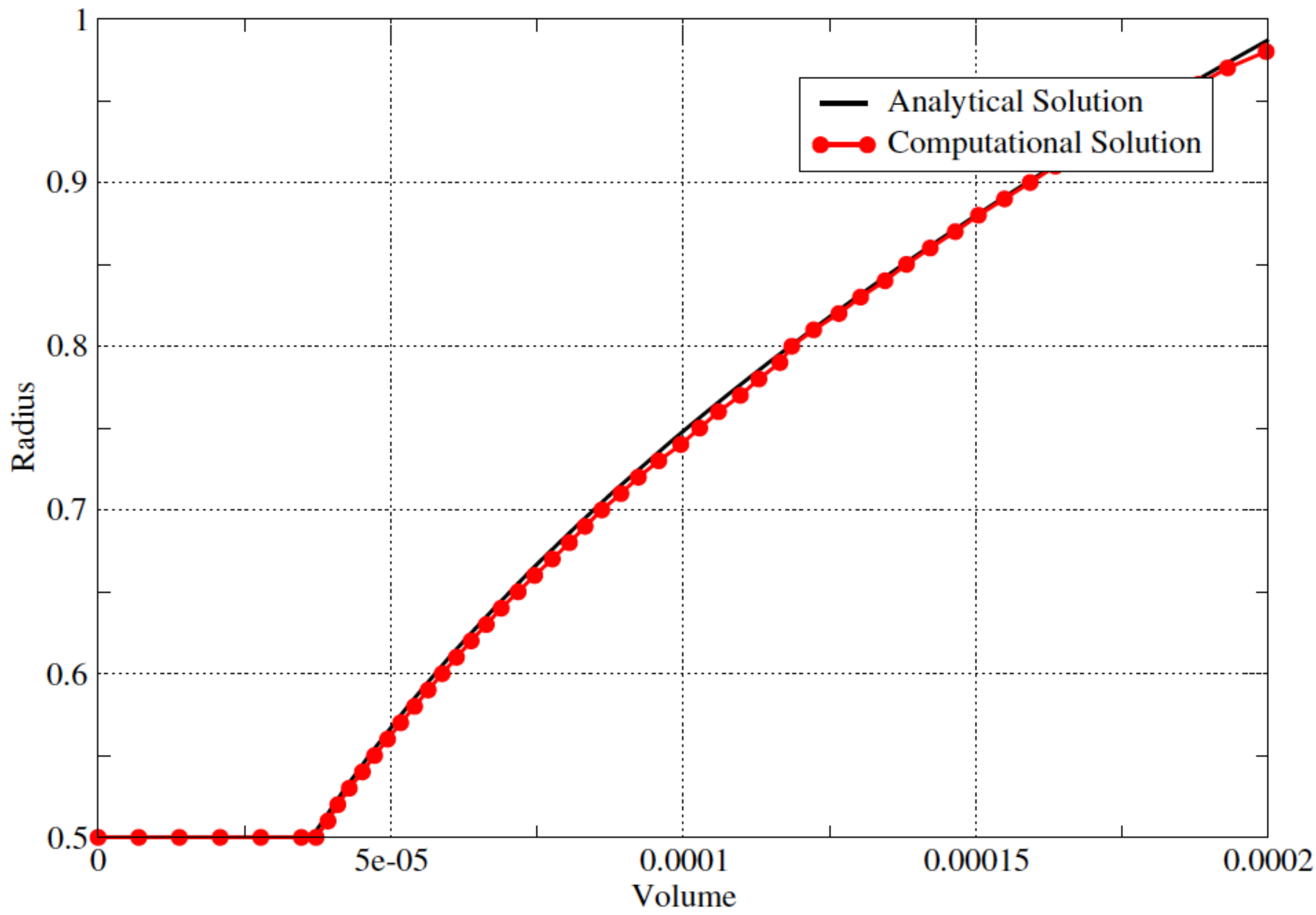
$$e_r(p_c) = 1.15\%$$

# Propagation of Penny Shaped Crack

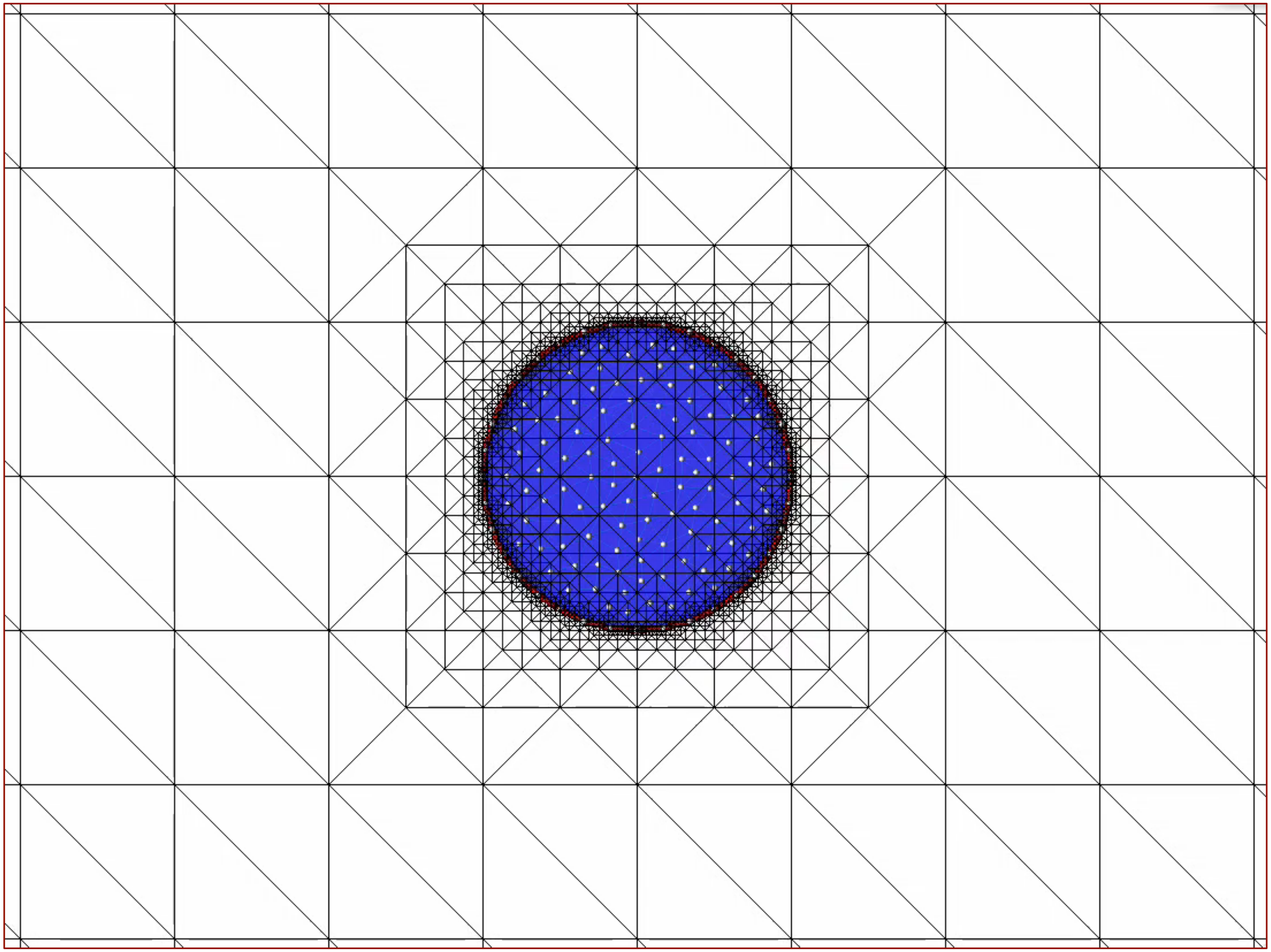
Repeating for each step of crack propagation



# Propagation of Penny Shaped Crack

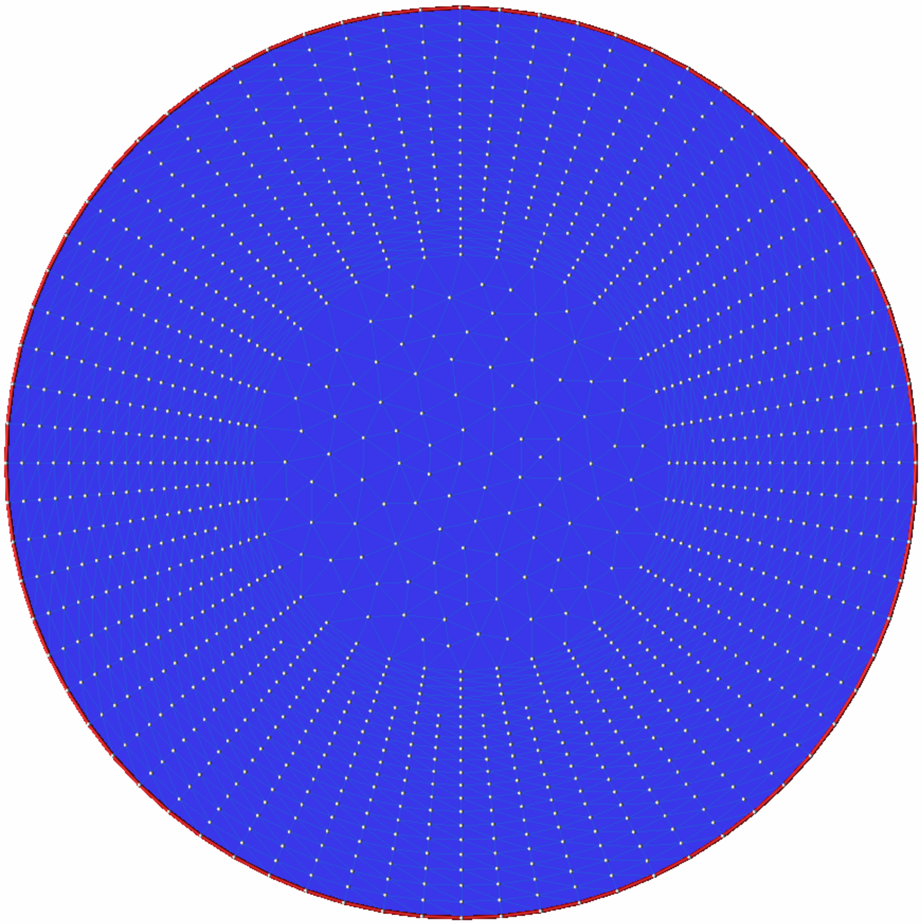


# Adaptive mesh refinement follows crack front

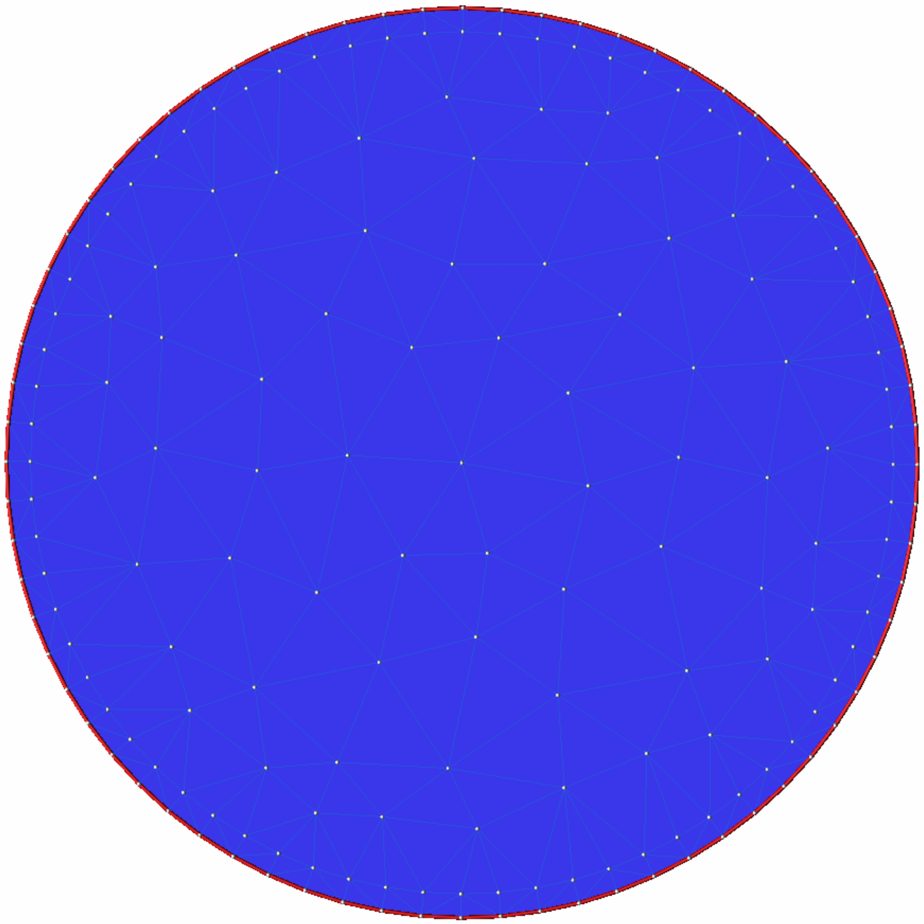


# Propagation of Penny Shaped Crack

Crack surface at step 20

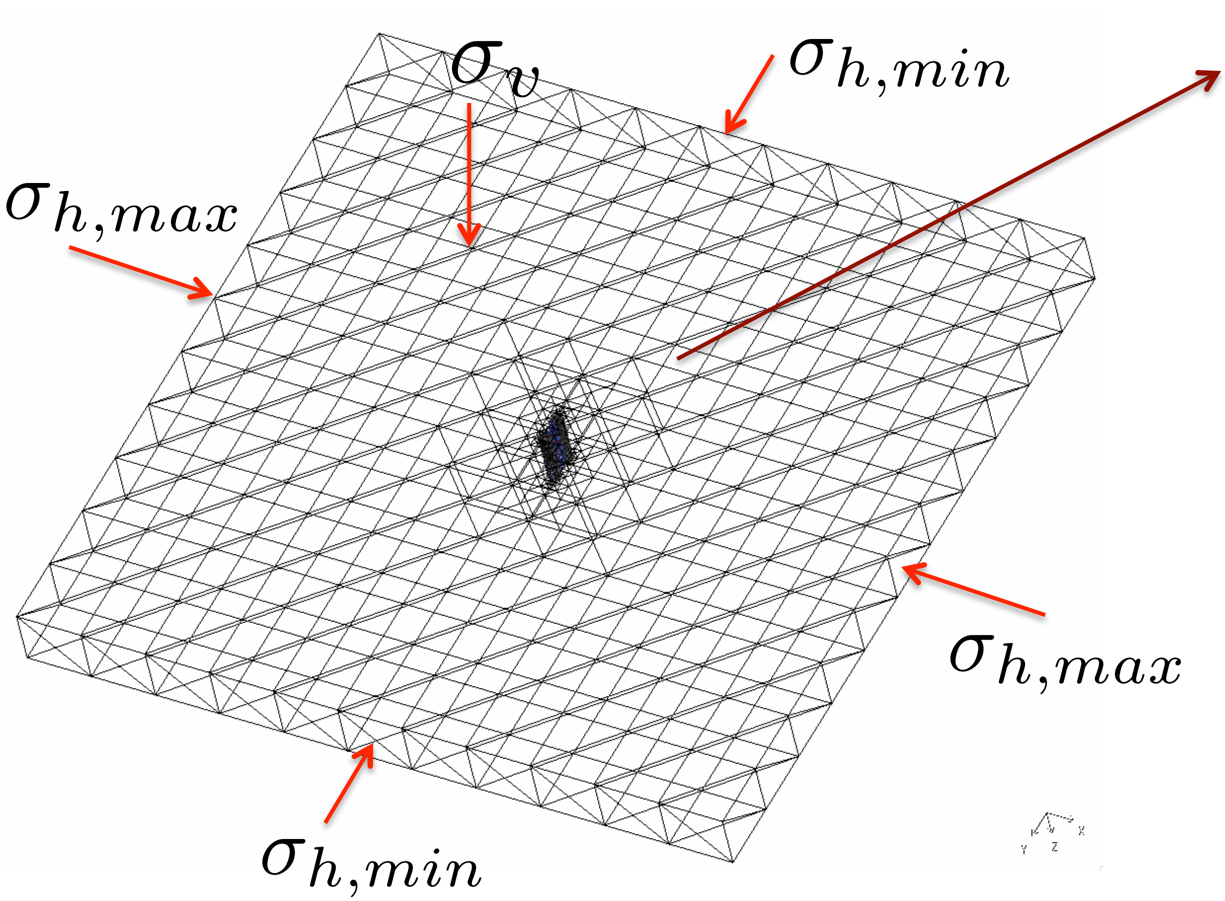


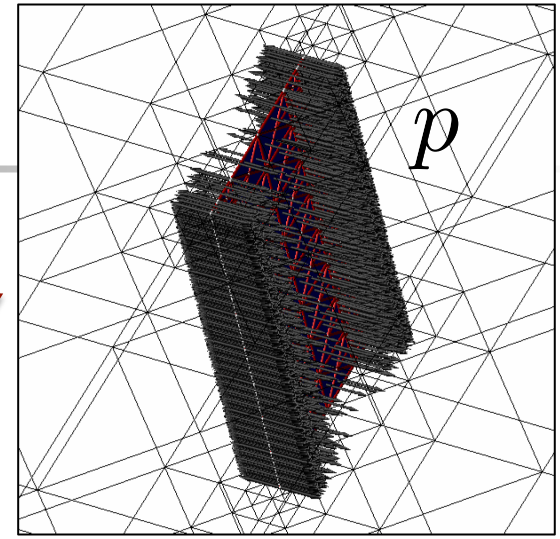
Adaptive update of crack surface

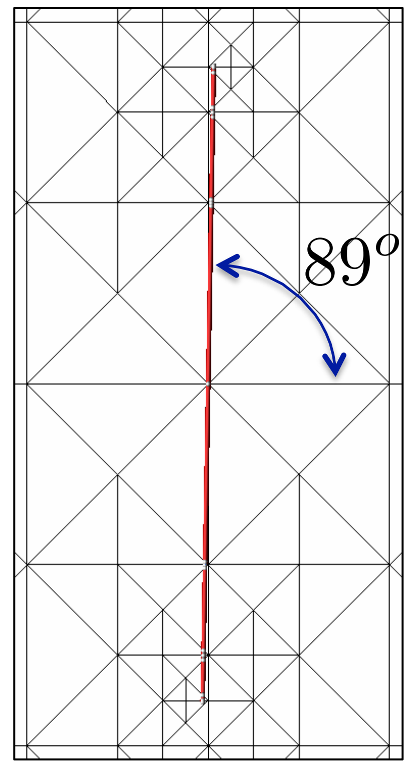


Remeshing of crack surface

# Verification: Crack Reorientation

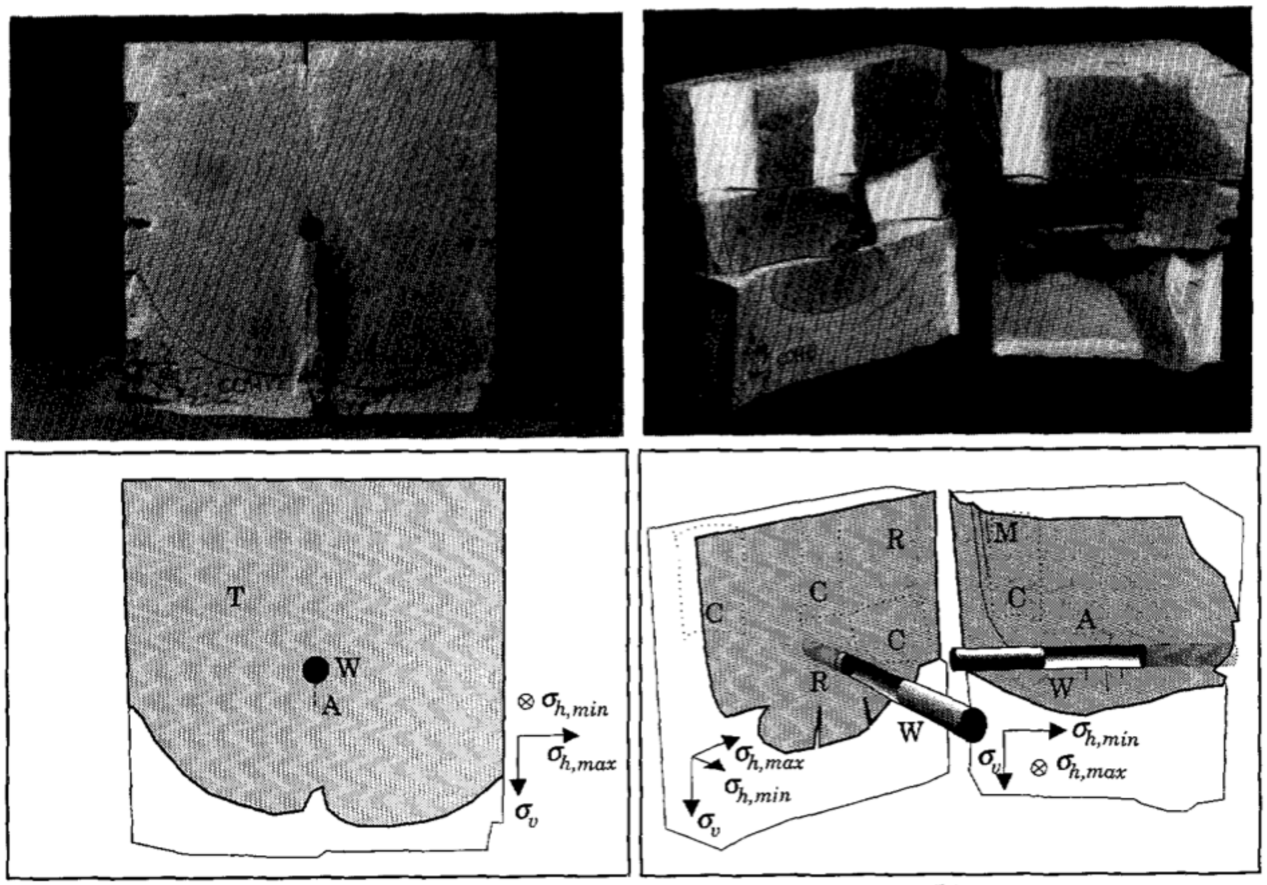


# Crack Reorientation

Same parameters as those used by Weijers [1] in experiment COH13

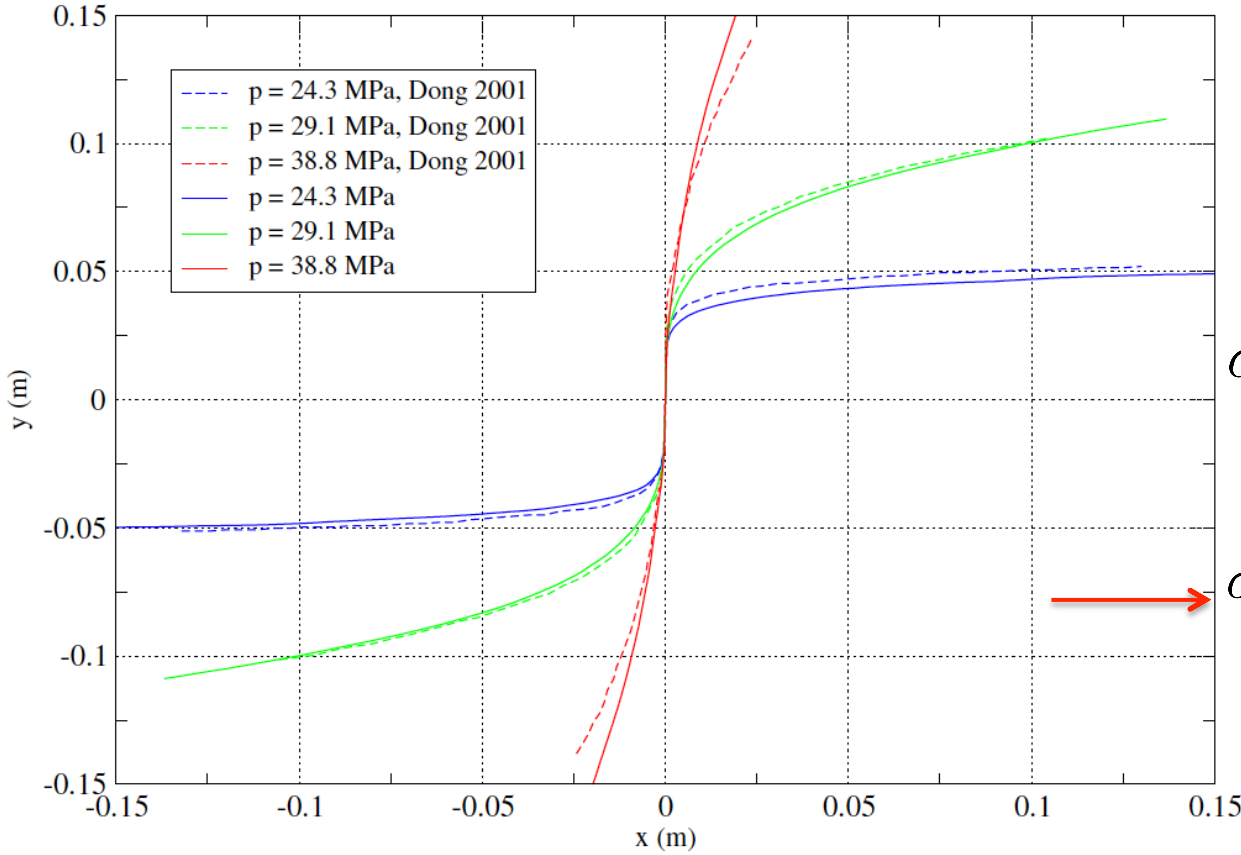


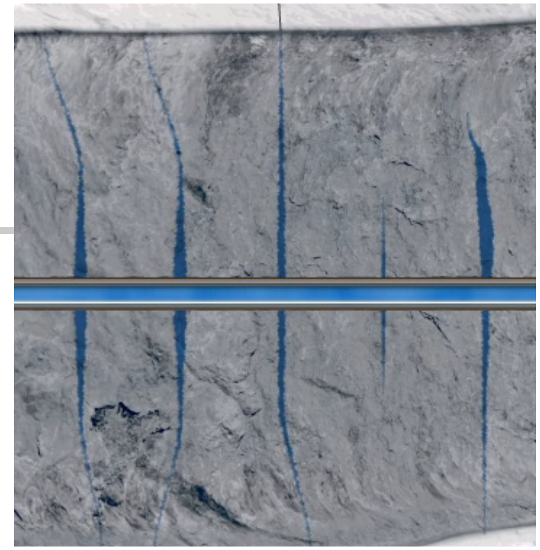

(a) (b)


[1] Weijers L. The near-wellbore geometry of hydraulic fractures initiated from horizontal and deviated wells. Ph.D. Dissertation, Delft University of Technology, 1995.

# Crack Reorientation

Crack paths for different pressures on crack



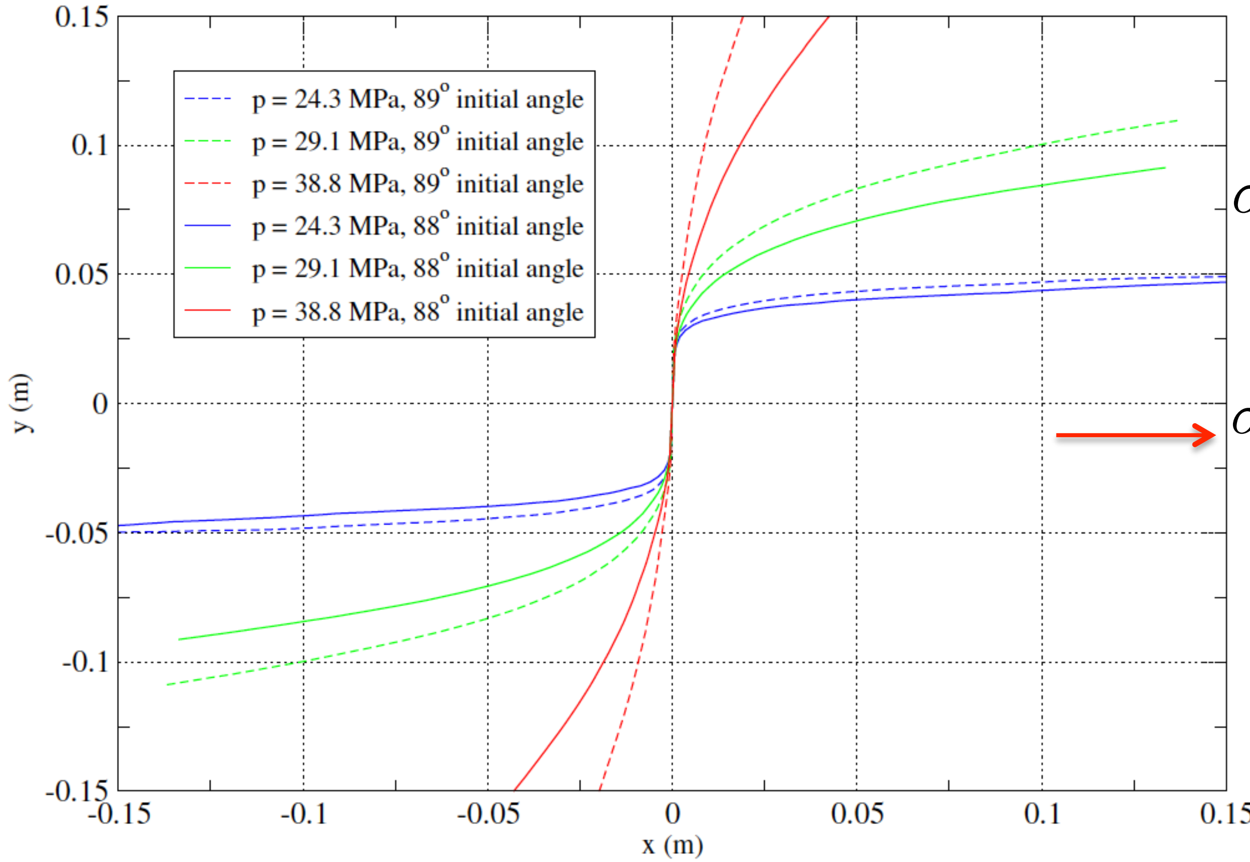
$$\sigma_{h,min} = 9.7\ \text{MPa}$$

$$\sigma_{h,max} = 19.4\ \text{MPa}$$

[2] C.Y. Dong and C.J. de Pater. Numerical implementation of displacement discontinuity method and its application in hydraulic fracturing. *Computer Methods in Applied Mechanics and Engineering*, 191:745–760, 2001.

# Crack Reorientation

Crack paths for different pressures on crack and initial orientation




$$\sigma_{h,min} = 9.7 \text{ MPa}$$

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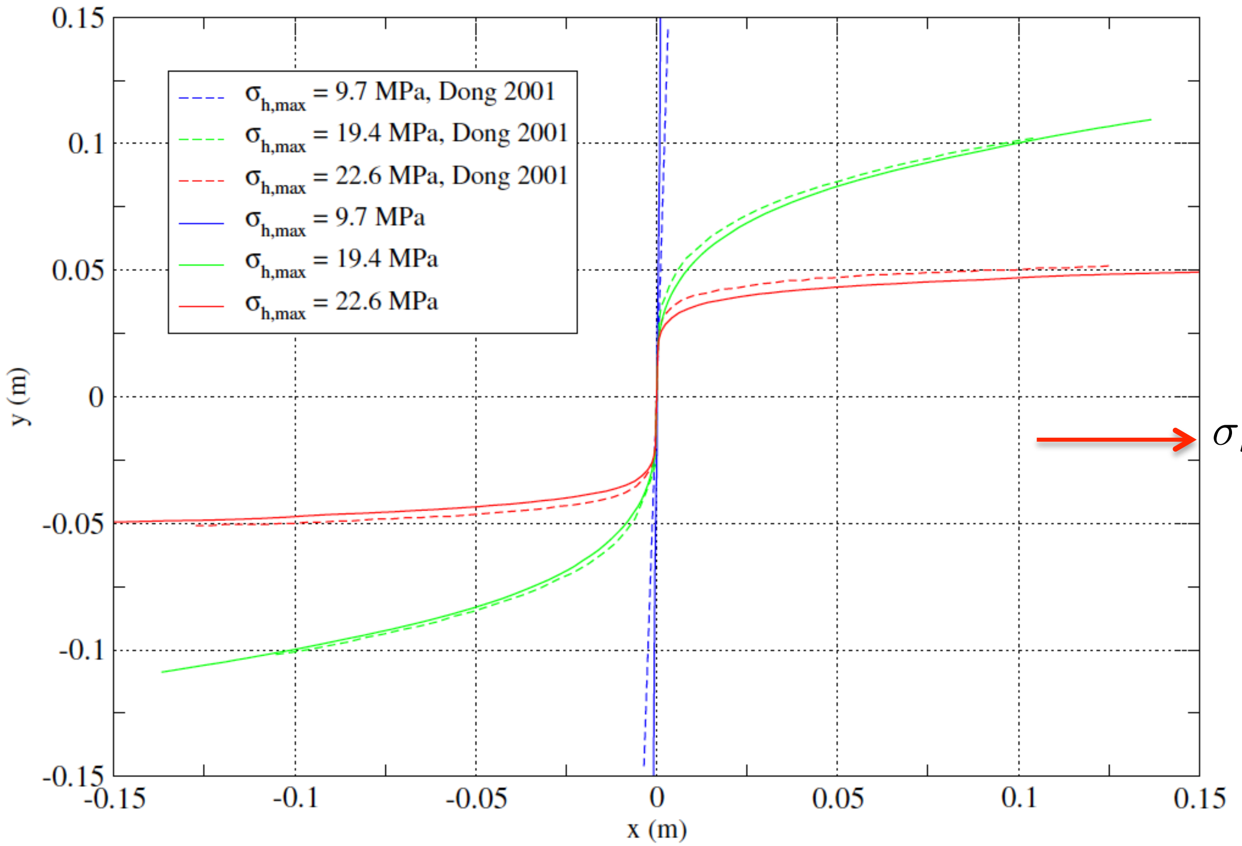
C.Y. Dong and C.J. de Pater. Numerical implementation of displacement discontinuity method and its application in hydraulic fracturing. *Computer Methods in Applied Mechanics and Engineering*, 191:745–760, 2001.

# Crack Reorientation

Crack paths for different $\sigma_{h,max}$

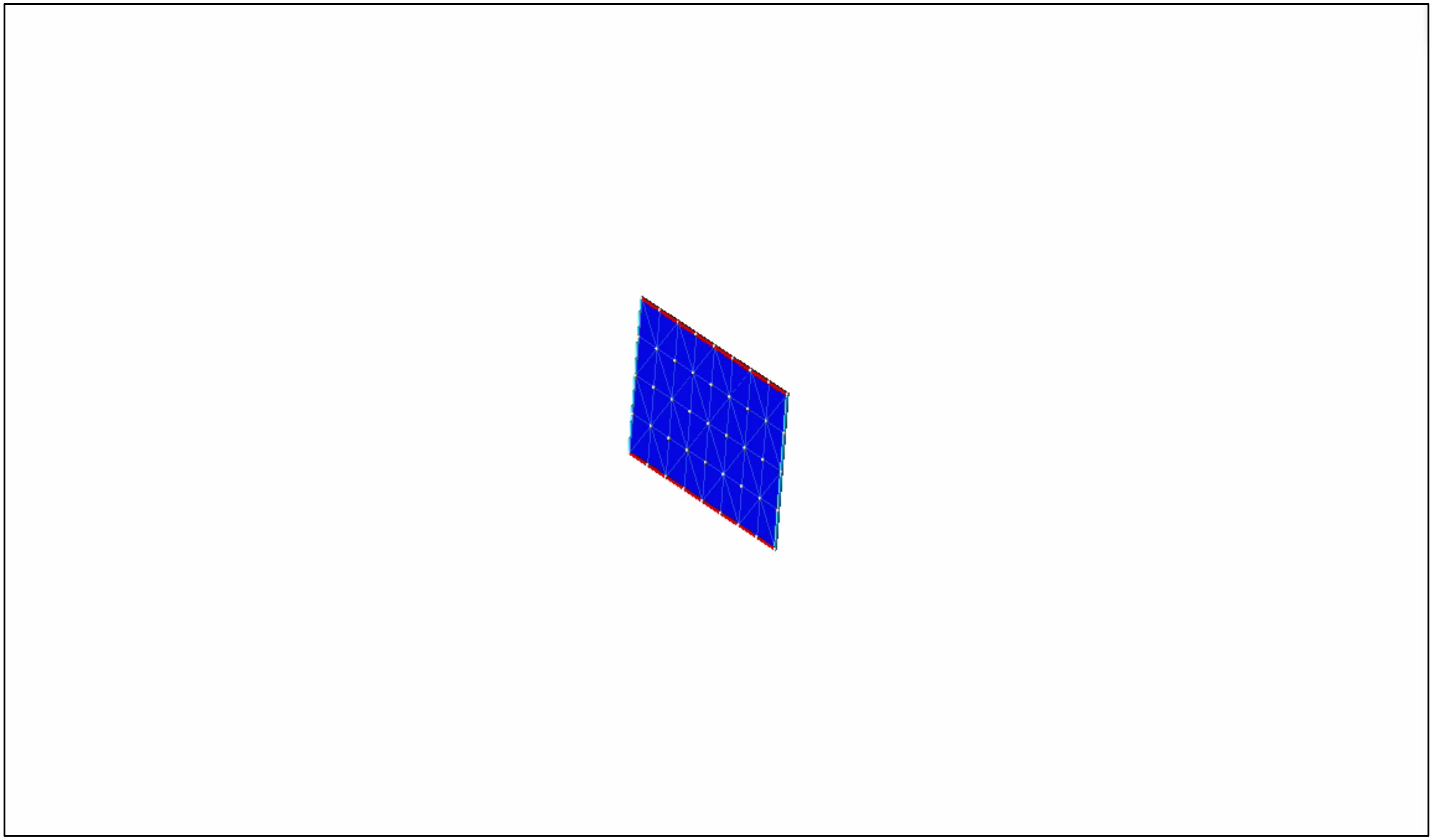



$$\sigma_{h,min} = 9.7 \text{ MPa}$$

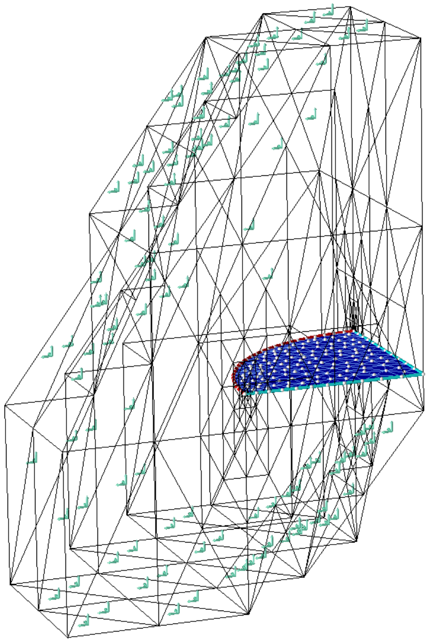
$$\sigma_{h,max} = 9.7 - 22.6 \text{ MPa}$$

$$p = 29.1 \text{ MPa}$$

# Crack Reorientation



$\sigma_{h,min} = 9.7$ MPa $\quad \sigma_{h,max} = 19.4$ MPa $\quad p = 24.3$ MPa



# *Questions?*

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